# GHOST: A time-reversible mixture model for recovering phylogenetic signal from heterotachously-evolved sequence alignments. 

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## Signed Statement

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name, for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

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## Dedication

For Emily and Daniel, may you find happiness wherever life takes you.

## Abstract

The accuracy and reliability of phylogenetic inference is compromised by the adoption of models of sequence evolution that don't adequately reflect the dynamic nature of evolution by natural selection. Heterotachy refers to variation in the rate of evolution of a particular site across lineages on a tree. We carry out simulations, showing that phylogenetic inference using popular methods and models is unreliable when the data evolved under the influence of heterotachy. We carry out a theoretical analysis of these methods and models, concluding that their failure was inevitable given the nature of the data.

To remedy this we introduce the General Heterogeneous evolution On a Single Topology (GHOST) model. We implement the GHOST model under a maximumlikelihood (ML) framework in the phylogenetic inference program IQ-TREE. We perform extensive simulation studies, showing that the GHOST model can successfully recover the tree topology, branch lengths and substitution model parameters from heterotachously-evolved sequences. We apply our model to a real dataset and identify a subtle phylogenetic signal linked to the convergent evolution of the electric organ in two geographically distinct lineages of electric fish. Furthermore, we use the model to successfully identify specific sites in the alignment that are pivotal to the effective function of the electric organ.

The GHOST model and its implementation in IQ-TREE provide the most flexible mixture model currently available for performing phylogenetic inference in a ML framework. This increased flexibility better equips the GHOST model to represent the process of evolution by natural selection. We show that the GHOST model is
able to highlight subtleties in evolutionary relationships that coarser models cannot. We foresee the GHOST model having potential uses in a variety of applications: helping to resolve disputed topologies; focusing the efforts of biologists by identifying alignment sites of functional importance; bringing to light evidence of convergent evolution; and investigating the coevolution that occurs between disease and immune cells, or hosts and parasites. As computing resources continue to grow and phylogenetic algorithms are revised and improved, the GHOST model will be applicable to ever larger MSAs, ultimately assisting in illuminating the history of life on earth.

