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STUDIES IN CROP VARIATION. VI. EXPERIMENTS ON THE RESPONSE OF THE POTATO TO POTASH AND NITROGEN

Author's Note (CMS 18.200a)*

This paper is now of interest principally as showing the gradual development of ideas on experimental design appropriate to agricultural experiments.

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STUDIES IN CROP VARIATION.

VI. EXPERIMENTS ON THE RESPONSE OF THE POTATO TO POTASH AND NITROGEN.

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(With Four Text-figures.)

In a previous paper (1) the authors have described in detail the design, statistical calculations, and advantages of a method of field experimentation which, on its theoretical side, is based upon the analysis of variance (2). The method is capable of expansion and elaboration in several directions, and the purpose of this paper is to put on record three further examples of experiments in which the new technique has been employed.

As before, it has seemed to us best to present the details of this method in the form of an actual description of experiments themselves rather than as abstract examples. Such a procedure has in this case an added advantage since the three examples chosen follow one another logically, and were each the result of a realisation of both the limitations and the advantages of prior attempts. A consideration of these trials will, it is hoped, enable the experimenter to appreciate the advantages of planning his experiments so as not only to embody an agricultural question, but also to ensure the most accurate decision possible.

When the time was opportune for applying in practice some of the advances in experimental method then available, two of the most important investigations being carried out in the field at Rothamsted, were concerned with

(1) the qualitative aspect of potash manuring,
(2) the interaction of potash and nitrogen.

The crop selected was the potato, and it was to these investigations that the new methods were first applied. In the first two years the variety was "Kerr's Pink" and in 1927 "Arran Comrade." The qualitative aspect of the investigation gave rise to a design which may be designated Type I.

Type I.

The problem at hand was to design an experiment capable of distinguishing the differential effects (if any) of potash applied as sulphate, muriate, and low grade salt (*i.e.* a sylvenite containing, in addition to potassium chloride, a high percentage of sodium chloride). There were thus three treatments, to which a fourth was added with no potash, this being in the nature of a safeguarding plot to ensure that an apparent equality in the efficacy of the three forms of potash was not in reality due to non-effectiveness and consequent lack of response.

The necessity of a high standard of accuracy to distinguish between equivalent dressings of various forms of the same nutrient together with the smaller number of comparisons attempted, led to the adoption of the Latin square arrangement of plots. The principal features of the design have been described elsewhere (2) but may be repeated here.

Each Latin square experiment contains as many replications as there are treatments. In each row and in each column of the square, each treatment occurs once and once only. It is in this respect that it is a square, for in actual shape it may vary from a true square to a rather pronounced rectangle. The actual allocation of the position of any treatment within its row or column is, apart from this one restriction, determined by chance. It thus follows that for a 4 by 4 Latin square, for instance there are 576 alternative arrangements. An actual experiment is a random choice from the total possible arrangements. Though not completely randomised with respect to plot arrangement, this design possesses complete randomness with respect to the elements of variation used in testing significance. Fig. 1 shows the actual arrangements employed during the two years of the trial. The details of the treatment are as follows:

Basal manuring: Superphosphate 6 cwt. per acre; sulphate of ammonia 2 cwt. per acre.

Potash in the form of sulphate, muriate or low grade salt: The equivalent of 2 cwt. per acre of sulphate of potash.

1925

M 444	P 422	O 173	S 398
O 279	S 439	M 423	P 409
P 436	M 428	S 445	O 212
S 453	O 237	P 410	M 393

1926

M 584·0	S 557·0	O 461·5	P 498·5
S 519·5	P 485·5	M 477·0	O 389·0
P 474·5	O 378·5	S 467·5	M 491·5
O 464·0	M 511·0	P 507·0	S 492·0

Fig. 1. Arrangement of two Latin squares with yields in lb. per plot.

Applying the analysis of variance to this arrangement it is possible to obtain a variance due to:

(1) Treatment.
(2) Position.
(3) Random variation of parallels.

Table I. *Total yields* (1925).

Rows		Columns		Treatments		
1	1437	1	1612	(O)	No potash	901
2	1550	2	1526	(S)	Sulphate	1735
3	1521	3	1451	(M)	Muriate	1688
4	1493	4	1412	(P)	Potash manure salts	1677

Table II. *Analysis of variance* (1925).

Variance due to	Degrees of freedom	Sums of squares	Mean square
Treatments:			
Potash *v.* no potash	1	119,700	119,700
Potash manures	2	475	237
	— 3	—— 120,175	40,058
Rows	3	1,740	580
Columns	3	5,841	1,947
Parallels	6	1,995	333
Total	15	129,751	—

Table III. *Total yields* (1926).

Rows		Columns		Treatments		
1	2101·0	1	2042·0	(O)	No potash	1693·0
2	1871·0	2	1932·0	(S)	Sulphate	2036·0
3	1812·0	3	1913·0	(M)	Muriate	2063·5
4	1974·0	4	1871·0	(P)	Potash manure salts	1965·5

Table IV. *Analysis of variance* (1926).

Variance due to	Degrees of freedom	Sums of squares	Mean square
Treatments:			
Potash *v.* no potash	1	20,254	20,254
Potash manures	2	1,278	639
	— 3	—— 21,532	7,177
Rows	3	12,055	4,018
Columns	3	3,989	1,330
Parallels	6	2,065	344
Total	15	39,641	—

The positional variance is here of the utmost importance since it can be calculated in two directions, as variance of the rows—*i.e.* variability from top to bottom, and variance of columns (variability from side to side). In this experiment the number of degrees of freedom assigned to each positional factor is three, and since any variation in column totals does not affect the variation in the row totals, the estimates of the variances of the two are independent, and for the purposes of elimination of positional variances are additive. In other words, a considerable amount more of positional variance can be taken out by using this arrangement as a Latin square than by using it merely as four blocks of four treatments. The actual magnitudes of the variances are given in Tables II and IV.

One point may be mentioned here, which, although not demonstrated by these analyses, is sometimes likely to arise. Treated as a Latin square, the data provide six degrees of freedom for the estimation of error. If the variance of either rows or columns had been sacrificed, there would however have been not six but nine available, and the sums of squares corresponding to the rejected three degrees of freedom would have been absorbed in the remainder sums of squares. The question will sometimes arise as to whether the gain effected by eliminating the sums of squares of either rows or columns will counterbalance the loss entailed by having three less degrees of freedom with which to estimate the random error. In both the examples given it will be seen that even though in 1925 the positional variance of the rows, and in 1926 that of the columns, was small, the error is reduced by eliminating them from the error calculations. But, in cases where this is not so, the fact that the Latin square gives a larger error than blocks with only a one directional variance, has sometimes been held to imply a disadvantage. This is not altogether a fair criticism. One of the merits of the method is the recognition that the arrangement of plots has a real effect upon error estimation, and it makes use of that knowledge. If it were possible to say beforehand that in one particular direction, owing to the absence of a large component of soil heterogeneity in that direction, the positional variance was negligible, then it would be advantageous to use the simpler block arrangement. In point of fact, it is seldom, if ever, with annuals, possible to rely on such a contingency, even where a preliminary uniformity trial has been carried out, and the Latin square arrangement is adopted to make sure that the residual variance, on which hangs the precision of the experiment, shall not be inflated from either source. When, in this way, certain elements of error have been eliminated from the field results, the statistician has no choice but to eliminate them in his estimate of error. To include a portion of these because they make his estimate smaller would be to miss the point of making an unbiassed estimate.

These experiments have one defect which in some cases may be hard to overcome. The inclusion of the no-potash plot (for the reasons specified) does in a year of pronounced response to potash contribute very largely to the treatment variance. In applying tests of significance, particularly what is known as the z test, significance may be claimed for treatment as a whole which is really due entirely to the one degree of freedom potash versus no potash. A further analysis of the variance due to treatment by separating this component, as in Tables II and IV, will, of course, settle the point, but the need of this precaution, and the possibility of carrying it out, requires, at the present time, some emphasis.

The benefit of the elimination of the disturbing element of soil heterogeneity is clearly seen in Tables II and IV, if the positional variance is amalgamated with the random variance as it would be if the old methods of designing field experiments were followed. Table V shows this advantage in terms of the standard error per cent., and of the precision figure based thereon.

Table V. *Advantage of eliminating a portion of the soil heterogeneity.*

	Soil variation eliminated	With soil variation	Soil variation eliminated	With soil variation
Standard error %	2·4	3·8	1·9	4·0
Precision	17	7	27	6

In 1925 soil variation would have increased the error by more than 50 per cent. and in 1926 by more than 100 per cent. The influence on the accuracy of an experiment which such an increase of error entails is shown by the figure for precision. This value is arrived at as follows. On a precision scale a 10 per cent. error, the approximate error of a single plot with many crops, is assigned the value 1 and a 1 per cent. error then has a precision value 100, this latter being the value at which with our present resources it is reasonable to aim. The precision index I will then be

$$I^{-1} = 100\left(\frac{\sigma}{m}\right)^2,$$

where σ is the standard deviation of the mean yield of each treatment, and m is the mean yield of all.

TYPE II.

The Latin square form of experiment answered admirably for the foregoing qualitative distinctions between potash manures, but was unsuited to the investigation into the quantitative relationships between potash and nitrogen.

To serve any useful purpose, this had to include several increments of potash with corresponding increments of nitrogen combined in as many ways as the size of the experiment would allow. To have carried out an experiment involving so large a number of treatments in the form of a Latin square, would have been very wasteful of space and effort. Past a certain point with the Latin square the increase in replication does not bring about a decrease in error commensurate with the labour involved. There are indications that comparisons of more than seven treatments or varieties can be made more precisely with other arrangements. Accordingly, when in 1925 twelve treatments were contemplated, positional variance was eliminated by assigning each treatment to each of four similar blocks, the arrangement of which was substantially the same as that of the top dressing series previously referred to (1). Within the blocks, however, the arrangement of the plots was not a random one.

M 491	N 328	C 340	R 508	D 388	A 322	Block I
L 437	J 217	Q 487	P 464	T 272	S 516	
P 450	S 464	R 461	C 320	N 298	M 482	Block II
T 252	D 352	A 281	L 438	Q 515	J 315	
C 341	P 439	S 456	M 466	J 247	N 344	Block III
T 226	L 393	D 338	R 519	A 198	Q 501	
M 449	A 191	N 185	P 472	D 342	T 234	Block IV
Q 461	R 475	J 157	L 377	C 298	S 441	

Fig. 2. Quantitative experiment of 1925, yields in lb. per plot.

The actual arrangement of the blocks as shown in Fig. 2 was determined by the knowledge that there is a high correlation between adjacent plots.

In 1926 a similar experiment was carried out (Fig. 3). The actual treatments involved in these trials are shown in the following plan where the letters indicate the treatments employed.

1925

Sulphate of ammonia, cwt.	Sulphate of potash, cwt. 0	2	4	6
0	A	C	D	—
2	J	L	M	—
4	N	P	Q	R
6	—	—	—	S

1926

Sulphate of ammonia, cwt.	Sulphate of potash, cwt. 0	1	2	4
0	A	B	C	D
1	E	F	G	H
2	J	K	L	M
4	N	O	P	Q

N 332·0	J 302·5	F 383·0	A 317·5	D 439·0	O 533·5	K 544·5	A 404·5
K 444·5	Q 568·0	O 450·0	D 381·5	L 483·5	B 308·0	F 434·0	N 468·0
B 363·0	C 368·0	M 449·0	L 471·5	H 422·0	P 500·0	G 402·0	E 318·0
H 447·5	E 314·0	P 527·0	G 434·5	M 504·0	Q 561·5	C 356·0	J 456·0
A 351·5	L 495·5	J 443·0	C 383·5	P 559·0	Q 550·0	B 359·0	E 395·5
K 472·5	B 367·5	G 455·5	O 502·5	C 328·5	H 390·5	J 483·0	O 512·0
E 357·5	F 381·5	Q 531·0	D 316·0	N 522·0	M 444·0	A 325·0	D 259·0
N 385·5	H 354·0	P 496·5	M 474·5	F 410·5	G 351·5	K 430·0	L 394·5

Fig. 3. Quantitative experiment of 1926, yields in lb. per plot.

It was realised that the 1925 experiment was inadequately designed with respect to the treatments included, and the distribution of plots within the blocks, consequently in 1926, by using every possible combination and a strictly random arrangement, the experiment was greatly improved. The analyses of variance are set out in Tables VI and VII and the plan of the arrangements in randomised blocks in Figs. 2 and 3.

Table VI. *Analysis of variance*, 1925.

Variance due to	Degrees of freedom	Sums of squares	Mean square
Treatment	11	464,251	42,205
Blocks	3	22,030	7,343
Parallels	33	34,285	1,039
Total	47	520,566	—

Table VII. *Analysis of variance*, 1926.

Variance due to	Degrees of freedom	Sums of squares	Mean square
Treatment	15	261,497	17,433
Position	3	11,303	3,768
Parallels	45	97,361	2,164
Total	63	370,161	—

The 1926 results call for some explanation. For an experiment in which care has been taken to reduce to a minimum disturbing factors contributing to error, the errors are disconcertingly high. This can be traced to the very small amount of positional variance which has been eliminated. The variance due to position is largely caused by soil heterogeneity, as is also the random variance. The difference between the two lies in the fact that the former is due to systematic changes in fertility affecting whole blocks (inter-block variance), whilst the latter is sporadic in its incidence (intra-block variance). So long as the size of the block is such that the changes of fertility which must occur even in one block are systematic, the variation will be reflected in a large positional variance which is all to the good. If, however, the blocks get so large that within the blocks there is local heterogeneity which is not systematic in incidence, such heterogeneity will increase the remainder or random variance. The question as to how much soil heterogeneity variance makes its appearance in the one or the other sections into which the analysis of variance is divided, depends entirely upon the inter-relation of plot size with block size and the type of soil heterogeneity encountered.

In the present instance it would appear that as only some 10 per cent. of the sum of squares contributable by soil fertility variation is assignable to systematic changes, the blocks have been too large to fulfil their function. Greater replication of smaller blocks would have improved the experiment.

It will be noticed that every experiment of this type really constitutes a sort of uniformity trial in addition to answering the normal agricultural purpose. From a number of experiments carried out on one field over a variety of seasons, a very much fuller knowledge of the behaviour of the field is obtained than could be gained from a similar series of the older type.

Type III.

The failure to realise the standard of accuracy desired in the 1926 experiment led to further discussion of experimental design and the evolution of a further elaboration. The simple expedient mentioned above of increasing the replication in order to ensure greater accuracy, was not possible on the potato crop. To have done so would have brought the number of plots in potatoes above the number which could be successfully harvested in the interval between maturity and the onset of bad weather. More plots would have rendered lifting either impossible or at any rate unsatisfactory from the experimental point of view. Any improvement to be effected had to be accomplished without a large increase in plot number, because of this very practical and relevant restriction. The difficulty was overcome by amalgamating the qualitative and quantitative trials. In 1926, these two totalled 80 plots, a Latin square of 16, and 4 blocks of 16. In 1927, the two investigations were combined in an experiment of 81 plots. In order to do this the quantitative side had to be cut down to three increments of nitrogen and potash, but as will appear later there was a marked improvement of accuracy and a much greater fund of information available from the new design.

4 0 333·5	2 0 379·5	2 P4 382·5	2 0 379·0	4 P2 381·0	0 0 382·0	2 0 380·5	0 P4 335·5	2 M2 389·0
0 M4 308·5	2 S2 421·0	4 M2 430·5	4 P4 396·0	4 0 413·5	2 S4 424·5	2 M4 409·5	4 S4 436·0	0 0 348·5
4 S4 403·0	0 0 356·5	0 P2 365·0	0 S2 401·0	2 M2 420·0	0 M4 364·0	4 0 399·0	4 S2 408·0	0 P2 354·0
2 P2 404·5	0 S2 357·0	4 0 412·5	2 P2 408·5	4 S2 438·5	4 0 428·0	4 S4 412·0	2 S2 411·0	0 M2 361·0
4 M2 440·0	2 P4 323·5	0 S4 362·5	2 P4 403·5	2 0 409·5	0 M2 360·5	0 P4 319·0	2 0 402·5	2 M4 369·5
4 M4 436·5	2 0 394·5	0 0 395·0	4 M4 465·5	0 0 366·5	0 S4 395·5	0 0 349·5	4 0 400·5	4 P2 358·5
0 0 337·5	0 M2 345·0	4 0 440·0	2 0 446·5	2 M4 455·0	4 P4 405·5	0 P4 333·0	2 S4 405·0	4 M2 390·5
0 M4 302·0	2 0 377·0	2 S2 467·5	4 S2 473·0	0 P2 395·5	4 0 411·5	0 0 351·5	0 S2 344·0	4 0 369·0
4 P4 356·5	4 P2 388·0	2 S4 463·5	2 M2 474·0	0 0 411·5	0 S4 401·5	2 P2 400·5	2 0 389·5	4 M4 436·0

Fig. 4. Qualitative and quantitative experiment of 1927.

The two numbers in the upper line represent the quantities of nitrogenous and potassic manures, the kind of the latter used being indicated by S for potassium sulphate in cwt. per acre, M for potassium chloride containing equal amount of potassium, and P for the equivalent low grade salt. The lower numbers represent the yield in lb. of a plot of one-fortieth of an acre.

The quantities of potash and nitrogen are shown below:

Sulphate of ammonia, cwt.	Equivalents of sulphate of potash, cwt.		
	0	2	4
0	0 0	0 2	0 4
2	2 0	2 2	2 4
4	4 0	4 2	4 4

and the arrangement in Fig. 4 where S, M, P indicate the source of the potash applied.

These nine treatments constituted the block, and of such blocks there were nine in all. The potash had however to be divided out amongst the three kinds, sulphate, muriate and low grade; there being three plots receiving double and three single potash, one of each in each block were allotted to each kind. The manner of allotting these qualitative differences amongst the varying quantities of nitrogen requires detailed description. The actual position of a plot considered only as representing potash and nitrogen interactions was determined entirely by chance. The element of chance also operated largely in the disposition of the qualitative factor, but there was one restriction. The restriction provided that any particular variety of potash manure should occur in the total of the nine blocks in conjunction with every amount of nitrogen three times. In every other way the distribution was at random.

The amount of replication in this experiment varies with each factor or interaction of factors concerned. The number of independent comparisons which can be made is thus summarised:

	Number of comparisons
(1) Action of potash in varying quantites in combination with a standard quantity of nitrogen	27
(2) Action of nitrogen in varying quantites with standard potash (quantitative)	27
(3) Interaction of nitrogen and potash in every combination	9
(4) Between kinds of potash	18
(5) Differential response of kind of potash to quantity of potash	9
(6) Differential response of kind of potash to quantity of nitrogen	9
(7) Differential response of kind of potash to quantity of potash and nitrogen varying simultaneously	3
(8) Elimination of soil heterogeneity	9

The experiments of 1925 and 1926 gave information on sections 1–4, sections 5–7 are additional information and the accuracy of the comparison between kinds of potash is enormously enhanced, there being now 18 comparisons in place of 4.

The advantage of this type of survey experiment is, as has been pointed out, very great. For each comparison an appropriate error is obtained with respect to which interpretation can be made. Consideration of the mean yields in conjunction with their appropriate errors shows how greatly improved is the standard of accuracy of the qualitative side of the trial. In 1925, although the means showed the apparent order of efficiency to be sulphate, muriate, low grade salts, even by taking the maximum difference sulphate versus low grade salts (a not entirely fair method), the differences were only probably significant and not completely so, and a similar state of affairs is seen in 1926 with respect to the greatest difference, muriate versus low grade salts.

In the 1927 trial, a summary of which is shown in Table VIII, a much closer control is established and for double dressings the depression of the low grade salts is significant. The depressing effect of muriate rests as a probability and the results show that the effect is felt least where the dressings of nitrogen are high—*i.e.* where the manurial effect of potash would be more apparent. The sulphate appears to function normally at all values of nitrogen.

Table VIII. *Analysis of variance*, 1927.

Variance due to	Degrees of freedom	Sums of squares	Mean square
Potash and nitrogen	8	49,905	6,238
Quality of potash	2	14,458	7,229
Quantity *v.* quality of potash	2	1,005	503
Blocks	8	21,442	2,680
Error	60	33,919	565
Total	80	120,729	—

Average yield in tons per acre.

Potash, cwt.	Nitrogen, cwt.			Potash, kind		
	0	2	4	S	M	P
0	6·545	7·061	7·158	6·921	6·921	6·921
2	6·514	7·532	7·357	7·383	7·164	6·858
4	6·193	7·215	7·435	7·348	7·037	6·458

Standard error 0·141 ton.

The analysis of variance shows that differences of decided significance have been obtained both on the quantitative and qualitative questions. The table of average yields shows (i) the responses to increasing dressings of nitrogen, not very large in absolute amount, but capable of fair quantitative estimation in an experiment of the precision actually attained, (ii) a decided response to the first dose of potash, but much less, if any, to the second dose, (iii) that the second dose, when all three kinds of potassic manure are considered together, is deleterious in the absence of nitrogen, but probably becomes beneficial when the total yield is stimulated by heavy nitrogenous dressings.

The table showing the three kinds of potash separately is of special interest in providing unequivocal confirmation of the conclusions indicated, but without sufficient statistical significance, by the earlier experiments; all levels of nitrogenous manuring are here thrown together. With sulphate we have a decided increase from the first dose, and no appreciable decrease due to the second dose. With muriate the yield with double potash is about midway between those obtained with none and with single potash; while with a source of potash which contains much additional sodium chloride, the first dose has on the average no appreciable effect, while the second dose produces a decided loss of yield.

If we contrast the yields at the same level of abundance of potash, we find sulphate beating muriate by 0·22 ton at the single level, and by 0·31 ton at the double level; while it beats the potash manure salts by 0·525 at the single level, and by 0·890 at the double level, the difference being two and one-half to three times as great. It is clear that we must interpret these results, not as due to any difference in availability of the potash, but as due to other effects, presumably the presence of chloride, which effect a quantitative depression of the yield nearly proportional to the quantity of chloride present. The use of no-potash plots designed to show that the crop is really ready to respond to available potash, while essential if availability is in question, are quite superfluous for the examination of effects of this kind, which are most clearly seen with the second dose of potash, to which there is in the present experiment no appreciable response.

Summary.

The development is recorded of the series of experiments with potatoes at Rothamsted during 1925–27, designed to examine the quantitative response of yield to varying quantities of nitrogenous and potassic manures, and to test the relative value with this crop of different sources of potash.

While rather precise comparisons were obtained on the qualitative question by means of Latin squares in 1925–26, the reality of the depression ascribable to chloride could not be demonstrated in these years, but became clearly apparent when in the following year, the qualitative experiment was merged with the quantitative one.

In the earlier quantitative experiments, although satisfactory responses were obtained, the precision of the results left much to be

desired, since only four replicates could be used. When by merging the experiments this was increased to nine replicates, much smaller responses were clearly measurable.

The large and complex type of experiment finally adopted thus supplied more precise information on both heads than could previously be obtained, and in addition to a more thorough exploration of the different combinations possible.

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