# A Residual Strength Approach for the Fatigue Analysis of Welded Components

**ABIR GHOSH**

B.E.(Hons), Jalpaiguri Government Engineering College, INDIA

Thesis submitted for the degree of **Doctor of Philosophy**

in

The University of Adelaide
(Faculty of Engineering)

October, 1996

AUSTRALIA

*Dedicated to*
*My Grandmother*
*Mrs. Sureshwari Ghosh*

# ABSTRACT

At present welded components subjected to fatigue loadings are designed and assessed using curves of stress-range against number of cycles as well as Palmgren-Miner's summation. These techniques concentrate on the endurance of the component and do not take into account the variation of the strength of the component during the fatigue life. Thus present design techniques assume that the strength of the component remains equal to the static strength throughout the fatigue life, and the variation in strength is independent of fatigue loading until the component fails at the end of the fatigue life. Present assessment techniques give the life of a component that has been expended as a portion of the total life and do not give any idea of the strength or crack length of the component.

A fundamental procedure of design and assessment has been developed in this thesis which takes into account the variation of the residual strength during the fatigue life of the component. A simple and easy method using linear curves has been determined to find the reduction in strength from fracture mechanics. Hence a hand method of assessment and design has been developed which can easily be used by practising engineers. Using the new technique the residual strength, the crack length and the remaining life of a component can be found. Furthermore, the procedure can be used to design components based on the residual strength variation and hence is more accurate and versatile compared to present methods where design is based on endurance.

# ORIGINALITY STATEMENT

This work contains no other material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I consent to this copy of my thesis, when deposited in the University Library, being available for loan or photocopying.

ABIR GHOSH

October, 1996

# ACKNOWLEDGMENTS

The author wishes to thank his supervisor Dr. D.J.Oehlers for his valuable advice and assistance during the project. His suggestions and corrections during preparation of the thesis are gratefully acknowledged. The author wishes to thank his Co-supervisor Dr. M.A. Wahab for his suggestions during the project, his support during the programme, for going through the thesis and giving valuable ideas.

The author wishes to thank the secretarial staff, the computing officer and laboratory staff for their help during his stay in the Department of Civil and Environmental Engineering. The author is indebted to all other academic members and fellow post graduate students of the department for their assistance and support. The author especially wishes to acknowledge the support provided by students John Kenyon, Gregg Klopp, Derek Henekar, Anton Bergant, Wade Stevens, Weimin Luo and Wen-gang Hua during the initial days of the project.

The author wishes to acknowledge the financial support provided by the Department of Civil and Environmental Engineering and Mechanical Engineering of the University of Adelaide during the project.

The author wishes to make a special mention of Mr. Zahrul Islam, Mr. Rangini Jha, Mr. Miguel Davilla, Dr. Himangshu Ganguly and the Chakraborty family who were helpful to the author in difficult times. The author wishes to thank Dr. Sanjay

Mazumdar for going through the thesis. Lastly the author is greatly indebted to his father and mother for their support during the last days of the project.

# LIST OF PUBLICATIONS

The following is a list of publications which are related to the research reported in this thesis:-

**Journal Publications:-**

1) **Oehlers, D.J., Ghosh, A., Wahab, M.A.(1995):** A residual strength approach to fatigue design and analysis. ***J.Strut.Engrg. American Society of Civil Engineers, Sept.***

2) **Ghosh, A., Oehlers, D.J., Wahab, M.A.(1996):** Linear fracture envelopes for fatigue design and assessment of welds. ***Structural Engineering and Mechanics, An International Journal, July.***

**Conference Publications:-**

3) **Oehlers D.J.,Wahab M., and Ghosh A.(1993):** "Fatigue design and assessment based on the residual strength of a component" 13th Australian Conference on the Mechanics of Structures and Materials, Wollongong, July, 1993.

4) **Wahab, M., Oehlers, D.J., and Ghosh, A.(1993):** Residual strength approach for fatigue design of structural components.' 5th Australian Aeronautical Conference, Melbourne, Sept.

5) **Ghosh, A., Oehlers, D.J., Wahab, M.A.(1994):** Assessing the remaining strength and life of bridge structures that are subjected to fatigue loads. Australasian Structural Engineering Conference, Sydney, Sept.

6) **Ghosh, A., Oehlers, D.J., Wahab, M.A.(1995):** Adapting S/N data for fatigue design and assessment. 14th Australasian Conference on Mechanics of Structures and Materials, Hobart.

7) **Ghosh, A., Oehlers, D.J., Wahab, M.A.(1995):** Fatigue assessment of welds using linear envelopes. International Conference on Structural Stability and Design, Sydney.

8) **Ghosh, A., Oehlers, D.J., Wahab, M.A.(1995):** Fatigue design of steel and composite bridges for greated reliability and reduced inspection periods. 5th East Asia-Pacific Structural Conference, Wollongong.

9) **Oehlers, D.J., Ghosh, A., Wahab, M.A.(1996):** Assessment of the performance of welds in existing structures subjected to fatigue loads. Third International Conference on Bridge Management, U.K.

**Departmental reports:-**

10) **Ghosh, A., Oehlers, D.J., and Wahab, M.A.(1993):** A New Fatigue Design and Analysis Approach Based on Residual Strength'. Research report no.R108, October, Department of Civil and Environmental Engineering, University of Adelaide.
11) **Oehlers, D.J., Ghosh, A., Wahab, M.A.(1994):** Fatigue design and analysis of structural components based on residual strength. Research report no.R114, March, Department of Civil and Environmental Engineering, University of Adelaide.

# TABLE OF CONTENTS

## Chapter 3. Residual strength variation of fundamental components



## Chapter 4. Determining the reduction in strength

## Chapter 5. Residual strength variation of welded components subjected to constant amplitude loading



## Chapter 6. Adapting S/N curves for determining the initial crack length

## List of figures

## List of Tables

# ORIGINALITY STATEMENT

This work contains no other material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

ABIR GHOSH

October, 1996

# GLOSSARY OF SYMBOLS

## VARIABLES AND PARAMETERS

| | |
|---|---|
| a | -Crack length for a single edge crack or a half crack length for a central crack, crack in a direction perpendicular to the thickness of the component |
| $a_i$ | -Initial crack length, crack existing in a component before fatigue loading |
| $a_f$ | -Final crack length |
| $a_m$ | -Intermediate crack length of an idealised weld |
| $a_x$ | -Intermediate crack length for an infinite plate |
| A | -Fatigue damage |
| b | -Half the distance between splitting forces |
| B | -Probability of occurance of vehicle |
| C | -Constant in Paris' equation |
| $C'$ | -Intercept of S/N curve |
| d | -Constant in S/N relation |
| E | -Endurance of a component |
| $f_u$ | -Tensile stress |
| $F_E$ | -Correction factor for elliptical crack |
| $F_S$ | -Correction factor for crack at free surface |
| $F_W$ | -Correction factor for finite width |
| $F_G$ | -Correction factor for geometry |
| K | -Stress intensity factor |

$K_C$ -Critical stress intensity factor or fracture toughness

$K_I$ -Opening mode stress intensity factor

$K_O$ -Constant in S/N relation

m '-Slope of S/N curve

m -Constant in Paris' equation

M -Magnification factor

$M_m$ -Magnification factor corresponding to crack length $a_m$

N -Number of cycles applied

$N_e$ -Number of cycles at which assessment is carried out.

$N_{en}$ -Endurance obtained from codes

$N_{in}$ -Number of cycles required for a crack to propagate through an increment

$N_f$ -Asymptotic endurance

$P_m$ -Unstable crack propagation residual strength

$P_{mp}$ -Plastic deformation failure load

$P_{mx}$ -Residual strength of a component after being subjected to multiple load patterns

$P_S$ -Initial unstable crack propagation strength

$Q_m$ -Unstable crack propagation strength of idealised weld after application of fatigue loading

$Q_{of}$ -Unstable crack propagation strength of idealised weld required to resist fatigue loads and maximum overloads at the end of the fatigue life

$Q_s$ -Unstable crack propagation strength of idealised weld required to resist maximum overload at end of fatigue life

r -Distance of material element from crack tip

$r_p$ -Radius of plastic zone

| | |
|---|---|
| $t$ | -Thickness of component |
| $T$ | -Number of vehicle traversals |
| $W$ | -Ratio of weight |
| $\varepsilon_z$ | -Strain in the z-direction |
| $\Delta$ | -Constant in S/N relation |
| $\Delta K$ | -Range of stress intensity factors |
| $\Delta Q_e$ | -Shear flow corresponding to effective stress range |
| $\Delta\sigma$ | -Stress range |
| $\Delta\sigma_e$ | -Effective stress range |
| $\sigma_b$ | -Stresses acting on a uncracked body along the line where crack is formed |
| $\sigma_f$ | -Stress at which failure occurs due to unstable crack propagation |
| $\sigma_i$ | -Initial unstable crack length for an infinite plate |
| $\sigma_p$ | -Peak stress applied |
| $\sigma_x$ | -Stress in the x-direction |
| $\sigma_y$ | -Stress in the y-direction |
| $\sigma_z$ | -Stress in the z-direction |
| $\upsilon$ | -Poisson's ratio |

# Chapter 1
# Introduction

## 1.1 Introduction

Structures in use today such as bridges, buildings, cranes, pipelines and offshore platforms are subjected to repeated loadings continuously. Components which form part of such structures are subjected to repeated stresses when in service. Such components, which are subjected to repeated application of stresses, often fail at a strength much below their original static strength (Gurney, 1979). This type of failure due to repeated application of stresses, is termed as fatigue failure. Fatigue analysis is generally carried to find out when a component will fail so that failure during service can be prevented.

Structural components are often fabricated using rivets, bolts or welds. Welded components are generally weaker under fatigue compared to unwelded components (Maddox, 1991). Such components deteriorate rapidly under fatigue and have shorter lives compared to unwelded components (BS 5400, 1980). Ever since welds have been used in structural construction, failure of welds during service have often occurred throughout history. Recent failures include the fracturing of nine-T2 tankers and seven liberty ships during the Second World war (Rolfe and Barsom, 1977). In 1962, the Kings bridge in Melbourne failed, while in 1967 the Point pleasant bridge in the U.S.A. failed with a loss of 46 lives (Rolfe and Barsom, 1977). Fatigue analysis of welds, therefore, is extremely important so that their deterioration in strength and endurances can be monitored, and failure during the service life of welded components can be prevented.

Fatigue analysis of components involve assessment and design. Present Civil Engineering techniques concentrate on the endurance while assessing or designing a component. Components are presently assessed using Miner's cumulative damage law (Miner, 1945) which gives the fatigue life expended compared to the total life. Miner's law is based on a empirical approach and gives a rather vague idea of the condition of the structure. In this thesis a new technique of fatigue assessment is proposed which not only concentrates on the endurance but takes into account the variation of strength during the fatigue life. Using the variation of strength during the fatigue life one can assess a component more accurately as well get much more information about the condition of the component. Thus one can predict the residual strength, the remaining life and crack length of a component at any intermediate period of the life of the component.

Presently in Civil Engineering practise, the design of welded components involves a calculation to find the static strength of a component and another calculation to find the endurance of a component. These designs are carried out separately and any dependence of strength on endurance is neglected. It is assumed that the strength of a component does not change during the fatigue life and remains equal to the static strength (Oehlers, 1992). The variation of strength with endurance as followed by present methods can be represented in a graphical form by the curve ACF as shown in Fig.1.1.

A particular type of welded component is the welded stud shear connector. This is a welded protrusion on the flange of a steel section required to connect the steel portion of a composite beam to the concrete section. Experimental research carried out with this connector (Oehlers, 1990) showed that its residual strength varied along a straight line as shown by Fig.1.2 below.

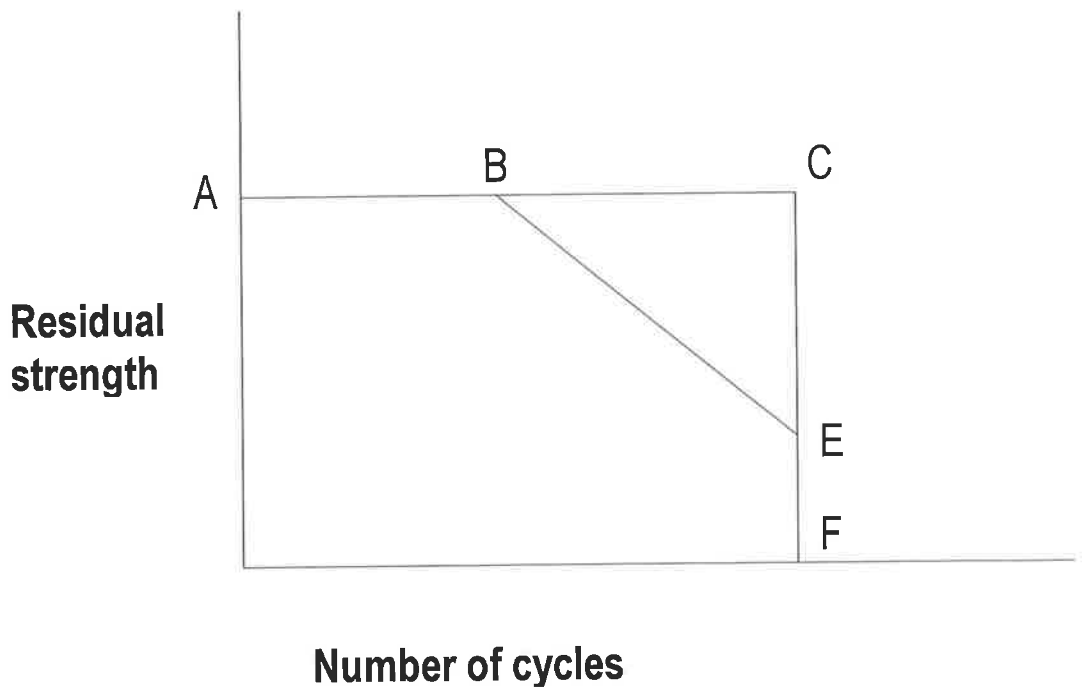



Fig.1.1 Residual strength variation of structural components

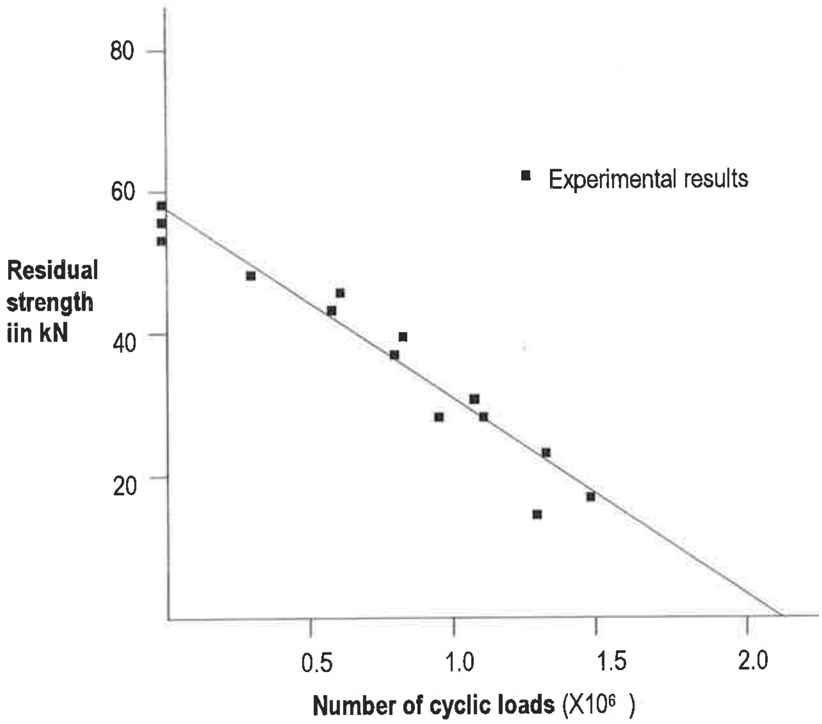



Fig.1.2 Residual strength variation of stud shear connectors (Oehlers, 1990)

Thus even though at present we use the residual variation of strength in Fig.1.1 to design structural components at least in the case of stud shear connector, which is a particular type of welded component, the actual variation is different. This raises the question whether the residual strength variation of other welded components follow the variation ACF shown in Fig.1.1. It is quite likely that the component may follow any other variation such as ABEF in Fig.1.1. If the actual variation of the residual strength of a component is known, then components can be designed based on this actual variation in strength and such design will be more accurate compared to present methods.

## 1.2 Scope of thesis

In Chapter 2 of the thesis the concepts and information used in the rest of the thesis is discussed. In Chapter 3 of the thesis a theoretical approach for determining the variation of residual strength with number of cycles has been determined for constant amplitude loading. The technique developed is based on fracture mechanics principles. The relation between residual strength and number of cycles may be linear as in the case of stud shear connectors shown in Fig.1.2 or may be non-linear as in the case of the curve ABEF shown in Fig.1.1.

While Chapter 3 of the thesis deals with constant stress ranges, components in service are generally subjected to variable stress ranges. It has been shown in Chapter 4 that when a component is subjected to variable stress ranges, the reduction of strength due to any number of stress ranges can easily be determined if the constant amplitude residual strength variation is linear and in such a case, the reduction in strength does not depend on the sequence in which stress ranges are applied. However when the residual strength variation due to a constant stress range is non-linear, the reduction in strength due to number of stress ranges depends on the sequence in which loads are applied.

In Chapters 3 and 4 the basic techniques required to determine the strength of components have been developed. In the Chapters 5 to 8 this technique is developed for welded components in particular. A procedure of determining the constant amplitude residual strength variation of welded components is discussed in Chapter 5. However in order to determine the residual strength variation of a given component during the fatigue life the initial crack length of a component must be known. This initial crack has been determined in Chapter 6 of this thesis from standard curves of stress range against number of cycles to failure or S/N curves.

In Chapter 7, a procedure of assessing welded components using linear residual strength curves is developed. In Chapter 8, a procedure of designing welded components using linear residual strength curves is discussed. In Chapter 9, a component is assessed and designed in order to illustrate the procedures developed in Chapters 7 and 8. Finally in Chapter 10 the conclusions and recommendations for future work is discussed.

## 1.3 Aim of research

The primary aim of this research is to develop a fundamentally new fatigue design and assessment procedure for practising engineers. The new analysis procedure takes the present standard practise, of curves of stress range against number of cycles, that is based on endurance design, one step further as it allows for the variation in residual strength as well as endurance. Hence the new procedure can be used to predict the variation in crack size, strength and inspection periods as well as endurances. Basic fracture mechanics and fatigue procedures has been deliberately used in this thesis to illustrate the development of this fundamentally new procedure. Hence the examples given are purely there to illustrate the new technique. However the new fatigue procedure can be used to represent the results of advanced fatigue and fracture mechanics when required.

# Chapter 2
# Literature review

## 2.1 Introduction

Fatigue analysis and design of structures came with the advent of the industrial age. The industrial age brought in the use of structural components. Such components when subjected to repeated loading failed due to fatigue. Failure of axles, wheels or rails of railway coaches occurred frequently in the middle of the last century. In 1842, there was a railway accident in Versailles, France which resulted in the loss of 80 lives. This accident led authorities to engage in the first ever detailed study of metal fatigue. Soon studies on fatigue were being carried all out over Europe. Thus laboratory experiments were designed to study how long a component can be used without the danger of them failing by fatigue (Suresh, 1992).

The first systematic fatigue experiments were carried out by August Wohler in the period 1852 to 1870 for the German railway industry (Klesnil and Lukas 1980). Wohler's work led to the characterisation of fatigue behaviour in terms of curves of stress range against number of cycles to failure. These curves were called S/N curves. S/N curves have been the mainstay of fatigue design for nearly a hundred and fifty years now and are still used for design. A lot of experiments have been done with different materials and different loading conditions to establish S/N relationships to be used in design. The effect of other factors such as mean stress, stress concentration and environment surface finish on S/N curves have also been documented.

Even though S/N curves are hugely successful in design they are however of purely empirical nature without any understanding of the total fatigue process. In the middle of this century there was an attempt to understand the whole fatigue process and the various stages it undergoes when subjected to repeated loading. The whole fatigue process has been divided by Klesnil and Lukas (1980) into the following three broad stages based on irreversible changes caused by cyclic loading:

i) Fatigue hardening or softening:- This is the first stage in the fatigue process and involves rearrangement of the substructure of the whole material.

ii) Crack initiation:-This involves initiation of a crack from any point within the structure of a component.

iii) Crack propagation:-The crack that has initiated propagates with the application of fatigue loading leading to failure.

The reason for the failure of a component subjected to fatigue loading is the formation and propagation of a crack until the strength of the component becomes less than the maximum load applied. In order to be able to predict such failures, studies have been carried out to find out how cracks would propagate and also what strength a cracked structure would have after it has propagated a distance. Such studies have been collectively grouped under fracture mechanics. Fracture mechanics is primarily used to analyse structures with known defects in them and to find out their strength and life.

The residual strength method of design and assessment proposed in this research involves both fracture mechanics and S/N curves. The procedure developed is based on monitoring the progress of an initial crack. When a component is subjected to external loads it creates stresses within the structure of the component. Repeated application of such stresses causes the initial crack of the component to propagate. The propagation of the initial crack finally leads to failure of the component. In Section 2.2, the external loads acting on the structure is discussed. Section 2.3 deals with the internal stresses in the structure caused due to the application of external loads. Section 2.4 explains the formation of an initial crack in the structure while Section 2.5 discusses the propagation of this crack. This crack propagates until the component fails and such failure is explained in detail in Section 2.6. The fatigue life at which such failure occurs is discussed in Section 2.7. The variation of residual strength during the fatigue life is explained in Section 2.8. The residual strength approach to fatigue design and assessment is compared to other methods in Sections 2.9 and 2.10.

## 2.2 Stresses applied to a component

The fatigue process starts with the application of an external loads to a component. Components used in engineering service are subjected to such repeated loads continuously. The external loads induces internal stresses within the component to cause progression of a crack and hence variation in residual strength. Apart from the internal stresses induced in the component due to external loads welded components are also subjected to built-in stresses which already exist within the component.

### 2.2.1 Stresses in a Welded Component

The built-in stresses that exists in a welded component before application of external loads are called residual stresses. Residual stresses are introduced in welded components during the process of welding. When welding is carried out, the part of the component that is close to the heat source becomes hotter than the material away from the heat source. The welded component goes through a heating and cooling cycle during which the natural expansion and contraction of material close to the heat source is restrained by the cooler portion of the welded component away from the heat source. This causes the welded component to have internal stresses. Thus even when they are not loaded, welded components generally contain in-built residual stresses. However in the absence of any external force acting, these stresses, which are both of a tensile and compressive nature, are distributed in such a way that the whole body is in equilibrium (Fig.2.1). The magnitude of such stresses are near the yield point of the material, $\sigma_{ys}$. (Maddox 1991).

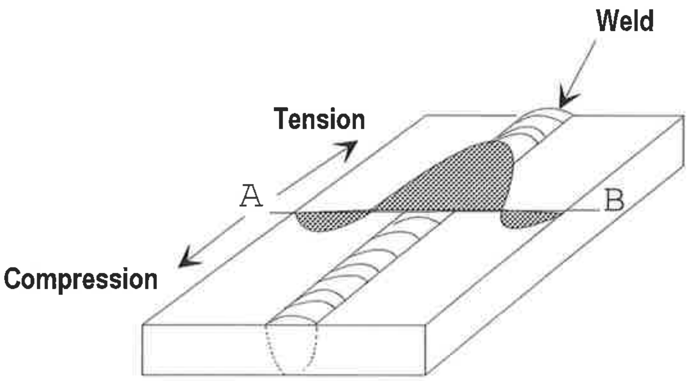



Fig.2.1 Residual stresses in a welded component. Stress distribution shown along Section AB. (Maddox, 1991)

Fig 2.2a shows the variation of stresses in a welded component when subjected to external loads. The residual stresses in the region which is under tension are at yield point. Thus further application of loading increases the strain in this region but the stresses remains at the yield point of the material. However when the stress range applied $\Delta\sigma$ is removed the stresses in this zone becomes $\sigma_{ys}$ $-\Delta\sigma$ (Fig.2.2a). Thus if the stress range $\Delta\sigma$ is repeatedly applied the stress in the portion of the material having tensile residual stress will vary from $\sigma_{ys}$ to $\sigma_{ys}$ $-\Delta\sigma$. If instead of a tensile stress a compressive stress of $-\Delta\sigma$ is applied it will cause the stress to vary between $\sigma_{ys}$ and $\sigma_{ys}$ $-\Delta\sigma$ as in the case of a tensile stress (Fig.2.2b). Thus it can be concluded that irrespective of what magnitude of external stresses are applied, the tensile portion of the component will always be stressed near the yield stress of the material. Also since cracks do not propagate in the region having compressive residual stress (Maddox 1991) it is the portion under tensile stress where the cracks will propagate leading to the failure of the component. Hence to find the number of cycles to failure for welds the effect of the mean of the range of stress applied is not considered to be important. Present codes for designing welds in bridges such as BS 5400 (1980) and EUROCODE (1992) do not take the effect of the mean stress into account. The fatigue analysis and design method developed in this thesis also considers the endurance to be independent of the mean stress.

## 2.3 Stresses in a cracked component

When external loads are applied to a component that does not contain a crack, the internal stresses can be determined using solid mechanics. However when external loads are applied to a component with an existing crack the stresses around the crack is obtained using fracture mechanics principles. The magnitude and distribution of stresses around a crack and its variation with thickness is discussed in this Section. Repeated application of external loads will cause the propagation of a crack and the reduction of the residual strength. However if the applied stresses are high enough so that a component cannot withstand the stress applied, the component will fail.

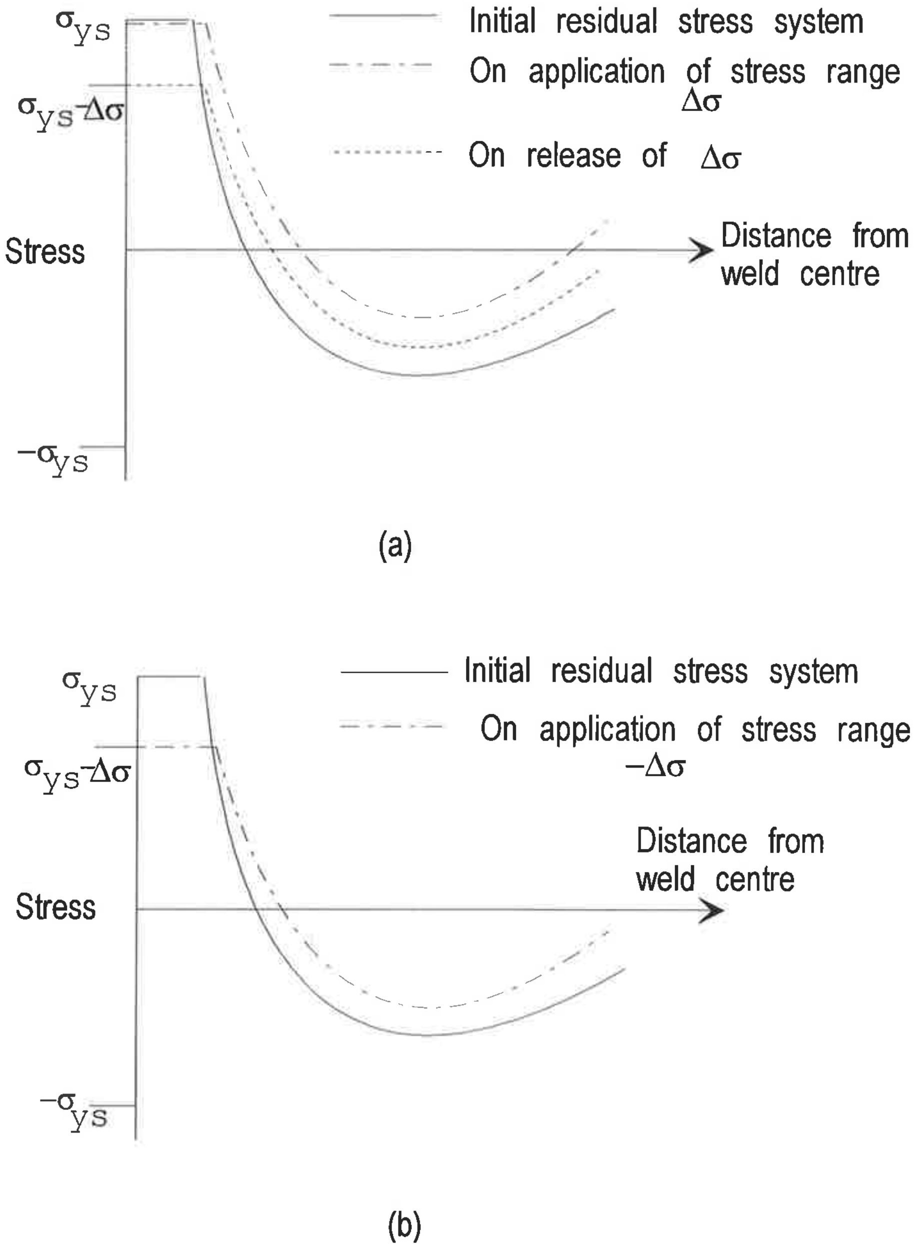



Fig.2.2 Variation of stresses in a weld due to application of stress ranges.

(Maddox, 1991)

### 2.3.1 Stress around a Crack

A component with a crack can be stressed by external loads in three different modes. They are called the opening mode, the tearing mode and the sliding mode as shown in Fig.2.3. The opening mode is the most common type of mode in which the component is stressed, in most service conditions. It is the only mode considered in this thesis and all further discussions in this Chapter and in the rest of the thesis will consider the component to be stressed in this mode.

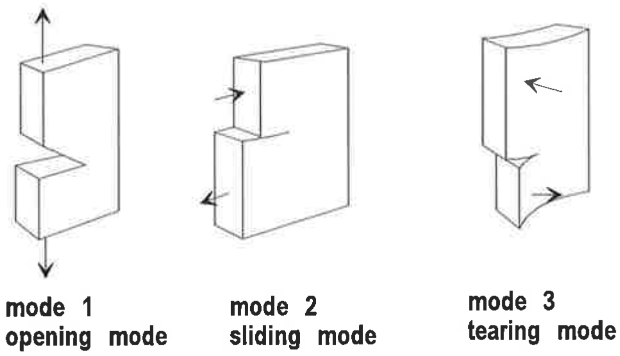



Fig 2.3 Different modes of applying load

When a load acts on a cracked structure in the opening mode, fracture mechanics uses concepts of equilibrium, compatibility along with boundary conditions to relate the distribution of stress around the crack to the external load acting. The stress in a given plane around a crack for an elastic body of any general shape and size (Fig.2.4) subjected to external stress was given by Paris and Sih (1965) as

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\Pi r}} f_{ij}(\theta) \tag{2.1}$$

where $\sigma_{ij}$ are the stresses acting on a material element of dimensions dx and dy at a distance r from the crack tip and at an angle $\theta$ from the crack plane and $f_{ij}$ is a function of $\theta$. The factor $K_I$ in the above equation is called the stress intensity factor and gives the magnitude of stress around the crack while the function $f_{ij}$ gives the distribution of stress. Since the function $f_{ij}$ is constant for the opening mode of stressing shown in Fig.2.3, it can be seen from Eqn.2.1 that any two cracked components which have the same stress intensity factor will have the same distribution of stress.

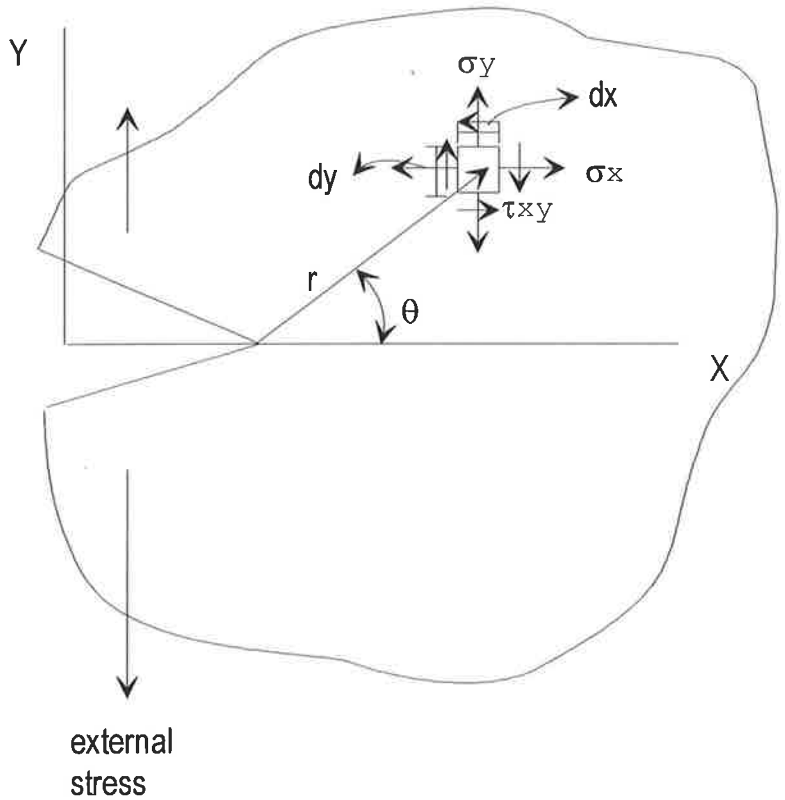



Fig.2.4 Stress around a crack

Equation 2.1 gives an elastic solution and as the value of r becomes small the stress tends to infinity at the crack tip. This does not occur in practice, as the region around the crack tip where the stress calculated according to Eqn.2.1 is more than the yield stress turns plastic. The area of the plastic zone can be found by replacing $\sigma_{ij}$ in the above equation by the yield stress, $\sigma_{ys}$. Thus the length of the plastic zone $r_p$ along the plane $\theta = 0$ can be calculated from Eqn.2.1 as

$$r_p = \frac{K^2}{2\Pi\sigma_{ys}^2} \tag{2.2}$$

If the angle of the plane is varied from $\theta = 0$ the length of the plastic zone changes. However a simplification can be made by considering the plastic zone to be circular in shape (Fig 2.5) with $r_p$ as the radius (McClintock and Irwin 1965).

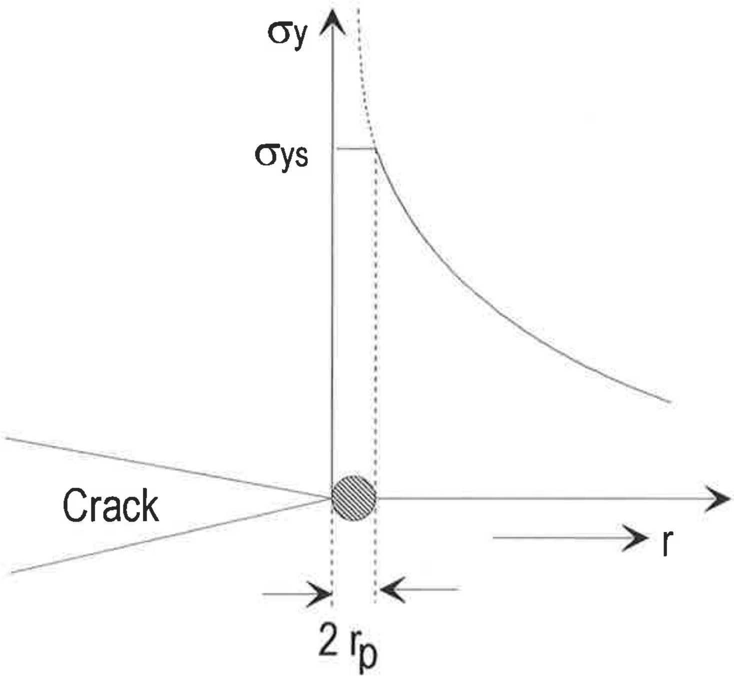



Fig.2.5 Circular plastic zone in front of crack tip (Broek, 1974)

Thus when a cracked component is subjected to external loads, it would develop a small plastic zone in front of the crack while the rest of the component will be elastically stressed.

### 2.3.2 Variation of stress and strain with thickness

The above discussion was concerned with stress in the given direction only. When a solid component is subjected to tensile stresses in a given direction it causes strain in that direction (Fig.2.6). Such longitudinal strain will also cause a transverse strain along the thickness so that there is a tendency to make the component thinner. The transverse strain is obtained by multiplying the longitudinal strain by the Poisson's ratio. Thus for steel with a Poisson's ratio of 0.33 the transverse deformation is one third of the longitudinal strain for an elastic case. However for the plastic case transverse deformation is about half of the longitudinal deformation and hence much greater than the elastic case.

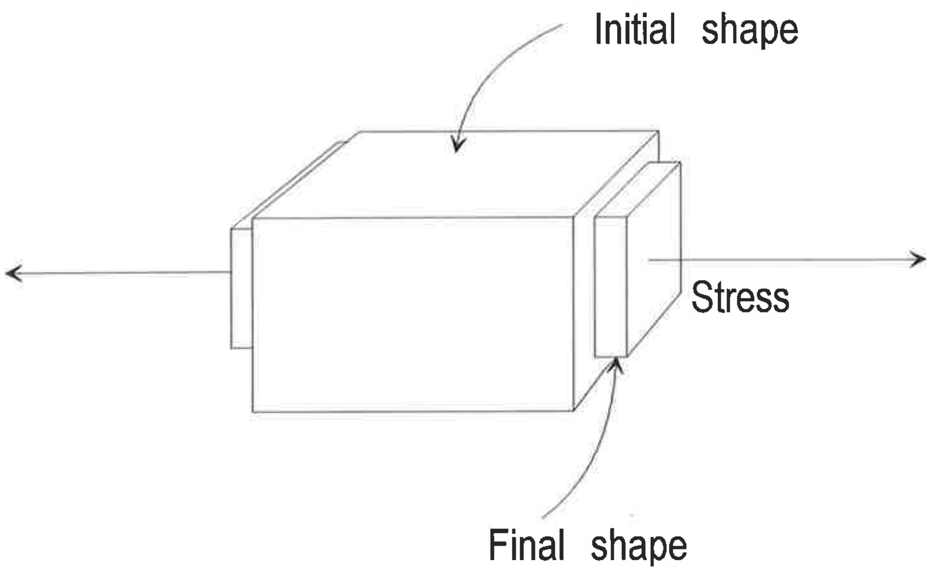



Fig.2.6 Deformation in the transverse direction

Thus when a cracked component is loaded, the plastic zone which forms around the crack, as explained in section 2.3.1, deforms more severely to the rest of the component (Fig.2.7). If the component is thick, then the large volume of material around the plastic zone will exert a stress in the transverse direction on the plastic zone and prevent it from contracting when it is subjected to loads. In such a case the component does not undergo any strain in the transverse direction and as such is said to be in plane strain or to have strain in only one plane. When the material is thinner, the volume of material surrounding the plastic zone is not large. Therefore it cannot exert a transverse stress on the plastic zone and as such cannot prevent it from contracting rather freely. In such cases there is a strain in the transverse direction but since the material contracts freely there is no stress. The stress therefore is only along the plane on which the component is subjected to external loads and hence the component under such conditions is referred to be under plane stress.

A component under plane strain will have zero strain in the transverse direction. Therefore, if the stress acting on such a component is $\sigma_X$ and $\sigma_Y$ in the x, y direction (Fig.2.7) and the stress induced in the transverse direction z is $\sigma_Z$ then equating the strains in the transverse direction we get for plane strain

$$\varepsilon_z = \frac{\sigma_z}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} = 0 \qquad (2.3)$$

where $\upsilon$ is the Poisson's ratio and E is the Young's modulus. Thus the stress in the z direction can be written as

$$\sigma_z = \upsilon(\sigma_x + \sigma_y) \qquad (2.4)$$

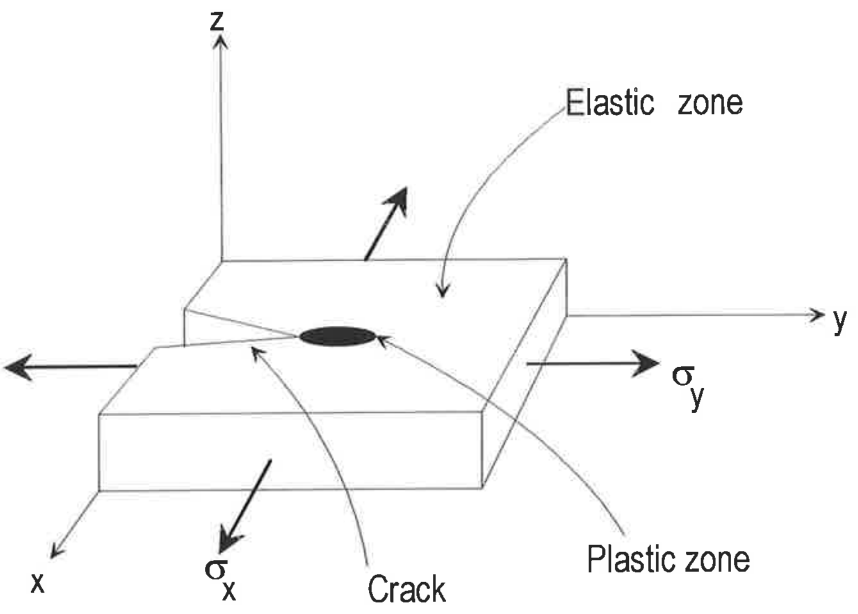



Fig.2.7 Direction of stresses acting on a component

Consider two cracked components of different thicknesses that are in plane strain. Let these two components be subjected to the same stresses $\sigma_x$ and $\sigma_y$. If the stress intensity factor of these two components are the same then the magnitude and distribution of stress will be the same along the xy plane. The stresses $\sigma_x$ and $\sigma_y$ being the same the stress $\sigma_z$ in the transverse direction (Eqn.2.4) will also be the same. Hence for components in plain strain having the same stress intensity factor, the magnitude and distribution of stresses will be identical when the external stresses are equal.

This similarity between magnitude and direction of stresses is true only for components in plain strain. For components that are not in plain strain, even if the stress intensity in a given plane is the same, the stress in the transverse direction depends on the thickness of the component. Thus two components having the same stress intensity factor will not have the same magnitude and distribution of stress if they are of different thicknesses.

The method of fatigue design and assessment developed in this thesis considers the component to be under plain strain. Thus if two components in plain strain have the same stress intensity factor, when subjected to the same stresses these components will have identical magnitudes and distribution of stress and are therefore expected to behave identically under fatigue loading.

### 2.3.3 Stress Intensity Factor for Components

As explained in Section 2.3.2 components in plain strain and having the same stress intensity will behave identically under fatigue loads. Hence fatigue behaviour of components is determined in terms of its stress intensity. Stress intensity is used to find the strength at failure in Section 2.3.4 and used to determine the propagation of a crack in Section 2.5. Here the stress intensity of welded components is discussed first for a simple idealised component and then for real welded component.

#### 2.3.3.1 Stress intensity factor for a simple idealised component

The stress intensity solution of a welded component is derived from other solutions of more simpler components. The most simplest component whose stress intensity was first derived is an infinitely wide plate with a centre crack 2a subjected to a remote stress $\sigma$ (Fig.2.8). The results which were obtained by Irwin (1962) can be understood using simple dimensional analysis on Eqn.2.1. The left hand side of Eqn.2.1 has the dimensions of stress. In the right hand side the function of angle $\theta$, $f(\theta)$ is dimensionless while $K/\sqrt{2\Pi r}$ in the equation must have the dimensions of stress. Substituting the only stress available which is the remote stress applied and the only length available which is the crack length '2a' we have

$$K = \beta\sigma\sqrt{a} \qquad (2.5)$$

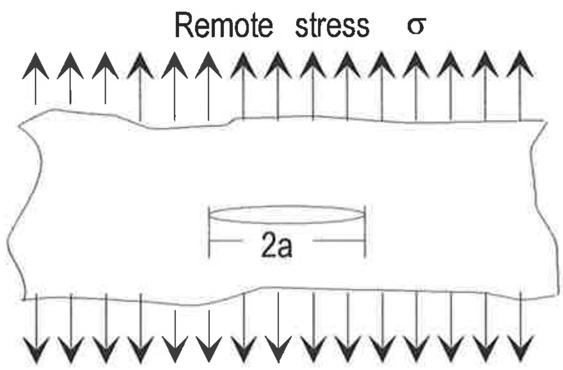



Fig 2.8 Remote stresses applied to a centrally cracked infinitely wide plate

Calculations (Broek 1974) have shown that the value of β in the case of an infinite plate is $\sqrt{\Pi}$. Hence the above equation becomes

$$K = \sigma\sqrt{\Pi a} \qquad (2.6)$$

**2.3.3.2 Stress intensity factor for welds**

Equation 2.6 gives the classical solution for an elementary structure. This fundamental solution needs to be modified for more practical structures and this is carried out by introducing a magnification factor M in Eqn.2.6 (Skorupa, et al 1987) to give

$$K = M\sigma\sqrt{\Pi a} \qquad (2.7)$$

Closed form analytical solutions for the stress intensity factor K exist for only a few idealised geometries such as a plate subjected to uniform tensile stresses with a centre crack or an infinite solid subjected to uniform tension with a circular crack (Sneddon, 1946). Practical components such as welds geometries cannot be idealised and applied stress patterns are not always uniform. Solutions for the stress intensity factor K are determined for such components using numerical or analytical methods. Numerical solutions for K are available in the form of finite elements and boundary element methods. The finite element solutions are more commonly used. Even though finite element solutions are more accurate than analytical solutions, computations of stress intensity using finite element is quite laborious when used for fatigue purposes. This is because as the crack advances and the crack length changes, stress intensity computations has to be done separately for each different crack length. The less accurate analytical solutions are therefore generally used to determine the values of K.

The accuracy of analytical solutions for K of welds in a structural member was tested by Skorupa, Braam and Prij (1987). Several analytical solutions by Smith and Miller (1977), Albrecht and Yamada (1977), Jergeus (1978), Karlsson and Backlund (1978), and Schijve (1981) were compared with more accurate finite element solutions for a cruciform welded joint (Fig.2.9). It was found that the solution by Albrecht and Yamada provided the best results with the highest discrepancies less than 10%. Albrecht and Yamada gave the value of M as

$$M = F_E \, F_S \, F_W \, F_G \tag{2.8}$$

where $F_E$, $F_S$, $F_W$ and $F_G$ are known as correction factors. $F_E$ is a correction factor for an elliptical crack front (Fig.2.9) since in welds the cracks are elliptical in shape (Gurney 1979). $F_S$ is a correction factor to account for the fact that the crack is no longer a central crack (Fig.2.9) but starts at a free surface. $F_W$ is a correction factor that is introduced to deal with a component having a finite width and $F_G$ is a correction factor accounting for the geometry of the welded component. The procedures for determining these correction factors are explained here.

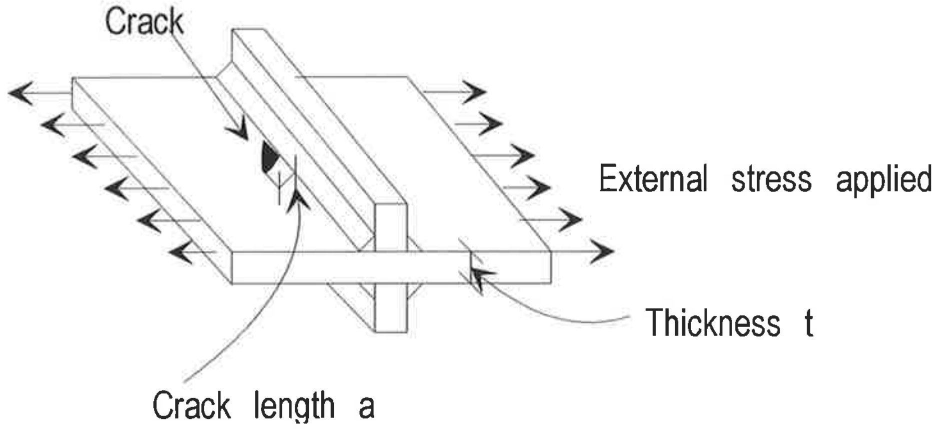



Fig.2.9 Crack formation in a stiffener weld

The correction factor $F_G$ was determined by Albrecht and Yamada using earlier work obtained by Tada, Paris and Irwin (1973). Tada, Paris and Irwin developed a closed formed solution for the stress intensity factor of a centrally cracked component of infinite width subjected to two equal pairs of splitting forces (Fig 2.10). These equal pairs of splitting forces are the forces acting on an uncracked body at the position where the crack is expected to form.

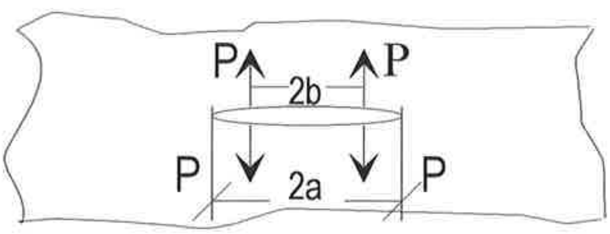



Fig.2.10 Central crack subjected to equal pairs of splitting forces

This stress intensity solution is given as

$$K = \frac{2P}{\sqrt{\Pi a}} \frac{a}{\sqrt{a^2 - b^2}} \tag{2.9}$$

where 2b is the distance between the splitting forces, and 2a is the length of the centre crack. Eqn.2.9 is for a single pair of splitting forces which can be extended for more splitting forces. Thus if there were other equal pairs of splitting forces ($P_1$, $P_2$ and $P_3$ in Fig.2.11) acting on the component, the stress intensity factor for different forces can be superimposed to give the resulting solution for stress intensity factor as

$$K = \frac{2P_1}{\sqrt{\Pi a}} \frac{a}{\sqrt{a^2 - b_1^2}} + \frac{2P_2}{\sqrt{\Pi a}} \frac{a}{\sqrt{a^2 - b_2^2}} + \frac{2P_3}{\sqrt{\Pi a}} \frac{a}{\sqrt{a^2 - b_3^2}} \tag{2.10}$$

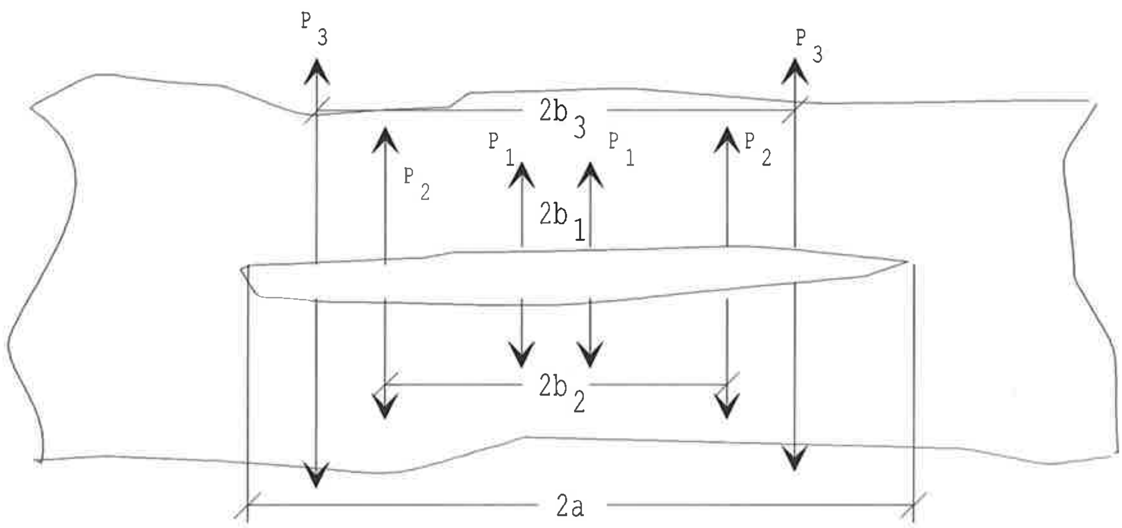



Fig.2.11 Central crack subjected to three equal pairs of splitting forces

where $2b_1$, $2b_2$, and $2b_3$ are the distances between the forces $P_1$, $P_2$, and $P_3$. Tada et al's solution can further be extended to determine the stress intensity for a distribution of stresses $\sigma_b$ acting along the line where the crack is expected to form. These stresses can be represented as forces per infinitesimal length db (Fig.2.12) and the value of K can be given as a integral

$$K = \sqrt{\Pi a}\,\frac{2}{\Pi}\int_o^a \frac{\sigma_b}{\sqrt{a^2 - b^2}}\,db \qquad (2.11)$$

The distribution of stresses $\sigma_b$ can be determined from a finite element analysis of the geometrical configuration for which the K solution is being found. If the continuous stress formation as in Fig.2.12 is divided into discrete sections as shown in Fig.2.13 then the solution for a pair of discrete stresses is a summation given by

$$K = \sqrt{\Pi a}\,\frac{2}{\Pi}\sum_{i=1}^{n}\frac{\sigma_{bi}}{\sigma_m}\left(\arcsin\frac{b_{i+1}}{a} - \arcsin\frac{b_i}{a}\right) \qquad (2.12)$$

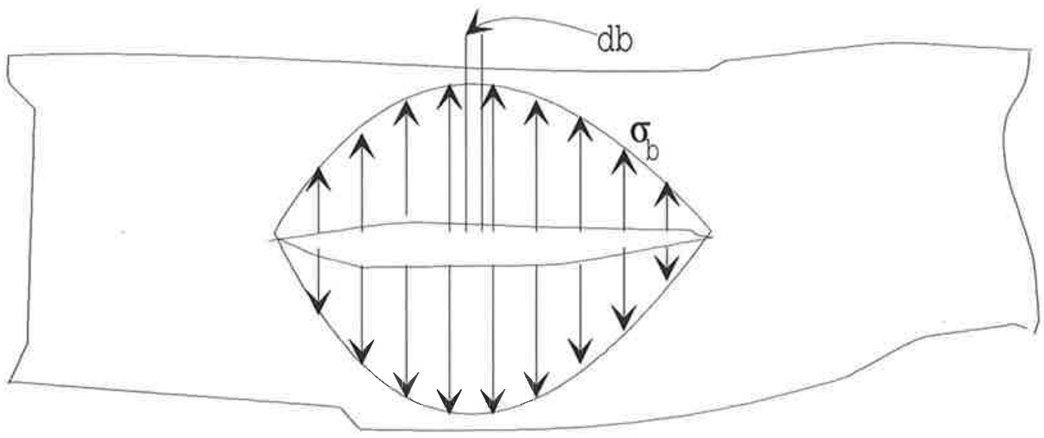



Fig.2.12 Continuous distribution of stresses acting on a cracked component

where $\sigma_{bi}$ is the stress applied over the element between $b_i$ to $b_{i+1}$ and $\sigma_m$ is the mean stress acting over the crack length 'a' divided into n elements (Fig.2.13).

$F_G$ is given by comparing Eqns 2.7 with Eqn.2.12 as

$$F_G = \frac{2}{\Pi}\sum_{i=1}^{n}\frac{\sigma_{bi}}{\sigma_m}\left(\arcsin\frac{b_{i+1}}{a} - \arcsin\frac{b_i}{a}\right) \tag{2.13}$$

Albrecht and Yamada suggested the use of Eqn. 2.13 to find the geometric correction factor of a component.

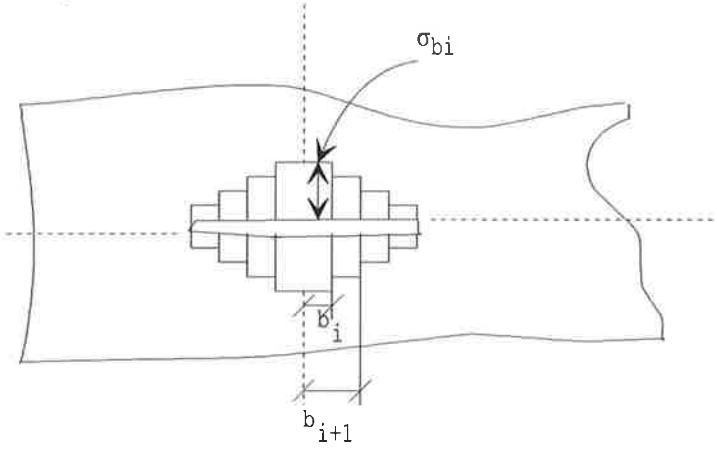



Fig.2.13 Discrete distribution of stresses acting on a cracked component (Albrecht and Yamada, 1977)

Apart from deriving the value of $F_G$, Albrecht and Yamada used standard work by other researchers for the other three correction factors (Eqns. 2.14, 2.17 and 2.18). The elliptical crack front correction, $F_E$ was developed from an expression for a stress field around an ellipsoidal cavity (Green and Sneddon 1950). Irwin (1962) used this information to derive a stress intensity factor solution for an embedded elliptical crack (Fig.2.14) as given below. The value of K varies along the crack front and at a point depends on the angle β as shown in Fig.2.15. Irwin's solution is given as

$$K = \frac{1}{E_k}\left(\sin^2\beta + \frac{a^2}{c^2}\cos^2\beta\right)^{1/4}\sigma\sqrt{\Pi a} \tag{2.14}$$

where

$$E_k = \int_0^{\Pi/2}\left[\left(1-\frac{c^2-a^2}{c^2}Sin^2\beta\right)\right]^{1/2} d\beta \qquad (2.15)$$

and a and c are the lengths of the minor and major semi axis. The integration is carried out for elements with angles dβ. The value of K is largest when $\beta=\frac{\Pi}{2}$. Substituting $\beta=\frac{\Pi}{2}$ in Eqn.2.15 gives

$$K=\frac{1}{E_k}\sigma\sqrt{\Pi a} \qquad (2.16)$$

The value of $E_k$ varies from $\Pi/2$ for a circular crack (a/c=1) to 1 for a crack with a/c=0. In practise the value of a/c would vary as the crack progresses however for simplification an average value of 0.67 has been used by Albrecht and Yamada in their calculations to find $F_E$.

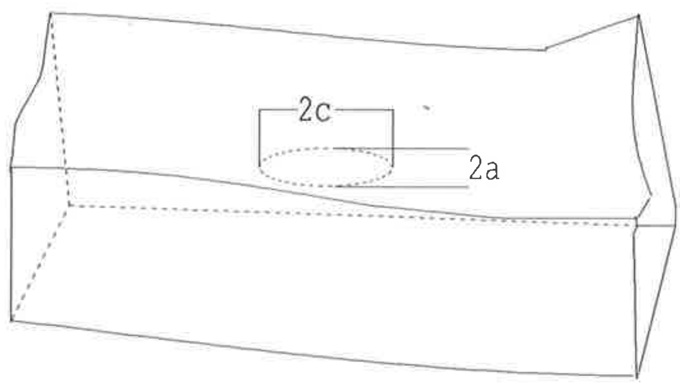



Fig.2.14 An embedded elliptical crack

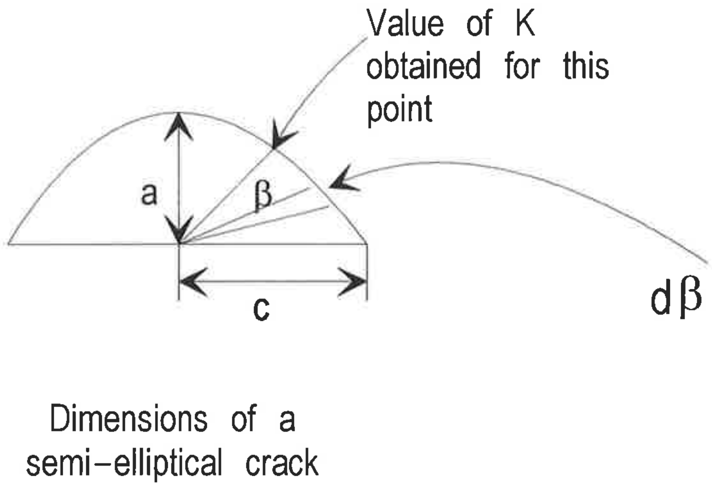



Fig.2.15 Dimensions of a semi-elliptical crack

The next correction factor $F_W$ accounts for the finite dimensions of the specimen. Several solutions exist for this finite correction factor, $F_W$. Feddersen (1967) developed a solution based on an empirical approach. Brown and Srawly (1967) and later Isida (1970) developed numerical solutions. Irwin (1958) developed a solution based on an analytic technique. Albrecht and Yamada chose Irwin's solution in their work arguing that the lower values given by Irwin's solution are suitable for edge cracks which commonly occurs in most components. The solution is given as

$$F_w = \sqrt{\frac{2t}{\Pi a}\tan\frac{\Pi a}{2t}} \tag{2.17}$$

where 'a' is the length of the edge crack (Fig.2.9) and 't' is thickness of the component in the direction of the propagation of the crack (Fig.2.9).

The last correction factor $F_S$ is for the edge crack in a welded component. Since an edge crack opens at the surface where there are no stresses, these edges would undergo a larger displacement compared to a central crack. This large displacement results in a higher value of K. The increase found out by Paris and Shih (1965) is about 12%. Thus the correction for a edge crack is given as

$F_S = 1.12.$ (2.18)

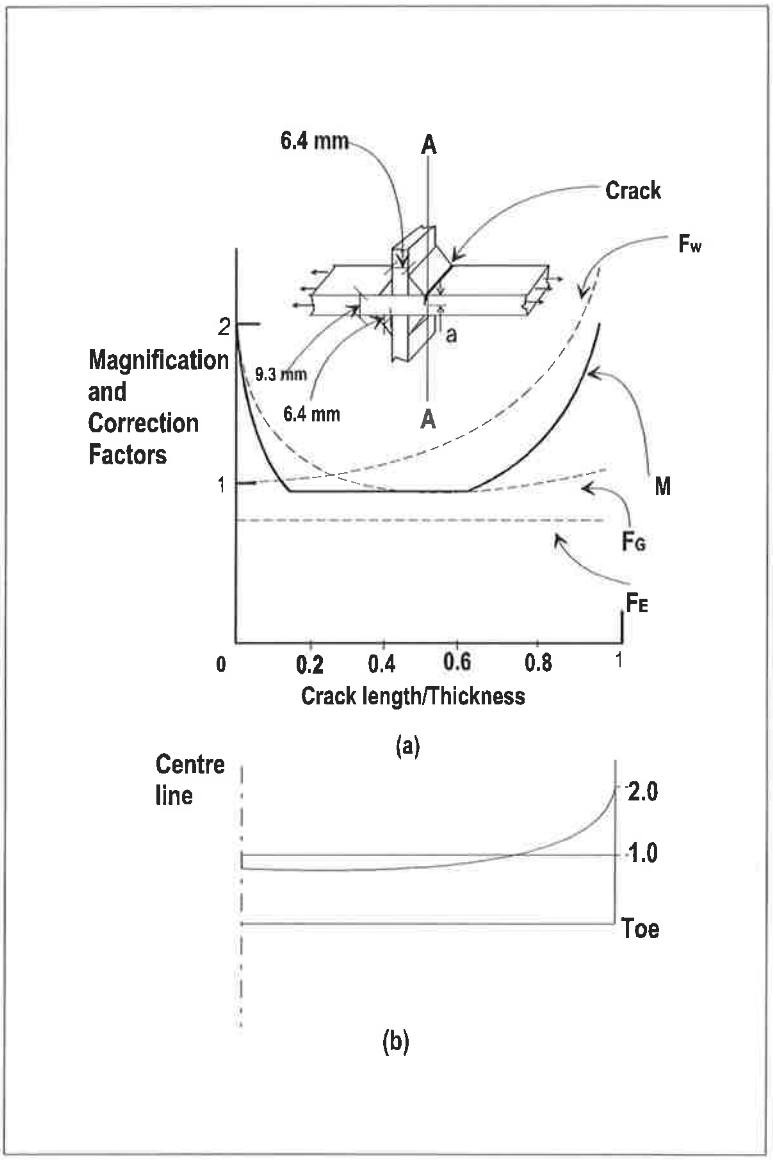



Fig.2.16 a) Figure showing variation of magnification factor with crack length for a stiffener weld. (Albrecht and Yamada, 1977)

2.16 b) Figure showing variation of stress concentration along Section A-A. (Albrecht and Yamada, 1977)

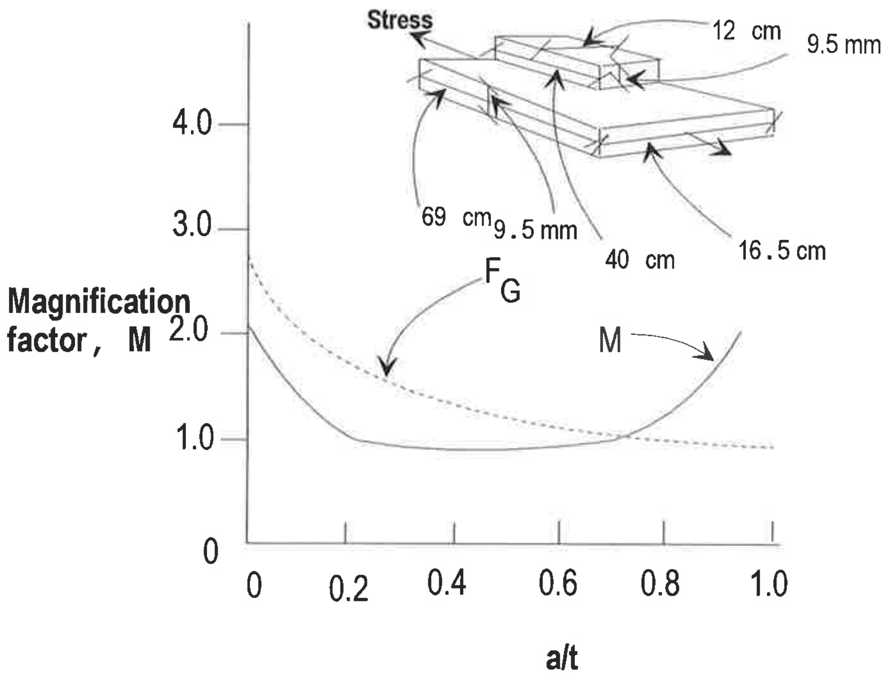



Fig.2.17 Figure showing variation of magnification factor with crack length for a cover plate

Albrecht and Yamada applied their proposed solution for the magnification factor M (Eqn.2.7) to a part-through crack in a stiffener weld, a cover plate and a weld containing a spherical pore. The stiffener assessed consists of a 9.53 mm thick plate to which 6.4 mm stiffeners are attached on either side with 6.4 mm fillet welds (Fig.2.16). Figure 2.16(a) shows the correction factors obtained. The elliptical correction factor was based on a constant aspect (a/c) ratio of 0.67. This value of a/c had earlier been obtained by a study of 42 part through cracks of stiffeners of various sizes (Irwin et al 1968). It can be seen from Fig.2.16a that the value of M is nearly constant between crack length to thickness ratio a/t of 0.2 to 0.6. The value of M is high when the crack length is small due to the geometric correction factor $F_G$. Also M again becomes high when the crack length is large due to the effect of finite width $F_W$. The cover plate assessed by Albrecht and Yamada is

40 cm long, 9.5 mm by 12 mm in cross section with a flange plate 69 cm long and 9.5 mm by 16.5 mm in cross section. Fig.2.17 shows the variation of the magnification factor. The variation is similar to that of a stiffener weld. The elliptical correction factor was taken as 0.67 as in the case of a stiffener weld. Fig.2.18 shows the variation in magnification factor for a component containing a spherical pore which are formed in the weld due to gases being trapped during welding (Hirt and Fisher 1973). The variation is again similar to that of a stiffener weld and a cover plate.

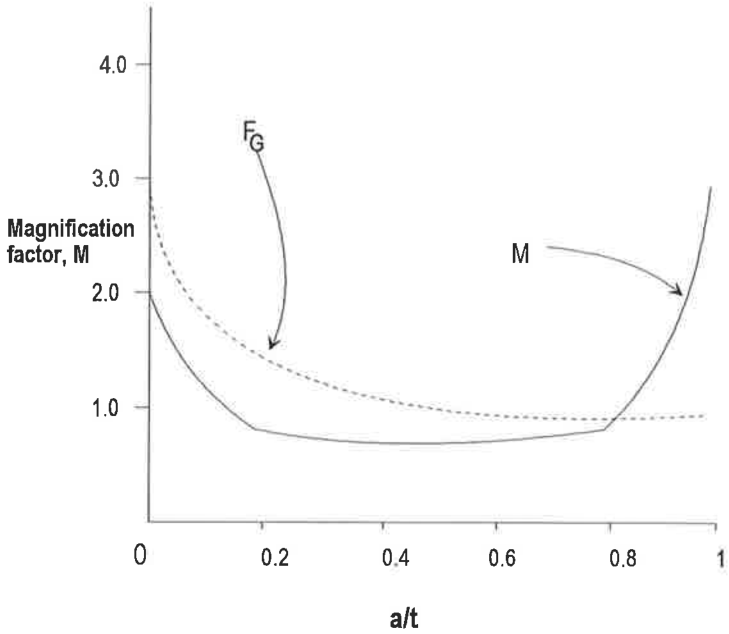



Fig.2.18 Variation of magnification factor for a weld with spherical porosity

It must be noted that among the four factors which comprise M in Eqn.2.8 the factors due to thickness, elliptical crack front and edge crack do not change for most welded components where the cracks start from the surface. However, the geometry correction factor will vary from weld to weld since it depends on the variation of stress concentration of the part of the welded component where the crack is assumed to grow.

The variation of stress of the stiffener weld is shown in Fig.2.16 b. For most components, the stresses on the surface are the highest and gradually decrease inwards. As long as the stress concentration continues in this pattern, Albrecht and Yamada's solution for the geometry correction factor and hence the M value will have a similar variation to Fig.2.16.

Albrecht and Yamada's solution has been used in this thesis to find the stress intensity of a welded component. The results provided by Albrecht and Yamada's solution are lower than finite element results for very small crack lengths (Crack length/Thickness, $a/t<0.02$) while for longer cracks the values are higher (Skorupa, Braam and Prij,1987). The overall fatigue life predictions using Albrecht and Yamada's solution are somewhat conservative and hence can be considered to be safe.

### 2.3.4 Fracture Toughness

The stress intensity factor determined for welded components (Eqn.2.7) depends on the external stress applied. Thus if the external stress applied to a component with an existing crack is increased, the stress intensity factor will also increase and ultimately this leads to the unstable propagation of the crack and hence the failure of a component. The stress intensity factor at which the component fails is called the fracture toughness or the critical stress intensity factor. The fracture toughness or critical stress intensity is a property of the material of the component.

Thus for a component in service it needs to be made sure that the service loads applied induce a stress intensity factor which is less than fracture toughness and the component does not fail.

## 2.4 Crack initiation

Service loads that are not high enough to fail a component can still cause damage to the component. The application of service loads to a component induces areas of elastic and plastic deformation. If a component contains no in-built stresses, when the applied loading during a load cycle are removed the zone of the component stressed elastically reverts back to its original condition. However the plastic zone remains in its deformed condition. Further application of other cycles of load causes repeated strain within the plastic zone. Gradual accumulation of permanent strain causes a crack to initiate in the structure (Klesnil and Lukas 1980).

At a micro level, cracks initiate in a portion of the structure where permanent strain will occur first. Permanent strain depends on the stress and is therefore likely to occur in the portion of the component with the highest stress concentration. This region is normally the surface of the loaded component. Even though the surface looks smooth under the naked eye it is actually quite uneven when studied under a microscope. The grooves and projections on the surface causes the stresses at the surface to be higher than the interior. Also, research (Argon 1972) has shown that an inclusion or precipitate will cause higher stresses on the surface compared to the interior.

Cracks will initiate in a component only when the stresses are quite high. As discussed in Section 2.2.1, welds have high tensile residual stresses. Also welds are subjected to severe stress concentration at regions like the weld toe and defects like undercuts. Welds therefore have relatively short or no crack initiation lives (Maddox 1991). Researchers often consider welded components to have no initiation life and a crack length of 0.25mm is often considered to be the initial crack (Skorupa 1992, Lawrence and Mattos, 1978) for welded components.

The fatigue design and assessment procedure developed in this thesis assumes a crack to initiate from the surface and neglects the crack initiation life. In this thesis a method for calculating the initial crack length of components for experimental data has been determined. However, where the initial crack length is not known a crack length of 0.25mm has been used for analysis.

## 2.5 Crack propagation laws

The crack initiation life of a component is followed by crack propagation life of the component which is the next phase it undergoes during its fatigue life. In order to predict the propagation of a given crack with the application of fatigue loading, crack propagation laws have been derived. The crack propagation laws can be used to determine the crack length at any intermediate period of the fatigue life and from the crack length the residual strength can be calculated. This section describes the theories of crack propagation, the material properties for propagation of cracks through steel and the rates of crack propagation through welded components.

It has earlier been discussed in Section 2.3.1 how the application of fatigue loading causes a stress field to develop around the crack in a component. It was shown that in an elastic case the stress intensity factor is an important parameter which describes the whole stress field. This is particularly true when the plastic zone at the crack tip is small compared to the rest of the component. It was shown by Paris, Gomez and Anderson (1961) that if two different cracks have the same stress intensity and hence the same stress variation in front of the crack tip they behave in the same manner and show equal rates of growth. Hence a relation between the rate of growth da/dN and the range in stress intensity $\Delta K$ was developed as follows

$$\frac{da}{dN} = C(\Delta K)^m \tag{2.19}$$

where a is the crack length, N is the number of cycles, da/dN is the rate of crack growth where $\Delta K$ is the variation in stress intensity factor for the range of cyclic load applied. C and m are material properties.

Equation 2.19, however, does not represent adequately the rate of crack growth for the entire fatigue life of a component (Ewalds and Wanhill 1984). A curve showing the variation of da/dN against log $\Delta K$ for the entire fatigue life is given in Fig.2.19. It can be seen that Eqn.2.19 gives the relation for only a portion of the life where the curve is linear (region 2). The curve is non linear and the rate of crack growth is slower at the start (region 1) and higher at the end of the fatigue life (region 3). In region 3 where the component is towards the end of its fatigue life there is an acceleration in crack growth which is exponential with number of cycles. This region therefore occupies an insignificant part of the fatigue life of the component and therefore is often ignored (Maddox 1991).

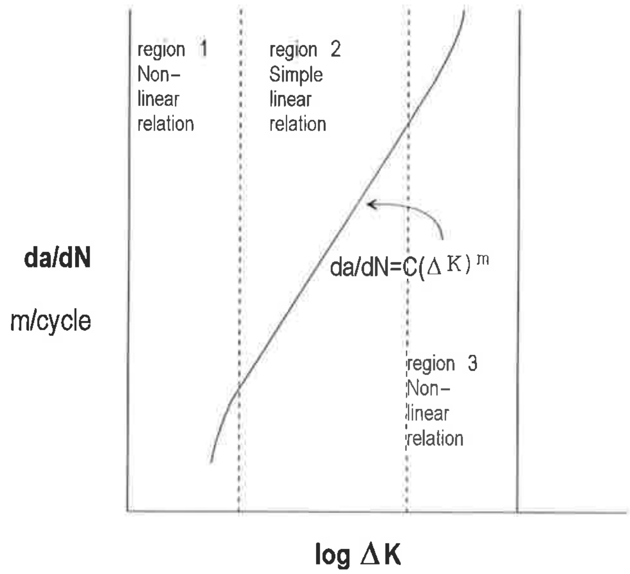



Fig.2.19 Variation of rate of crack growth with log of stress intensity

Equation 2.19 also does not account for such parameters as thickness, environment and frequency of application of stress, the ratio of minimum applied stress to the maximum applied stress, the critical stress intensity factor or temperature which are known to affect the rate of crack propagation (Broek 1974). There are more accurate crack propagation equations which take some of these factors into account. Thus Elber's (1971) and Schijve's (1979) relation takes account of the ratio of stress while Erdogan's (1967) relation takes account of the critical stress intensity. Foreman et al's (1967) relation takes account of both the ratio of stress and the critical stress intensity factor. The relation is given as

$$\frac{da}{dN}=\frac{C\Delta K^{n}}{(1-R)K_{c}-\Delta K} \qquad (2.20)$$

where R in the ratio of minimum to maximum stress, $K_c$ is the fracture toughness and n is a constant. Foreman's equation has been found to provide a better prediction of crack propagation in region 3 compared to Paris' equation (Ewalds and Wanhill 1984).

The rate of propagation of a crack is also affected by the variation in amplitude of loading. None of the above equations takes this variation into account. In the case of variable amplitude loading this rate depends both on the loads applied and the sequence in which they are applied (Meguid 1991). The rate of propagation is drastically affected if a overload is applied. An overload causes the rate of crack growth to decrease and the rate remains low for an extended period of time (Larsen and Annis 1980). The rate of crack propagation for variable amplitude loading has been proposed by Larson and Nicholas (1985) based on crack propagation data and by Wheeler (1972) based on stresses in front of the crack tip.

Among the various crack propagation equations discussed above, Paris's equation is the earliest and the simplest proposed crack propagation law and hence still the most frequently used (Burn et al 1987). The other equations are more complex in nature and require the use of a number of material properties. Such material properties are not well documented and are difficult to obtain. On the other hand the material properties required by Paris' equation are readily available. Hence Paris' equation has been adopted throughout the thesis.

### 2.5.1 Material Values for Paris' Crack Propagation Law

The material properties for Paris' equation have been found after extensive experimental research on steel and aluminium. The material properties for propagation of crack in steel was investigated extensively by Barsom (Barsom 1971 and Barsom et al 1971). Commonly used steels were broadly divided into three classes, ie. Martensitic, Ferrite-pearlite and Austenitic. It was found that for Martenistic steel (Fig.2.20) or high strength steel with a yield strength of more than 552 N/mm$^2$, Paris' Eqn. can be written as

$$\frac{da}{dN} = 2 \times 10^{-10} (\Delta K)^{2.25} . \qquad (2.21)$$

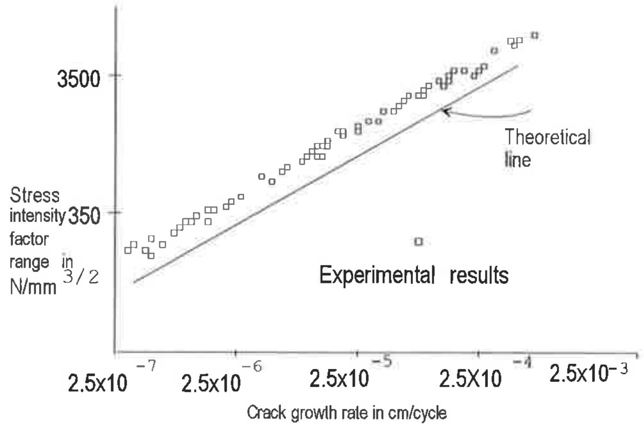



Fig.2.20 Crack growth in martensitic steel (Barsom et al 1971)

thus constants C and m being 2 x $10^{-10}$ and 2.25. The Ferrite-pearlite (Fig.2.21) type of steel includes all types of steel between the range of yield strength of 200 to 552 N/mm$^2$. Most steel used in structural work falls in this category. The relation between rate of crack growth and the stress intensity was found to be

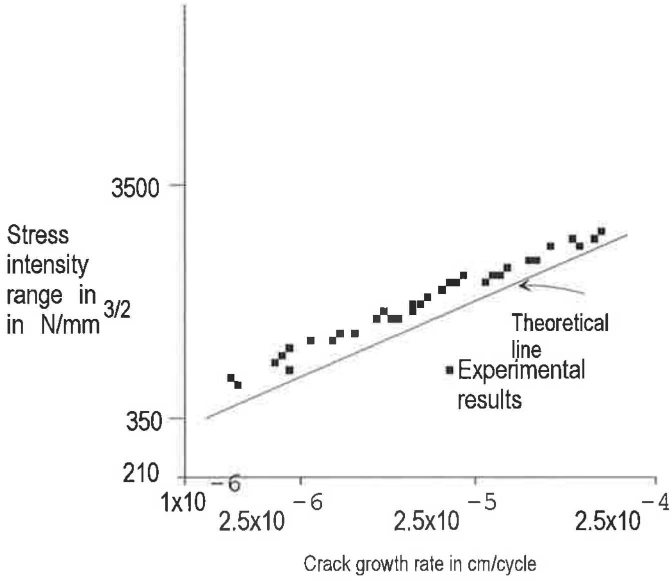



Fig.2.21 Crack growth in Ferrite-pearlite steel (Barsom et al 1971)

$$\frac{da}{dN} = 2.3 \times 10^{-13} (\Delta K)^3 \qquad (2.22)$$

The Austenitic type of steel (Fig.2.22) comprises of stainless steel and has a yield strength between 200 to 350 N/mm$^2$. The relation between the rate of crack growth and stress intensity range was found as

$$\frac{da}{dN} = 1.5 \times 10^{-11} (\Delta K)^{3.25} \qquad (2.23)$$

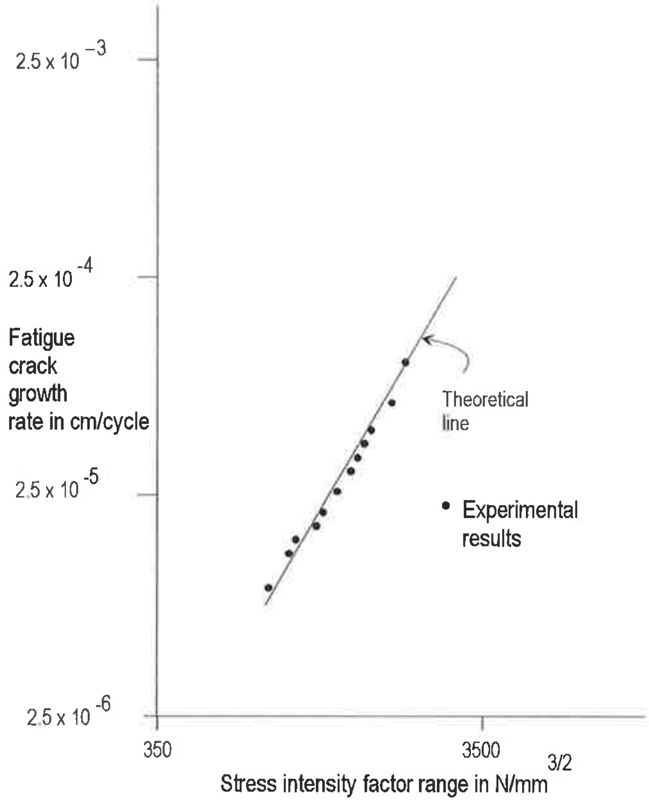



Fig.2.22 Crack growth in Austenitic steel (Barsom et al 1971)

Equations 2.21 to 2.23 gives the rates of crack growth through pure steel. However the process of welding steel involves heating, and a weld metal with different material properties compared to the material properties of the steel being welded is used when such heating is carried out. The material properties of welded steel therefore varies along the component.

### 2.5.2 Crack Propagation in Welded Components

The fatigue growth rate of welded steel members has also been investigated by various researchers (Bucci et al 1973 and Clark 1970). Welds have three regions with different material properties namely the parent metal which is the steel being welded, the weld metal which is used in fusion and the heat affected zone which separates the parent metal and weld metal zones (Fig.2.23). Experiments have shown that cracks propagate faster through the parent metal compared to the weld metal or the heat affected zone. Thus the material properties of the parent metal can safely be used to provide a conservative estimate of the rate of crack propagation through a weld. Hence the material properties relating to crack propagation of steel members described in Section 2.5.1 has been used in this thesis for developing a fatigue analysis method of welded component.

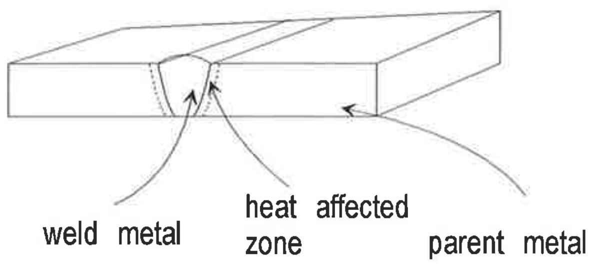



Fig.2.23 Regions of a welded component

## 2.6 Failure

The crack length of a component being known, the residual strength of the component can be determined. Failure will occur if the residual strength of a component is exceeded. Such failure occurs among all engineering structures such as tanks, pressure vessels, ships, bridges, airplanes etc. The different modes by which components can fail is discussed.

### 2.6.1 Cases of Fatigue Failure

A large number of well known cases of failure of components due to fatigue has been documented (Barsom and Rolfe 1977). Examples include the failure of several welded bridges before the second world war. There were several fatigue failures of ships during the second world war. These included at least nine tankers and seven liberty ships which completely fractured into two. These fractures are known to occur due to failure of welds at low temperatures. Such failure of welded components at low temperatures are quite common. Many of the ships which did not fail were severely damaged. After the war, at least two new cargo ships is reported to have broken into two due to brittle fracture of welds. In 1962 the Kings Bridge in Melbourne failed by brittle fracture at the temperature of 40 F. In 1967 the Point Pleasant Bridge at Point Pleasant, West Virginia failed with the loss of 46 lives.

### 2.6.2 Modes of Failure

Failure of welded components can occur by different modes. Thus components can fail due to buckling, large elastic deformation or jamming, large plastic deformation or deforming under tensile loads followed by reduction in cross sectional area leading to failure and unstable crack propagation or fast propagation of a crack (Knott 1974). The two main modes of failure for welded components (Cox 1987) are unstable crack propagation failure and plastic deformation failure. These two modes have been considered in this thesis. Failure will occur due to the mode which gives a lesser value of strength.

### 2.6.2.1 Unstable crack propagation failure

The criteria for failure of a component due to unstable crack propagation have been determined both from a energy and stress point of view. According to Griffith (1921) unstable crack propagation will occur when the energy released due to the propagation of a crack is more than the energy required or absorbed to form the crack. In such a case the energy available is always more than the energy required by the crack to propagate. Hence the crack will keep on propagating till the component fails.

As discussed in Section 2.3.4 unstable crack propagation will also occur when the stress intensity factor of a component reaches a material property termed as the fracture toughness. The fracture toughness is also referred to as the critical stress intensity factor and is denoted as $K_{IC}$. Substituting $K_{IC}$ into Eqn.2.7 gives the stress at which crack propagation failure occurs as

$$K_{IC} = M\sigma_f\sqrt{\Pi a} \qquad (2.24)$$

where $\sigma_f$ is the stress at which failure occurs.

### 2.6.2.2 Variation of fracture toughness

It was discussed in Section 2.3.4 that the fracture toughness varies primarily with thickness, loading rate and temperature and to a lesser extent with crack sharpness, orientation, homogeneity of the material etc. Since the strength of the component due to unstable crack propagation depends on the fracture toughness, the residual strength will also depend on these factors. The variation of these major factors has therefore been discussed here in detail.

### 2.6.2.2.1 *Thickness*

The fracture toughness of a component varies with thickness in a similar manner to the stress intensity (Pellini 1973). A curve of the variation of fracture toughness with thickness shows that the fracture toughness is high for thin plates and reduces as the plate gets thicker (Fig.2.24). When the plate is thick enough for the component to be under plain strain or a triaxial state of stress, the toughness acquires a constant value. Further increase in thickness does not cause the toughness to change since the component remains under plain strain or triaxial state of stress. The plane strain fracture toughness of A 36 steel corresponding to a strain rate of 10 strains/s is 1400 $N/mm^{3/2}$ which has been used throughout the thesis. The fracture toughness under such strain rate has been chosen since this represents the worst case scenario. The value of fracture toughness available for 4340 steel is 1470 $N/mm^{3/2}$ which has been used in this thesis.

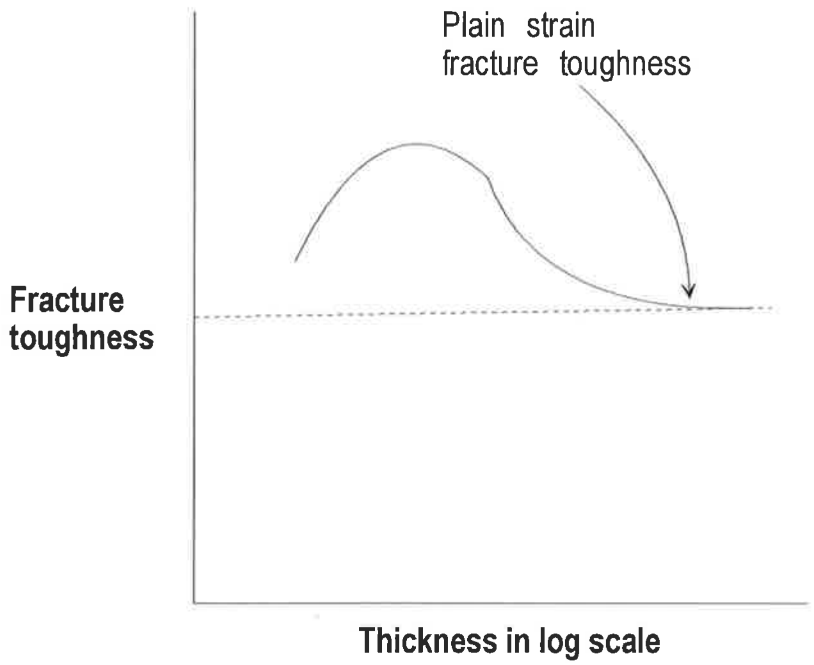



Fig.2.24 Typical variation of fracture toughness with thickness (Barsom 1978)

#### 2.6.2.2.2 *Temperature and Loading rate*

It has been found that the fracture toughness of structural steels increases with increasing temperature and decreasing loading rate. The fracture toughness of bridge steels were determined by Roberts et.al (1974) and Barsom and Rolfe (1971). Figs. 2.25, 2.26 and 2.27 show the variation of fracture toughness with temperature and loading rate for A36, A572 and 517F steels. There are three types of loading rates that are shown in the figures 2.25 to 2.27. The rates gives the increase in strain a component will undergo per second. There is a marked increase in the fracture toughness of a component with the rate of loading. Investigation has been carried out to find the loading rate of bridges and it has been found that their loading rates are greater than one second and would cause a maximum strain rate of $10^{-3}$ $sec^{-1}$(Barsom 1975). This corresponds to the intermediate rate of loading in the Figs 2.25 to 2.27. The use of the fracture toughness of components

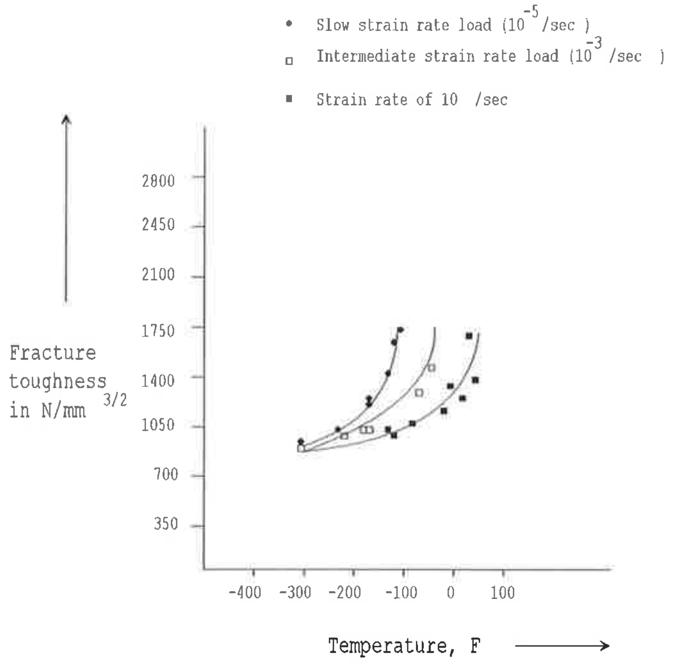



Fig.2.25 Variation of plain strain fracture toughness with temperature and loading rate for A36 steel (Roberts et al 1974)

corresponding to the dynamic rates can be considered to be conservative since they would give lower values of strength. The lowest temperatures to which bridge steels are subjected is around -65 C (Rolfe and Barsom 1977) which could be used to give a conservative estimate of the strength of the component.

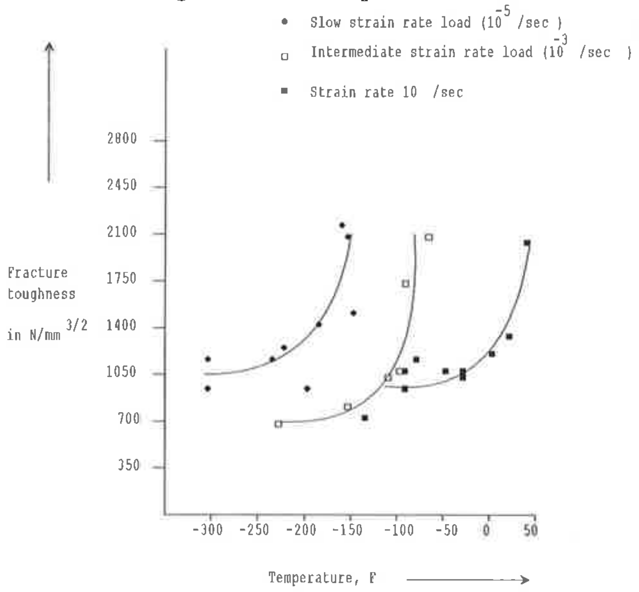



Fig.2.26 Variation of plain strain fracture toughness with temperature and loading rate for A 572 steel (Roberts et al 1974)

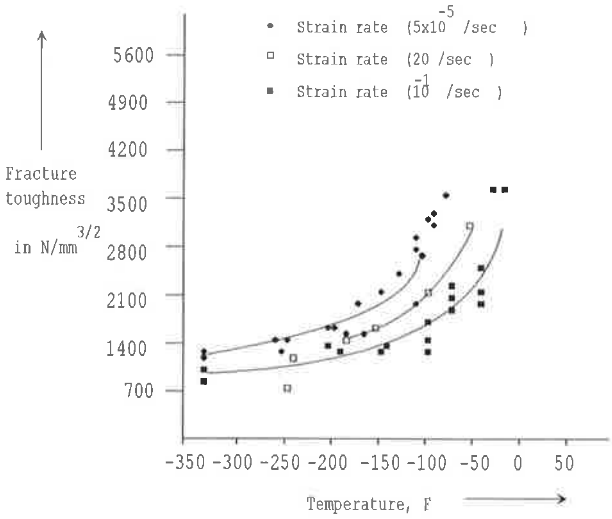



Fig.2.27 Variation of plain strain fracture toughness with temperature and loading rate for 517F steel (Roberts et al 1974)

### 2.6.3 Plastic Deformation Failure

Welded components can also fail by plastic deformation instead of unstable crack propagation failure. Thus a component subjected to tensile loads will first yield so that it enters the plastic stage. If the load is increased then this will cause the component to deform and fail. The most common example of plastic deformation failure of a component is the tensile test.

This test is performed to find the material strength of a component and involves subjecting a standard cylindrical specimen to a uniaxial stress. A diagram of a load verses deflection curve is shown in Fig.2.28. In Fig.2.28, the yield point the deformation is elastic and the deformation beyond the yield point is permanent and is termed as plastic. As the material plastically deforms the maximum load the component can undertake is reached. Due to the straining of the component the cross section of the component is much smaller than the initial cross section at this stage. If the load is kept on increasing a weak section of the material thins out further till the material fails.

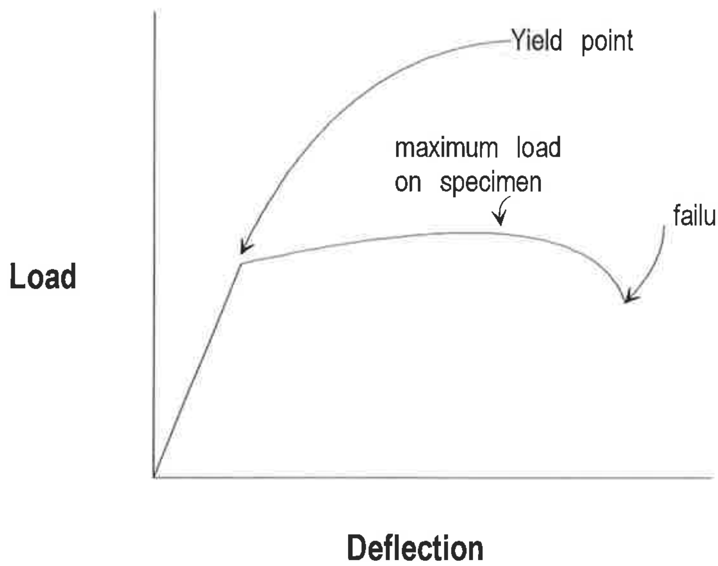



Fig.2.28 Idealised load deflection curve

If the load verses deflection diagram is divided by the initial cross section area of the specimen, a stress strain diagram can be obtained which is similar to the load verses deflection diagram.

The effect of an external load is to increase the length of the component and hence decrease the cross sectional area. This decrease in the cross sectional area remains uniform throughout the length of the component till the maximum load the component can undertake is reached. Any further application of load causes the weakest cross section to reduce in diameter considerably. Thus when failure occurs the cross section area of the component is considerably less than the initial cross section of the component. If the real cross sectional area at each point of the tensile test is noted than the load at each point of the load deflection curve will give the true stress strain variation of the component.

The ultimate tensile strength of a component is obtained by dividing the maximum load a structure can undertake in Fig.2.28 by the initial cross section of the component. The ultimate tensile strength is often used in design. The ultimate tensile strength of A 36 steel and 4340 steel are 412 N/mm$^2$ and 1450 N/mm$^2$ respectively which has been used in this thesis. It has to be noted however that when failure of the tensile specimen occurs the cross section of the component is much smaller than the initial cross section due to plastic deformation and hence the true strength of the material is higher than the ultimate tensile strength.

### 2.6.3.1 Tensile strength of a grooved specimen

Consider a component which apart from being cylindrical in shape also contains a groove (Fig.2.29). Let the cross sectional area of the component along the grooved portion be A. Such a component will be weakest along the cross section containing the groove. If the component is subjected to uniaxial stress the component will fail along the section containing the groove. When a grooved specimen is subjected to tensile stress reduction in the cross sectional area of the grooved specimen is prevented to a large extent by the larger cross sections D just below and above the groove.

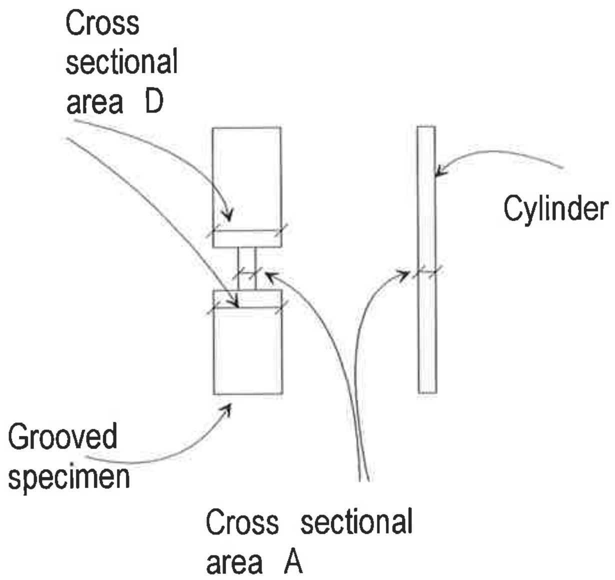



Fig.2.29 Deformation of grooved and non-grooved specimen

Hence when a grooved specimen fails the cross section area of a grooved specimen does not change appreciably. If instead a cylinder with the same cross section area A as a grooved specimen is failed, at the time of failure its cross section area will be considerably reduced. Since the true fracture strength is the same in both cases but the area at the point of failure is larger in the case of a grooved specimen the strength of the component will be larger.

### 2.6.3.2 Determining the strength

The strength of a notched specimen is given either by the net-section method or by dislocation theory.

The net section method:-

The strength of a notched component according to this method is given by this method by multiplying the remaining cross sectional area at the groove by the ultimate tensile strength (Fig.2.29). It is to be noted that if a cylindrical specimen of the same cross sectional area is taken and the cross sectional area is multiplied by the ultimate tensile strength it will give the true strength of the component (Fig.2.29). However the cross sectional area at failure for the cylindrical component is smaller than the cross sectional for the grooved component. Hence while multiplying the remaining cross sectional area by the ultimate tensile strength gives the true strength for the cylindrical component it will give a lower value of strength of a grooved specimen.

Dislocation theory:-

According to the dislocation theory plastic deformation failure occurs due to slip between layers of atoms. Thus atomic layers slip past one another for the component to fail. This slip between layers is caused by the shear stress acting between planes. For a component under uniaxial stress it is known that the maximum shear stress will occur along a plane at 45 degree to the vertical and horizontal. Also for a grooved component failure will occur through the cross section at the tip of a groove. Hence the slip planes has been given as shown in Fig.2.30. For the stress pattern shown it can also be seen that the shear stress along the 45 degree plane is half the uniaxial stress. If the shear stress at failure is k, then the uniaxial external stress is 2k.

Therefore the failure load $P_{mp}$ can be given by multiplying the cross section at the root of the crack tip by the uniaxial stress (McClintock 1961) as follows

$$P_{mp} = 2k\left(1 - \frac{a}{t}\right) \qquad (2.25)$$

where k is the shear stress at which failure occurs, a is the crack length and 't' is

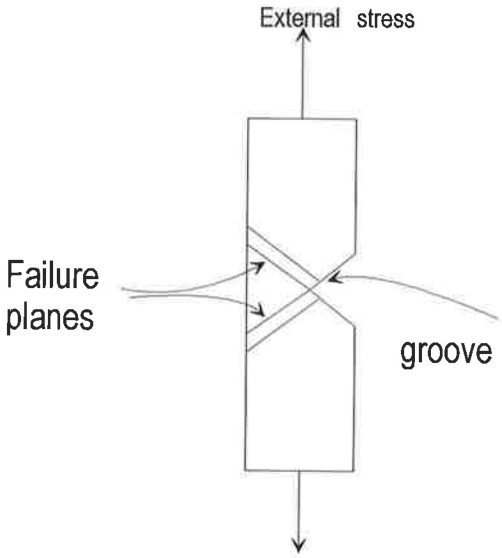



Fig.2.30 Grooved specimen under stress

the thickness. According to Tresca's theory when a component under uniaxial stress fails the shear strength of a component k, is equal to half its tensile strength $f_u/2$. Thus the load carrying capacity of the cracked specimen can be given according to Tresca's theory as

$$P_{mp} = f_u\left(1 - \frac{a}{t}\right) \qquad (2.26)$$

where $f_u$ is the tensile strength of the material. It can be noted that the above equation is the same at the equation provided by the net section theory. However according to Von-

Mises theory failure will occur when the shear strength of the component is equal to $1/\sqrt{3}$ of the tensile strength. The load carrying capacity is therefore given as

$$P = 2\frac{f_u}{\sqrt{3}}\left(1 - \frac{a}{t}\right) \tag{2.27}$$

Von-Mises theory thus gives a strength that is higher than the strength given by net-section theory.

It is to be noted that plastic deformation strength of a component increases with temperature (Hertzberg, 1983) and the effect of rate of loading on the plastic deformation strength is not known. It is assumed in this thesis for simplicity purposes that temperature and rate of loading of the component does not have any effect on the plastic deformation strength.

## 2.7 Fatigue life

Failure of a component subjected to fatigue loads signals the end of the fatigue life of a component. The fatigue life of a component is important for design purposes since we need to know how long a component can be used in service. Experimental tests are conducted to find out the endurance of components subjected to constant amplitude ranges. Thus curves of stress range verses number of cycles are formed to find the endurance of components. Such curves can be used to give the endurance of a component when it is subjected to a constant stress range. When a component is subjected to variable amplitude loading cumulative damage laws such as Miner's law is used to find the life. Miner's law can also be used to find the damage caused to a component. A particular technique of finding the damage caused to a component in a bridge is also discussed.

### 2.7.1 S-N Curves

The fatigue life of a component subjected to a given constant range of stress is obtained from S-N curves. The S-N curve was first used for design by Wohler between the period 1852 and 1870 for the German railway industry. These curves are still used by major codes in order to determine the fatigue life of components. In order to determine a relationship for a particular type of load between nominal stress range and number of cycles to failure a series of individual specimens were tested in the laboratory. The results obtained were generally scattered and used to produce the traditional S-N curve (Fig. 2.31).

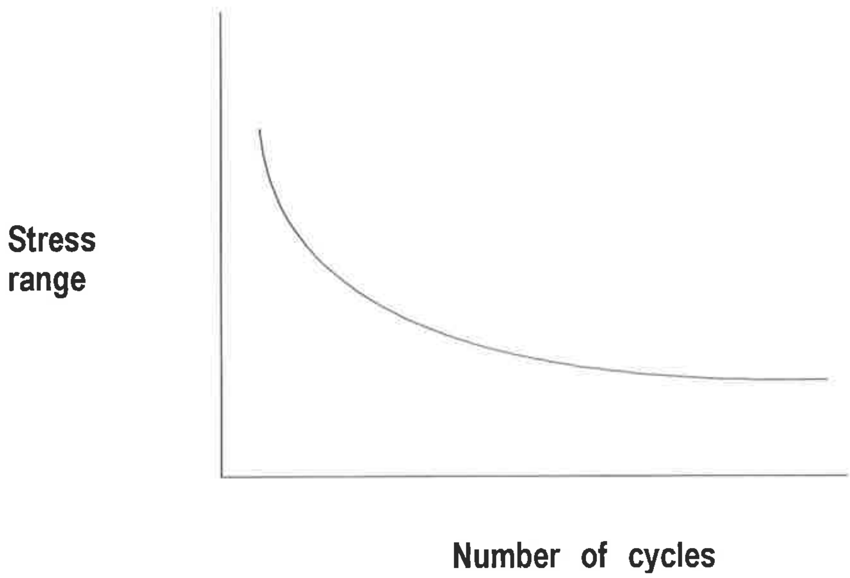



Fig.2.31 Typical S-N curve

When logarithmic scales were used the results fell in a straight line (Fig.2.32). If the slope of the linear curve of log of the stress range and log of the number of cycles is denoted as $m'$ and the intercept as $C'$, then the relation between log $N_{en}$, the log of the number of cycles at which failure occurs and Log $\Delta\sigma$, the log of the stress range applied can be given as

$$\log N_{en} + m' \log \Delta\sigma = C' \tag{2.28}$$

If we remove the log scale from both sides in the above Eqn.2.28 we get the equation

$$N_{en} \times \Delta\sigma^{m'} = C' \tag{2.29}$$

The slope of the logarithmic curve in Fig.2.32, $m'$ is found to be constant for a given class of component. Even though the value of $m'$ remains the same for a given component the value of $C'$ will vary depending on the probability of failure for which the component is drawn and also on the thickness of the component. The value of $C'$ is given for 50% failure of a component 22 mm thick by the constant $K_o$ in Table 2.1. If the probability of failure is different from 50% then the value of $C'$ is given for a component 22 mm thick by multiplying the constant $K_o$ by $\Delta^d$ where constant $\Delta$ is given by Table 2.1 and constant d is given by Table 2.2. For components whose thickness is different from 22mm it has been found that the value of $C'$ for a thickness t is given as $\left(\frac{22}{t}\right)^{0.25}$ times the value of $C'$ of a 22 mm plate. The value of $C'$ for all probabilty of failure and thicknesses can therefore be given as $K_o\Delta^d\left(\frac{22}{t}\right)^{0.25}$. Eqn.2.27 can therefore be written as

$$N_{en} \times \Delta\sigma^{m'} = K_o \times \Delta^d \times \left(\frac{22}{t}\right)^{0.25} \tag{2.30}$$

It is to be noted that $N_{en}$ in the above equation represents the number of cycles to failure of a component at maximum load applied which is also the endurance of the component.

Table2.1 Mean line $\Delta\sigma$-$N_{en}$ relationships (BS 5400, 1980)

| Detail class | $K_o$ | $\Delta$ | m |
|---|---|---|---|
| W | $0.37\text{x}10^{12}$ | 0.654 | 3.0 |
| G | $0.57\text{x}10^{12}$ | 0.662 | 3.0 |
| F2 | $1.23\text{x}10^{12}$ | 0.592 | 3.0 |
| F | $1.73\text{x}10^{12}$ | 0.605 | 3.0 |
| E | $3.29\text{x}10^{12}$ | 0.561 | 3.0 |
| D | $3.99\text{x}10^{12}$ | 0.617 | 3.0 |
| C | $1.08\text{x}10^{14}$ | 0.625 | 3.5 |
| B | $2.34\text{x}10^{15}$ | 0.657 | 4.0 |
| S | $2.13\text{x}10^{23}$ | 0.313 | 8.0 |

Table2.2 Probability factors (BS 5400, 1980)

| Probability of failure | d |
|---|---|
| 50% | 0 |
| 31% | 0.5 |
| 16% | 1.0 |
| 2.3% | 2.0 |
| 0.14% | 3.0 |

### 2.7.2 Cumulative Damage Law

The stress range verses life (S/N) curves predict the fatigue life of a component only when it is subjected to a single stress range. However in practise a given component will be subjected to a large number of stress ranges. We therefore need to find the cumulative damage caused to a component when subjected to a large number of stress ranges. There are about 15 different methods of finding the cumulative damage to which a component is subjected. The simplest one is Miner's law which is used by most codes. In

1945 Miner suggested that progress with methods of fatigue analysis has been slow because there was no method of handling any but the simplest problem where the component was subjected to a single stress range.

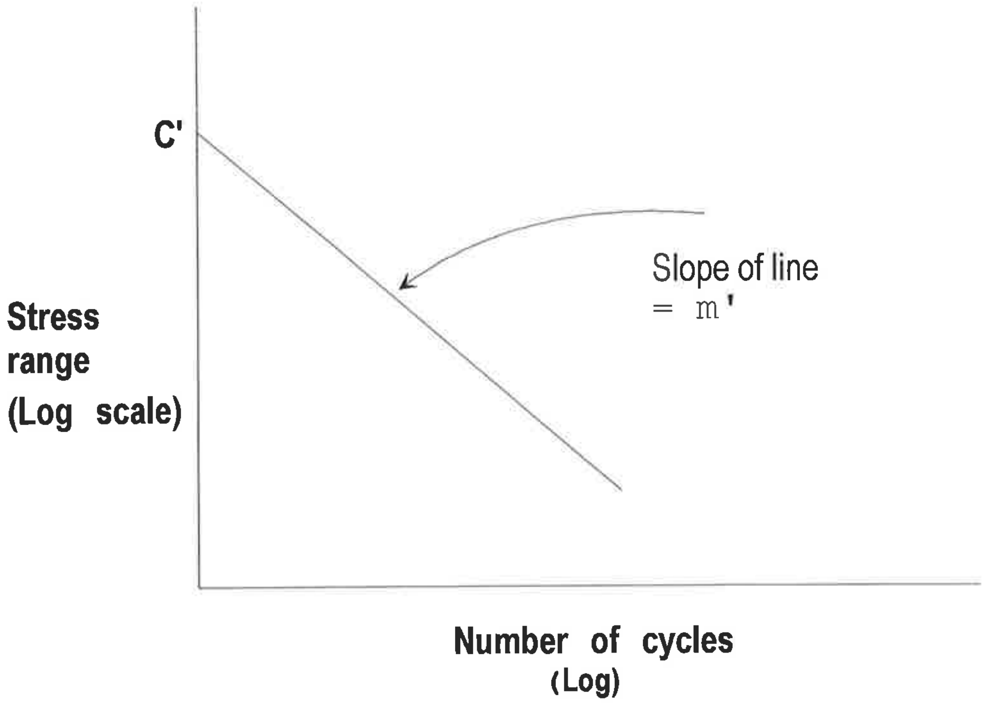



Fig.2.32 S-N curve in Log-Log scale

Miner formulated an expression that gives the cumulative damage that a component undergoes when subjected to a number of stress ranges. The expression was formulated based on the work to which a component has been subjected when under repeated loading. According to Miner the work done is proportional to the number of cycles when the component was subjected to a given stress range.

Let us consider that a component is subjected to a given stress range $\Delta\sigma_1$ for which the endurance, the number of cycles to failure, is $E_1$ and the work done after application of $N_1$ number of cycles is $w_1$. For another stress range $\Delta\sigma_2$, let the endurance be $E_2$ and the work done after application of $N_2$ number of cycles is $w_2$. Then since the work done is proportional to the number of cycles applied

$$\frac{w_1}{W} = \frac{N_1}{E_1} \text{ and } \frac{w_2}{W} = \frac{N_2}{E_2} \tag{2.31}$$

where W is the work required to cause failure.

If we consider the component to fail after application of n different stress ranges then

since

$$w_1 + w_2 + w_3 + \ldots + w_n = W \tag{2.32}$$

then dividing Eqn.2.32 by W we get

$$\frac{w_1}{W} + \frac{w_2}{W} + \frac{w_3}{W} + \ldots + \frac{w_n}{W} = 1 \tag{2.33}$$

then substituting from Eqn.2.31 we get

$$\frac{N_1}{E_1} + \frac{N_2}{E_2} + \frac{N_3}{E_3} + \ldots + \frac{N_n}{E_n} = 1 \tag{2.34}$$

or

$$\sum_{i=1}^{i=n} \frac{N_i}{E_i} = 1 \tag{2.35}$$

Consider a component which has not failed but has been subjected to X stress ranges. The work done to the component is $w_1 + w_2 + ... + w_x$. If W is the total work required to cause failure then the damage A to the component can be given as

$$(w_1 + w_2 + ... + w_x)/W = A \qquad (2.36)$$

Substituting Eqn.2.31 we get

$$\frac{N_1}{E_1} + \frac{N_2}{E_2} + ... + \frac{N_x}{E_x} = A \qquad (2.37)$$

or

$$\sum_{i=1}^{i=x} \frac{N_i}{E_i} = A \qquad (2.38)$$

### 2.7.3 Damage using a Load Model

Since there are a large number of stress ranges acting on a component, finding damage by considering each stress range one by one, can be extremely cumbersome. An easy solution needs to be determined where each individual stress range acting on the design point need not be considered one by one.

A model has been developed for finding the fatigue damage of a component in a bridge by categorising stresses acting on the design point in a bridge (Johnson, 1994 and Oehlers, 1992). The stresses which cause fatigue damage at the design point in a bridge is due to the loads acting when a vehicle traverses the bridge. Let us consider that the stresses acting at the design point due to the passage of a known weight 'O' is found out. If a vehicle with a weight $w_a$ times that of a vehicle with known weight 'O' crosses the bridge the magnitude of stresses acting at the design point will be $w_a$ times that caused by the weight 'O' assuming that vehicles having varying axle configurations will cause the same stress at the design point as long its weight remains the same. Thus if we know the stresses acting at the design point due to the passage of a standard vehicle of known weight we can find the stresses acting at the design point due to all other vehicles whose weight we know as a ratio of the standard fatigue vehicle.

As the stresses acting at the design point due to the passage of a standard fatigue vehicle is known the damage can be calculated for a standard fatigue vehicle. Given the ratio of weight of any vehicle compared to a standard fatigue vehicle the stress caused by this vehicle and hence the damage caused by this vehicle can be found. The procedure developed finds the fatigue damage due to a single vehicle termed as a standard fatigue vehicle (SFV). Knowing the weights of other vehicles and their number of applications, the fatigue damage of these vehicles can be calculated from the damage calculated due to a standard fatigue vehicle.

As can be seen from Eqn.2.30 the number of cycles to failure depends on the inverse of the stress range to the power $m'$. From Eqn. 2.37 the damage also depends on the number of cycles of stress range applied. Oehlers (1992) determined a damage term F for a standard fatigue vehicle traversing the bridge as

$$F = \sum_{k=1}^{k=z} \left( N_k (\Delta\sigma)_k^{m'} \right) \tag{2.39}$$

where N is the frequency of occurrence of a stress range $\Delta\sigma$ at the design point when a SFV crosses the bridge (Table.2.3). A particular range is denoted as k and the design point is considered to be subjected to 'z' types of ranges. A component in a bridge will be subjected to different vehicles both lighter and heavier compared to the standard fatigue vehicle. The average damage caused by these vehicles as a proportion of the damage caused by a standard fatigue vehicle has been given by the parameter L as (Johnson 1957)

$$L = \sum_{x=1}^{x=i} \left( B_x W_x^{m'} \right). \tag{2.40}$$

Table 2.3 Force spectrum

| Level (k) (1) | Range ($\Delta\sigma$) (2) | Frequency (N) (3) | N $(\Delta\sigma)^m$ (4) |
|---|---|---|---|
| 1 | $\Delta\sigma_1$ | $N_1$ | $N_1\Delta\sigma_1{}^m$ |
| 2 | $\Delta\sigma_2$ | $N_2$ | $N_2\Delta\sigma_2{}^m$ |
| | | | |
| Z-1 | $\Delta\sigma_{Z-1}$ | $N_{Z-1}$ | $N_{Z-1}\Delta\sigma_{Z-1}{}^m$ |
| Z | $\Delta\sigma_Z$ | $N_Z$ | $N_Z\Delta\sigma_Z{}^m$ |
| | | | $F = \sum N(\Delta\sigma)^m$ |

Table 2.4 Load spectrum

| Level (x) (1) | Weight (W) (2) | Probability (B) (3) | B $W^m$ (4) |
|---|---|---|---|
| 1 | $W_1$ | $B_1$ | $B_1W_1{}^m$ |
| 2 | $W_2$ | $B_2$ | $B_2W_2{}^m$ |
| | | | |
| i-1 | $W_{i-1}$ | $B_{i-1}$ | $B_{i-1}W_{i-1}{}^m$ |
| i | $W_i$ | $B_i$ | $B_iW_i{}^m$ |
| | | $\sum = 1$ | $L = \sum BW^m$ |

where W gives the ratio of weights of different vehicles with respect to the weight of a SFV and B gives the probability of occurrence of a vehicle with a weight ratio W (Table 2.4). A particular type of vehicle is denoted by x and the design point is considered to be subjected to 'i' different weights of vehicles. Thus the damage caused by the traversal of

T standard fatigue vehicles through the bridge is given as TF. The damage caused by T traversals of a spectrum of vehicles is given as TFL.

The load model proposed by Johnson and Oehlers is the simplest available. A more complicated load model proposed by Moses (1990) considers the effect of ten different random variables such as impact of loads, multiple presence of vehicles etc. The load model proposed by Oehlers and Johnson has been used in the thesis because of its simplicity. This load model has been linked to the residual strength curve as discussed in Section 2.9.3.

## 2.8 Residual strength

In the section above, either the endurance of a component was determined or the fatigue damage was calculated which gave the proportion of the fatigue life expended. These methods of fatigue analysis do not give the variation of strength of a component during its fatigue life. A discussion is carried out of the various efforts made by researchers to find the variation of residual strength of different components. They include experimental derivation of the residual strength of welded stud shear connectors (Oehlers 1990), thin centre cracked panels (Feddersen 1971) and incomplete penetration butt welds (Cox 1987).

### 2.8.1 Failure Envelope for Stud Shear Connectors

Stud shear connectors are welded steel protrusions on the steel flange of a composite beam of steel and concrete. Stud shear connectors are therefore a particular type of welded connection. The failure envelope of stud shear connectors, was obtained experimentally by Oehlers (1990). The results as shown in Fig.2.33 were obtained for constant amplitude loading. Three different series of tests termed as the S, F and M series were conducted on identical specimens. In all the fatigue tests a constant range of cyclic loads of 0.25 $P_s$ were applied. The specimens in series S were tested to determine the static strength $P_s$ of the stud shear connectors. In series F, cyclic loads were continually applied until the stud shear connector failed, the peak of the stress range was varied but the magnitude of the range was kept constant. In series M, blocks of cyclic loads were applied following which the component was failed. It was found that the residual strength envelope of the three series of tests can be given in the form of a straight line.

#### 2.8.1.1 Asymptotic endurance

Stud shear connectors subjected to fatigue loading will fail when its strength is equal to the maximum load applied. However if it is assumed that the component does not fail at the given load it will continue to lose strength due to crack propagation until its strength reduces to zero. This point ($N_f$) has been shown in the Fig 2.33 by extrapolating the linear curve until it touches the abscissae. The point has been termed the asymptotic endurance (Oehlers 1990) since it a theoretical endurance that can never be achieved in practise as the component will always fail at the peak of the load range applied. However the asymptotic endurance has been found to be extremely useful since a linear failure envelope can be defined in terms of the static strength and the asymptotic endurance (Oehlers 1990).

The asymptotic endurance for stud shear connectors was found for different stress ranges by Oehlers in 1990. Hence a relation between the stress ranges and the asymptotic endurances was determined.

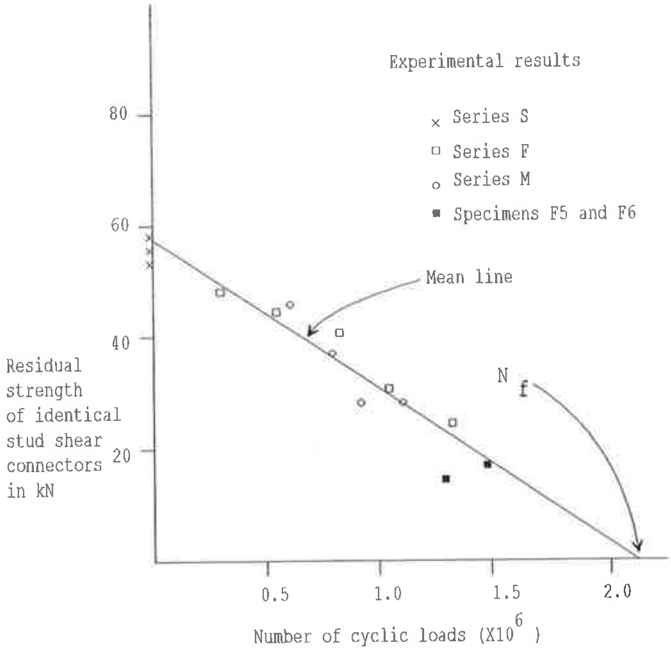



Fig.2.33 Experimentally obtained residual strength variation of stud shear connectors (Oehlers, 1990)

### 2.8.2 Feddersen's Envelope

Feddersen (1971) found the variation of residual strength against crack length for tension panels containing central cracks. He carried out experimental tests on centrally cracked panels and analysed the data. When such data was plotted with residual strength on the Y-axis and crack length on the X-axis it was found that the data behaved in a linear fashion both when the crack length was small or large and formed a saddle shaped central portion.(Fig.2.34).

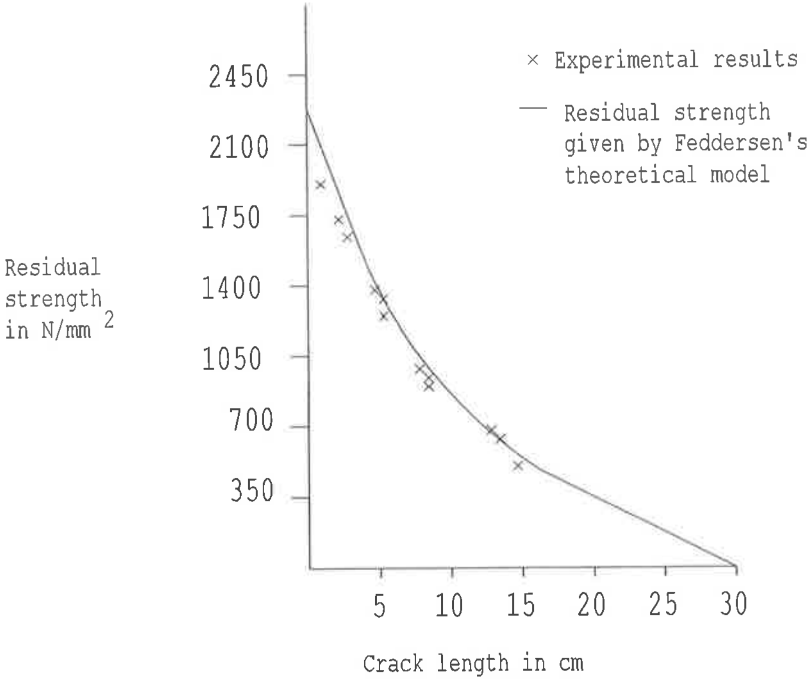



Fig.2.34 Comparison of experimental results with theoretical model (Feddersen, 1971) for a 0.15 mm thick, 30 cm wide, 2014-T6 alloy sheet at -195 C.

Feddersen tried to match experimental results obtained with a theoretical model. If the centre cracked tension panel failed by unstable crack propagation then the strength of the centre cracked panel would be given by Eqn.2.6 assuming the panel is of infinite width.

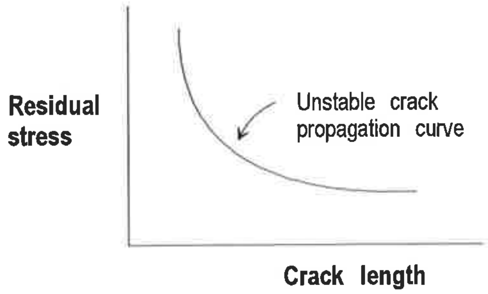



Fig.2.35 Variation of residual strength with crack length of a component failing by unstable crack propagation

If the panel failed according to Eqn.2.6 then the relation between strength of a component and crack length when plotted would be given by Fig.2.35. It is to be noted that the shape of the residual strength curve against crack length is in the form of a saddle in Fig.2.35. It was earlier discussed that the experimental results for intermediate cracklengths are in the form of a saddle. Thus a theoretical model which assumes that the component fails by unstable crack propagation matches experimental results for components having intermediate crack lengths. In order to develop a theoretical model which matches the experimental results at all crack lengths, Feddersen dropped tangents to the unstable crack propagation curve as shown in Fig. 2.36. The tangents were dropped from the point where the component has maximum strength and the crack length is zero and from the point where the crack length is maximum and the strength is zero. This two tangents along with the portion AB of the unstable crack propagation curve in Fig.2.36 gives a theoretical curve which matches the experimental data obtained by Feddersen. This is shown by Fig.2.34 where the experimental results are found to be close to the theoretical model. Apart from developing a model which closely matches the experimental results Feddersen also tried to analyse the reasons for the linear behavior of the experimental

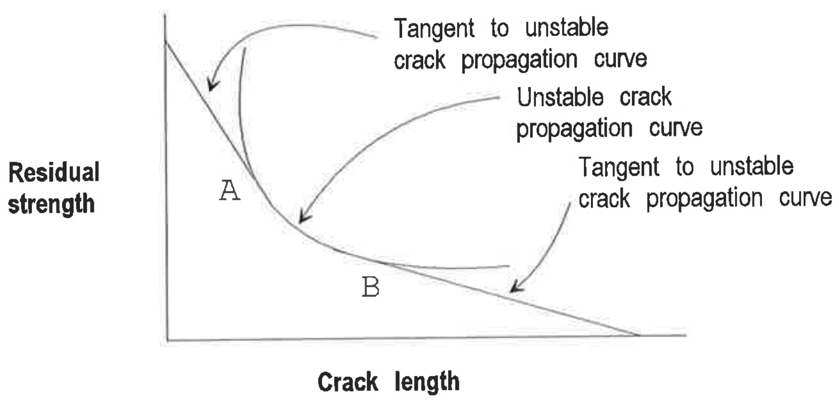



Fig.2.36 Determining the residual strength variation of tension cracked panels theoretically

data when the crack lengths are large or small. According to Feddersen the linear behavior shown by components at small crack lengths is due to plasticity while the linear behaviour shown by the component at large crack lengths is due to finite width effects.

### 2.8.3 Cox's Envelope

Cox (1987) similarly performed experiments on the incomplete penetration of butt welded joints (Fig.2.37). Thus components having different crack lengths were taken and their residual strengths determined by failing the components. Some of the cracks used in his tests were obtained by applying fatigue loads. A comparison of theoretical results with experimental ones showed that all components failed due to plastic deformation failure. Cox argues that often components are checked to make sure that they do not fail by unstable crack propagation but are rarely checked for plastic deformation failure. However as shown by his experiments welded components are quite likely to fail by plastic deformation. Cox provided a theoretical model which closely matches the experimental data. The model considers that when the crack length is small the presence of cracks can be neglected and the residual strength can be given by the plastic deformation strength of the weakest section of the base metal. However when the crack lengths are large a section through the weld metal can be used to determine the plastic deformation strength of the component and the presence of a crack needs to be considered. In this thesis the unstable crack propagation strength and plastic deformation strength of welded components have been compared to find the failure criteria.

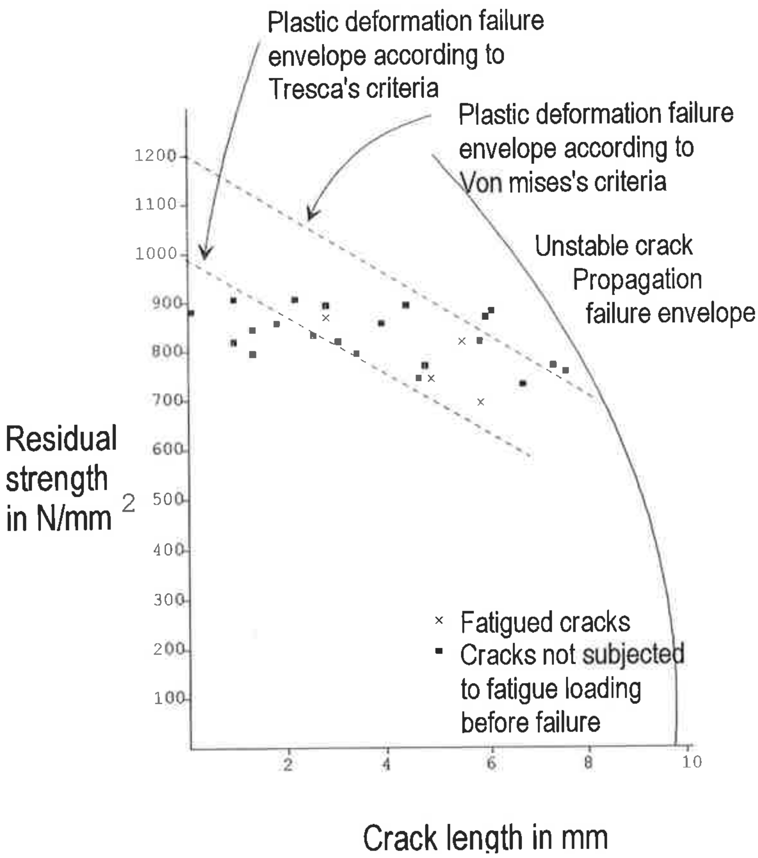



Fig.2.37 Figure showing residual strength variation of incomplete joint penetration butt weld (Cox, 1987)

## 2.9 Design Procedure

The stresses in a cracked structure, the initiation and propagation of a crack, their endurance and residual strength are all used to design welded components. The design procedures used at present can be broadly divided into the S-N curve method, the fracture mechanics method and the residual strength method. A basic knowledge of S-N curves, fracture mechanics and residual strength has been provided in earlier section. Here design approaches based on these methods have been described briefly.

### 2.9.1 S-N Curve Design

Design, using S-N curves is based on a two pronged approach. First a static design is done which gives the strength of the component before the application of fatigue loads. Next a fatigue design is carried out to give the fatigue life of the component, which is the information on the end of the fatigue life (Oehlers 1992). It is assumed that the strength of the component does not change during the fatigue life and remains equal to the original static strength until the component suddenly fails at the end of the fatigue life. Hence the failure envelope used for design using S-N curves is rectangular in shape as shown in Fig.2.38.

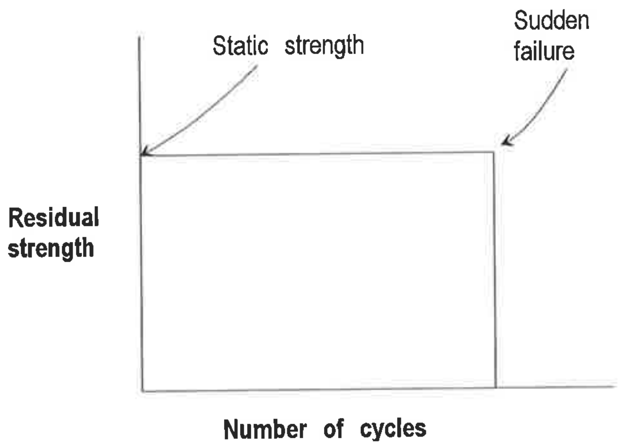



Fig.2.38 Present design techniques using S-N curve (Oehlers, 1990)

Standard codes such as the British standard BS 5400, Eurocode 3 and the AASHTO code uses S-N curves to design for fatigue (Fig.2.39). For the purpose of fatigue assessment the codes classify all constructional detail into several classes. The British code classifies all components into 9 groups S, B, C, D, E, F, F2, G & W (Table 2.1). The group most resistant to fatigue is S, while W is the group weakest under fatigue. For each group a S-N relation is given (Eqn.2.30) which can be used to find the fatigue life of the component provided we know the stress range applied.

Thus in order to design a component the class of the component is first chosen. Next, the fatigue life of the component is found from S/N curves. In case of components subjected to a number of stress ranges Palmgren-Miners summation is used (Eqn.2.38).

### 2.9.2 Use of Fracture Mechanics for Design

Design of components at present is nearly always carried out using S/N curves. However components can also be designed using fracture mechanics as has been suggested by researchers (Zhao Halder and Breen 1994). Techniques of designing components using fracture mechanics concentrate on the endurance limit of the component as in the case of S-N curves (Fuchs and Stephens 1980, Skorupa, 1992). Hence static design is still carried out independently of the fatigue design.

The total fatigue life of a component is divided into two parts, crack initiation and crack propagation. There are three general theories by which the fatigue life of components are predicted using fracture mechanics (Fuchs and Stephens 1980, Skorupa 1992). If the crack initiation life of the component is considered to form the larger part of the fatigue life, then fatigue life predictions are solely based on the crack initiation life. If the crack propagation life of the component is considered to occupy most of the fatigue life then fatigue life predictions are solely based on the crack propagation life. If the crack initiation and propagation life are both considered to form a important part of the fatigue life then fatigue life predictions take both into account. Since crack propagation forms most of the life of welds (Maddox 1991) the crack propagation life is widely used for the fatigue life prediction of a weld. In order to make a prediction

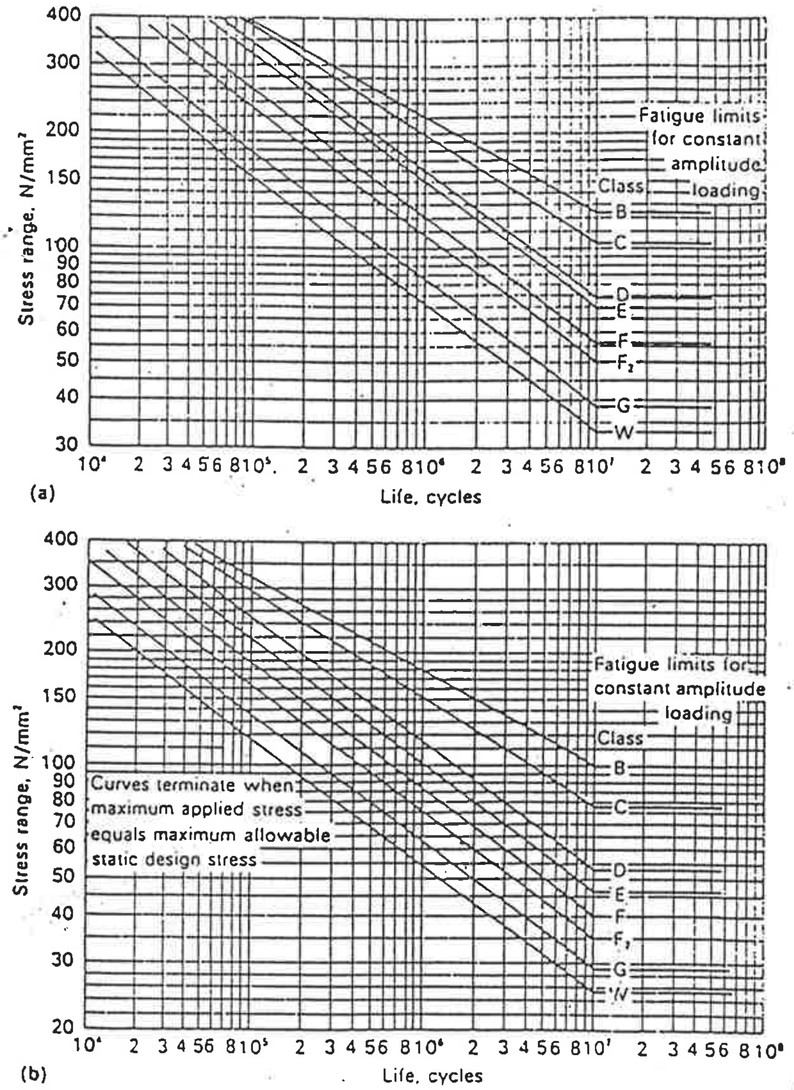



Fig.2.39 S-N curves used for design: (a) Mean lines (50% probability of failure); (b) Lines for 2.3% probability of failure (Maddox, 1991)

about the fatigue life of the weld using fracture mechanics the initial crack length of the component must be known. The value of the initial crack lengths has been often taken as 0.25 mm for fatigue life calculations by researchers (Section 2.4). The size and shape of acceptable initial flaws in welded components documented by BS PD 6493 (1991) have been determined using S/N curves. It has been suggested by Yazdani and Albrecht (1987) and Fisher et al (1970) that the initial crack length can be determined from S/N curves. A discussion on the calculation of the initial crack to be used for design has been carried out in this thesis.

### 2.9.3 Residual Strength as a Design approach

Design procedures using S-N curves and fracture mechanics as used at present do not consider any variation in the residual strength of a component. The strength of the component is considered to remain constant at the static strength and reduce suddenly at the end of the fatigue life (Oehlers, 1992). However, if the actual variation of the residual strength of a component with number of cycles is known, both the remaining strength and remaining life of the component can be determined at any period of the fatigue life. The use of residual strength curves for design has been suggested by Oehlers (1992) and Sedlacek (1990). Oehlers (1992) has proposed a general residual strength of three stages for a component subjected to a constant stress range (Fig.2.40). The component does not change strength during the first stage, reduces linearly in strength during the second stage and fails catastrophically during the third stage. The general residual strength variation has been linked by Oehlers (1992) to the load model for a bridge earlier described in Section 2.7.3. For a component in a bridge following

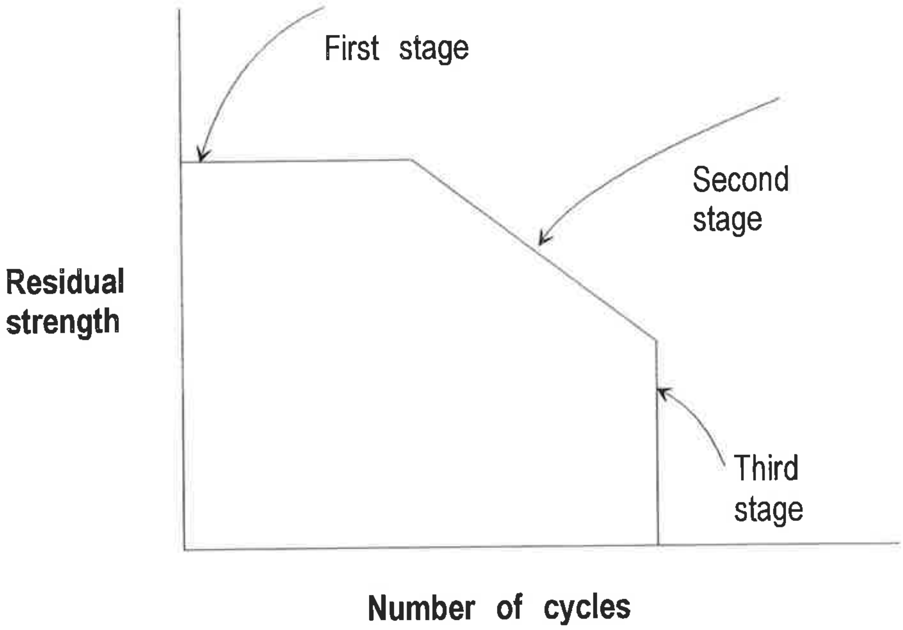



Fig.2.40 General residual strength variation (Oehlers, 1992)

the proposed general residual strength curve a relation has been established between the number of vehicles traversing the bridge and its residual strength. Thus if a component in a bridge follows Oehlers general residual strength curve then the component could be designed knowing the vehicles traversing the bridge.

### 2.9.3.1 Design approach used for stud shear connectors

The residual strength approach of fatigue analysis has been used to design stud shear connectors (Oehlers 1994). As earlier mentioned the variation of residual strength with life for stud shear connectors is linear for a given stress range. Thus stud shear connectors can be considered to be a special case of the general residual strength curve. The linear variation in strength of a stud shear connector could be expressed in terms of the static strength $P_s$ and asymptotic endurance $N_f$ (Fig.2.41) as

$$\frac{N_e}{N_f} = 1 - \frac{P_m}{P_s} \tag{2.41}$$

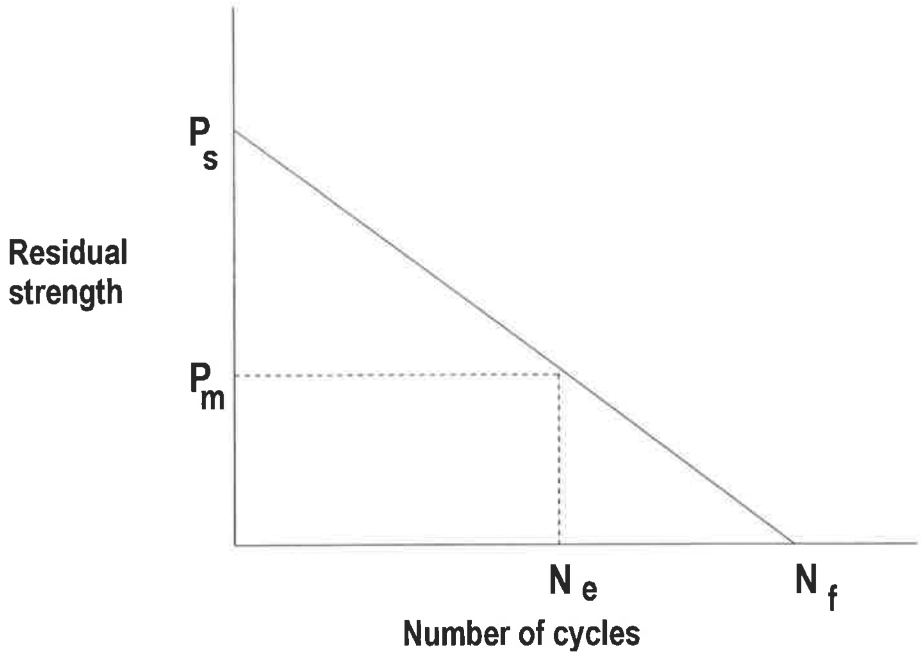



Fig.2.41 Schematic diagram showing variation of residual strength

where $P_m$ is the strength of the component after application of $N_e$ number of cycles.

Equation 2.41 can be used for design by considering $P_m$ as the strength of the component required to resist the maximum overload at the end of the fatigue life. The variation of the asymptotic endurance with different constant amplitude stress ranges has been found by Oehlers (1990) as described in Section 2.8.1.1. Thus given the stress range for which the component needs to be designed the asymptotic endurance $N_f$ can be determined. Hence considering the component is designed for N number of cycles the strength required at the beginning of the fatigue life or the static strength $P_s$ can be calculated.

The method of design and assessment of a stud shear connector explained for a single stress range above was extended (Oehlers 1995) to develop a method of design and assessment for the load model discussed in Section 2.7.3. Hence if the number of vehicles that will pass through the bridge during its fatigue life can be predicted design of a stud shear connector based on its residual strength variation can be carried out. A design method has been developed in the thesis for welded components using the above principle of designing stud shear connectors.

## 2.10 Assessment

Welded components are designed for a given life and strength. In order to find the condition of the component at any intermediate period of its life the component must be assessed. Assessment of components are carried out by using Miner's law, by inspection for cracks or using residual strength curves. The most preliminary form of assessment is carried out using Miner's law which give only the portion of the fatigue life used. Assessment using residual strength curves give the strength and life of a component expended. The most thorough form of assessment involves inspection of cracks. In this method on-site checks are carried out to detect cracks and advanced fracture mechanics techniques are carried out to find their remaining life.

Most of the infrastructure around the world is getting old and need to be repaired. Bridges are a typical example. Most of the bridges in use today were built after the second world war and have past a large portion of their fatigue lives. There are 578,000 bridges in the U.S.A. of which 40% require repair. Furthermore about 11,000 bridges are being added to this list each year (Zhao, Halder and Breen, 1994). The total cost of repairing bridges in the U.S.A. is estimated to cost 92 billion dollars (Ahlskog, 1990). In order to repair these bridges they need to be assessed first. Assessment therefore has assumed an important role today.

### 2.10.1 Assessment using Miner's Law

This form of assessment is used by standard codes such as the BS 5400. Thus if the stress ranges acting on a component are known, their corresponding endurance can be found from Miner's law. If we know the number of cycles of these stress range applied, the fatigue damage can be found from Palmgren-Miners's summation. This has been discussed in Section 2.7.2. The fatigue damage term A in Eqn.2.38 gives the portion of the fatigue life being used. This form of assessment thus gives a preliminary idea of the condition of the structure.

### 2.10.2 Assessment using Residual Strength Curves

If the variation of residual strength with number of cycles applied to a component is known, the residual strength and life of the component can be derived after application of any given number of cycles. Furthermore assessment procedures can be provided for any given load model.

#### 2.10.2.1 Assessment for stud shear connectors

Equation 2.40, that is used for designing stud shear connector in Section 2.9.3, can be interpreted in the following way in order to assess components (Oehlers 1994); Hence if the static strength $P_s$ and the asymptotic endurance $N_f$ is known, the strength of the component $P_m$ after application of N number of cycles can be determined. This procedure, of determining the variation in strength of a stud shear connector, has been linked to the load model in Section 2.7.2. Thus if the number of vehicles that has traversed the bridge is known at any time the reduction in strength can be determined. The method of assessing stud shear connectors by using residual strength curves has been included in the assessment version of BS 5400 (1980). The method of assessing welded components developed in the thesis uses the same principles as that of a stud shear connector.

### 2.10.3 Assessment by Inspection

A common form of assessment is to carry out inspection for cracks. Inspection manuals are provided by organisations such as the American Association of State Highway Officials describing the methods and devices used for inspection (Maintenance manual AASHTO, 1983). Therefore using different inspection methods cracks in a component are found. Such inspections are carried out after fixed time interval which is often about two years. Apart from looking for cracks the stresses acting on the structure are also found out. If the crack length of a component is known and the stresses acting on a component is known then the remaining life of a component can be determined using fracture mechanics (Rolfe and Barsom 1977). Assessment codes such as the BS PD 6493 (1991) provides the material properties and equations to be used for complex loadings so that it is possible to predict the remaining life of components.

Present research on assessment using inspection are based on improving the existing methods. Therefore research concentrates either on the methods or devices used for inspection (Fisher et al 1990), the methods and devices to find the stresses in the structure (Yamada, 1990) and accurate prediction of crack propagation of components (Dikstra et al 1990). Experimental research is carried out to find the rate at which crack propagates (Skallerud et al 1990 and Knesl et al 1990) through different components and theoretical laws are proposed to match the experimental data (Castiglioni, 1990).

Assessment using inspection gives an advanced idea of the condition of the structure but requires on-site checks and fracture mechanics calculations. On the other hand the residual strength technique is a much simpler theoretical technique. Using the residual strength approach the residual strength of a component can be determined at any period of its life. From the residual strength determined the crack length is calculated at any intermediate period of its life. Knowing the crack length the remaining life can be calculated. Though the residual strength approach does not give as good an idea of the condition of a structure as the inspection method it still gives a much better understanding of the condition of the structure compared to the S/N curve method. Also the residual strength approach can be used to complement inspection ie. the results of the residual strength analysis can be used to find out when inspection needs to be carried out.

## 2.11 Conclusions

The literature review discusses the stages in the fatigue life of a welded component and various methods of designing and assessing it. The material properties and existing laws used in the thesis and the reasons for using them are discussed. Thus in this thesis the approach adopted assumes that a welded component contains an initial crack; components analysed are considered to be in a condition of plane strain; Paris' crack propagation law is used and the types of failure considered is crack propagation failure and plastic deformation failure.

A residual strength approach for fatigue design and analysis has earlier been developed for stud shear connectors by Oehlers. The residual strength variation was found to be linear and hence the residual strength variation could be given in terms of the static strength and asymptotic endurance of components. Thus the stud shear connector was assessed and designed using its residual strength variation. Assessment of stud shear connectors using residual strength curve has been adapted by the assessment version of the BS 5400. The residual strength variation has been adopted for a bridge load model so that the variation in strength of the component can be given in terms of vehicle traversals.

The method of design and assessment of stud shear connectors has been developed using the residual strength technique. Stud shear connectors as earlier discussed are a particular type of welded component. In this thesis the technique has been expanded to cover all welded components. While for stud shear connectors the residual strength was found from experimental research, here the residual strength variation has been determined using basic fracture mechanics and S/N curve analysis. Since the technique presented in this thesis covers all welded components it can be considered to be a general technique of designing and assessing welded components.

Welded components in common practise is designed using S/N curves. The S/N curve technique as discussed earlier considers that the strength of the component remains equal to the static strength till the component fails. The residual strength approach of designing component developed in this thesis bases the design on the actual variation of residual strength and therefore is more accurate than the S/N curve technique.

The residual strength technique of assessing welded components is simpler compared to the inspection method but is complicated compared to the S/N curve method. The S/N curve method gives the fatigue life of a component expended while the residual strength technique gives the residual strength at any intermediate period of the fatigue life of a component, the crack length and the remaining life. Not only it gives more information about the condition of a component compared to the S/N curve method but also is more accurate. It can be used to complement the inspection method since it can be used to predict when inspection needs to be carried out.

# Chapter 3
# Residual strength of fundamental components

---

## 3.1 Introduction

When fatigue loading is applied to a component containing a crack it causes the crack to increase in size (Section 2.5). The application of fatigue loading also causes the residual strength of a component to change and the remaining life to decrease (Section 2.8). The change in both the residual strength and the remaining life of a component is related to the growth of the crack. For a given component the larger the crack size, the residual strength is lower and the remaining life is smaller.

Hence the fundamental effect of fatigue loading is the growth of a crack. It is only because fatigue loading causes the crack size to increase that there are secondary effects, such as the change in residual strength and remaining life. The first step in a fatigue analysis therefore must be to find the growth of a crack during the fatigue life. If the crack size is known at any intermediate stage then the residual strength and remaining life can be calculated.

The remaining life of a component depends on the residual strength because the residual life of a component is the number of cycles required for the residual strength of a component to become equal to the maximum load applied when the component fails. Therefore a curve showing the variation of the residual strength with number of cycles can be used to find the remaining life of the component. In conclusion, the fundamental effect of fatigue loading can be considered to be the growth of a crack. The growth of a crack leads to the variation in residual strength while the reduction in strength is responsible for the change in the remaining life.

Even though crack growth is the fundamental aspect of fatigue loading, a designer will not be directly interested in the crack length of a component at any intermediate stage. When a component is designed or analysed, one really wants to understand the performance of the component from the point of view of external loads that are applied. Therefore, the designer wants to know what residual strength the component has at any period so that one can specify what maximum load the component can withstand before it fails and also how long the component will last. A residual strength curve is extremely helpful to the designer in this respect. It provides the maximum strength and the maximum life the component can endure before it fails. It thus gives us the maximum load that can be applied and the maximum number of loads that can be applied for a given overload.

The variation of residual strength of a component with fatigue loading has been determined here using fracture mechanics. In this chapter the variation of residual strength has been found for a component subjected to constant amplitude loading and failing by unstable crack propagation failure. The residual strength has been found for an infinitely wide plate with a centre crack (Section 2.3.3.1). Then the residual strength variation has been determined for an idealised component having a constant value of the magnification factor M in Eqn.2.7 that does not change with a variation in crack length (Section 2.3.3.2).

## 3.2 Residual Strength from Fracture Mechanics

In the literature review (Section 2.3), it was discussed how fracture mechanics can be used to derive the internal stresses in a cracked component subjected to external loading. Thus the stress intensity factor of a component gives the magnitude of stresses in a component. As the external applied stresses increase the stress intensity increases. The component fails when the stress intensity attains a certain value termed as the critical stress intensity factor (Section 2.6.2.1). If the stresses are not high enough to fail a component but are applied repeatedly ie. the component is subjected to fatigue loading, it will cause a crack in a component to propagate (Section 2.5). The rate of such propagation is given by Paris' law (Eqn.2.19).

The theoretical approach for constructing a residual strength curve for unstable crack propagation is based on two fundamental fracture mechanics concepts: the critical stress intensity factor and the rate of crack propagation. The critical stress intensity factor gives the unstable crack propagation crack strength, at which failure occurs for a cracked component with a known crack length. Paris' law gives a relationship between crack length and number of cycles. Therefore, using these two relationships, one between residual strength and crack length and other between crack length and number of cycles, a relationship between residual strength and number of cycles can be found.

The relation here is first determined for a infinitely wide plate with a centre crack. The relation is then generalised for idealised components having a constant value of the magnification factor M.

### 3.3 Residual Strength for Infinite Plate

The stress intensity factor relation for an infinitely wide plate with a centre crack (Fig.2.8) to which a remote stress σ is applied is given by Eqn.2.6. Such a component will fail when the stress intensity factor is equal to the fracture toughness of the component. Hence at failure the stress intensity factor relation for an infinitely wide plate with a centre crack is given by

$$K_{IC} = \sigma_f \sqrt{\Pi a} \tag{3.1}$$

where $K_{IC}$ is the critical stress intensity factor at plane strain and $\sigma_f$ is the remote stress applied at which a an infinitely wide plate with a through crack length of '2a' fails. Since the critical stress intensity factor is a material property, Eqn.3.1 gives a relation between residual strength and crack length of a component. Thus if we know the crack length of a component from Eqn.3.1 we can calculate the residual strength.

Consider a component with an initial crack length $2a_i$. The unstable crack propagation failure strength of the component can be found for this crack length. Thus if the value $a_i$ is substituted for 'a' in Eqn.3.1, the value of $\sigma_f$ obtained is the residual strength of the component of a crack length $2a_i$. This value is denoted as $\sigma_i$ in Fig.3.1. If the component is subjected to constant amplitude loading this causes the crack length of the component to increase and the residual strength of the plate to decrease (Fig.3.1). The plate will finally fail when the strength of the component is equal to the maximum load being applied, which in this case is the peak of the stress range applied. If the peak is $\sigma_p$ we can calculate the corresponding crack length at failure. Thus if the value of peak stress $\sigma_p$ is substituted for $\sigma_f$ in Eqn.3.1 we can determine the crack length 'a' at which the component fails. This value is denoted as $a_f$ in Fig.3.1. Hence the corresponding crack lengths at the initial and final stages being known, one is able to calculate the fatigue life from Paris' law (Eqn.2.19), which is given as

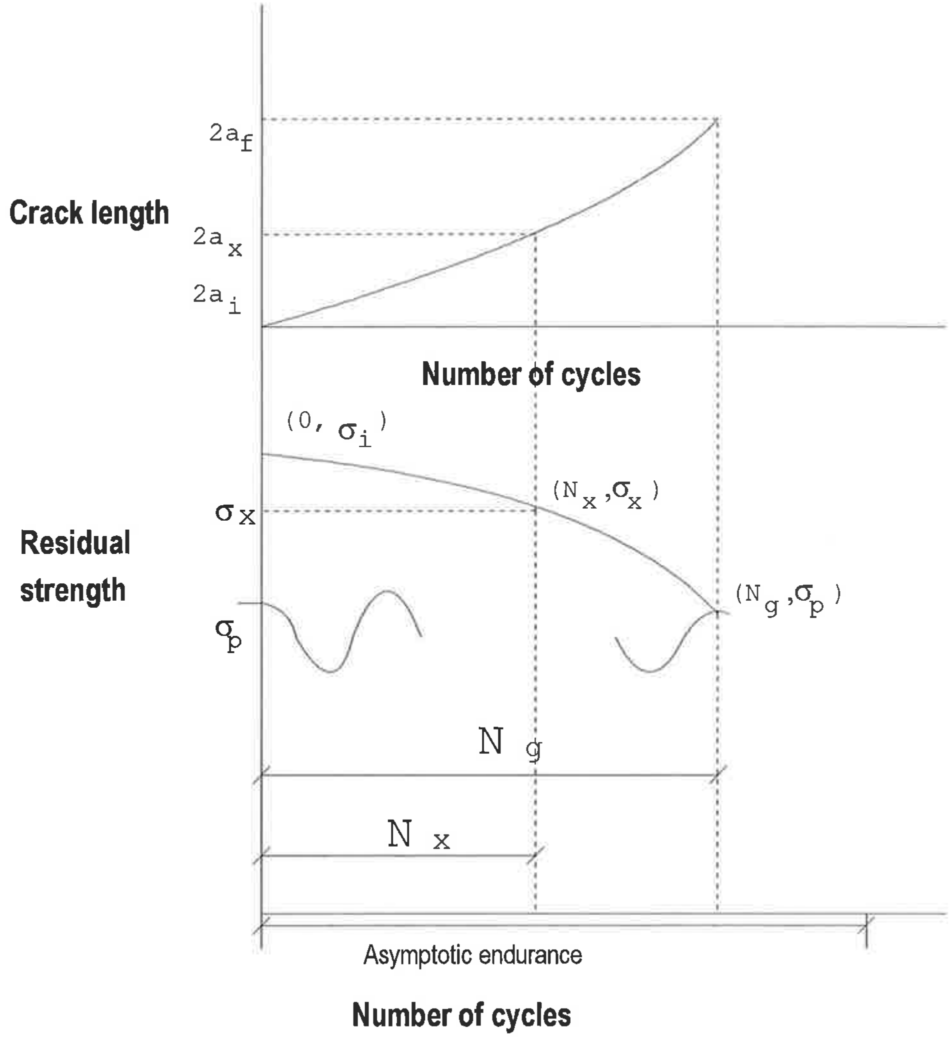



Fig.3.1 Determining the residual strength variation

$$\frac{da}{dN} = C(\Delta K)^m \tag{3.2}$$

where C and m are material constants and ΔK is the range of the stress intensity factor. ΔK depends upon the stress range being applied during fatigue loading and the crack length. Eqn.3.2 can be written in an integral form to give the number of cycles $N_g$ required for a crack length $2a_i$ to propagate to a final crack length $2a_f$.

$$N_g = \int_{2a_i}^{2a_f} \frac{da}{C(\Delta K)^m} \tag{3.3}$$

where $N_g$ is the fatigue life. Thus since the residual strengths at $2a_i$ (ie. $\sigma_i$) and $2a_f$ (ie. $\sigma_p$) and the fatigue life are known we can plot the initial and final points in the residual strength curve (Fig.3.1). In order to plot any intermediate point, let us consider the plate when it has a crack length $2a_x$. The residual strength $\sigma_x$ can be obtained from Eqn.3.1. The number of cycles required for an initial crack $2a_i$ to propagate to a crack length $2a_x$ is given by

$$N_x = \int_{2a_i}^{2a_x} \frac{da}{C(\Delta K)^m} \tag{3.4}$$

By varying the value of $2a_x$ the entire residual strength curve as shown in Fig.3.1 can be drawn.

### 3.3.1. Residual Strength Equation for a Fundamental Component

The general method of constructing a residual strength curve of a component failing by unstable crack propagation, just discussed, has been used here to formulate a mathematical equation for an infinitely wide plate with a centre crack.

The value of the range of stress intensity factor is given by

$$\Delta K = \Delta\sigma\sqrt{\Pi a} \tag{3.5}$$

where $\Delta\sigma$ is the constant stress range being applied. Substituting $\Delta K$ in Eqn.3.5 into Eqn.3.4 and integrating one can obtain

$$N_x = \frac{1}{C(\Delta\sigma)^m(\Pi)^{m/2}}\left[\frac{(2a_x)^{1-m/2} - (2a_i)^{1-m/2}}{1-m/2}\right] \tag{3.6}$$

Now in Eqn.3.1 consider the particular value of the crack length 'a' at which the component fails to be '$a_x$'. Let the particular value of the failure strength $\sigma_f$ at which the component fails be $\sigma_x$. Substituting these values in Eqn.3.1 one can get

$$\sqrt{a_x} = \frac{K_{IC}}{\sigma_x\sqrt{\Pi}}$$

which when substituted in Eqn.3.6 gives

$$N_x = \frac{1}{C(\Delta\sigma)^m(\Pi)^{m/2}}\left[\frac{\left(\frac{2K_{IC}^2}{\sigma_x^2\Pi}\right)^{1-m/2} - (2a_i)^{1-m/2}}{1-m/2}\right] \tag{3.7}$$

since m, C, the stress range $\Delta\sigma$, the critical stress intensity $K_{IC}$ and the initial crack length $a_i$ are all constants in this case the relation between $\sigma_x$ and $N_x$ can be written as

$$N_x = -S\left(\frac{1}{\sigma_x^2}\right)^{1-m/2} + A' \tag{3.8}$$

where S and A are the following constants

$$A' = \frac{(2a_i)^{1-m/2}}{C(\Delta\sigma)^m(\Pi)^{m/2}(m/2-1)} \tag{3.8a}$$

$$S = \frac{\left(\frac{2K_{IC}^2}{\Pi}\right)^{1-m/2}}{C(\Delta\sigma)^m(\Pi)^{m/2}(m/2-1)} \tag{3.8b}$$

discontinuous when m = 2, the relation between the number of cycles of loading and the residual strength is linear when m=3, quadratic when m = 4 and cubic when m = 5. Fig.3.2 shows the variation of residual strength with number of cycles for a infinitely wide plate with a centre crack having different values of m.

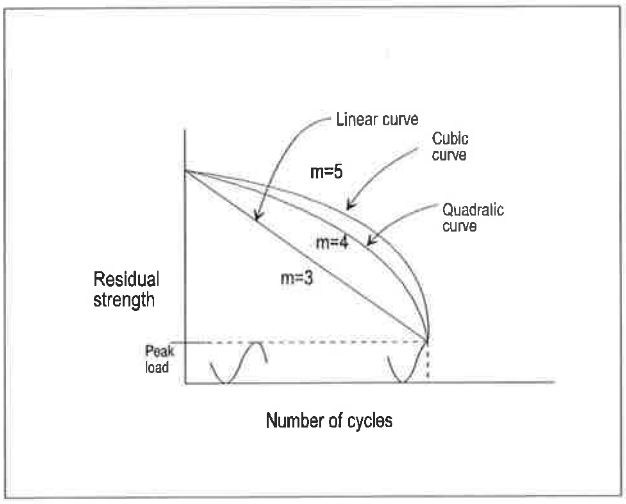



Fig.3.2 Variation of the shape of the residual strength curve of a fundamental component

### 3.3.2 Residual Strength of Structural Steel

Equation 3.8 derived in Section 3.3.1 has been used to determine the residual strength variation for structural steel. Common classes of steel are austenitic (stainless steel), ferrite-pearlite and martensitic steel. The value of m for austenitic steels is 3.25, for ferrite-pearlite 3 and 2.25 for martensitic steels (Section 2.5.1). The ferrite-pearlite group includes steel in the range of yield stress between 205 and 550 N/mm$^2$. The martensitic group includes steel with yield stress more than 480 N/mm$^2$. Hence structural steel would belong to the ferrite pearlite group. The value of m for structural steel is 3 and the relation between residual strength and number of cycles is linear as discussed in Section 3.3.1.

A relation between residual strength and number of cycles for structural steel can be obtained by substituting the value of m=3 into Eqn.3.8 and is given as

$$N_x = -G\sigma_x + H \tag{3.9}$$

where the constants G and H are

$$G = \frac{\sqrt{2}}{\Pi C(\Delta\sigma)^3 K_{IC}} \tag{3.9a}$$

$$H = \frac{\sqrt{2}}{C(\Delta\sigma\sqrt{\Pi})^3 \sqrt{a_i}} \tag{3.9b}$$

where $a_i$ is the initial crack length. Equation 3.9 can be rewritten with residual strength as the dependent variable as follows:

$$\sigma_x = -N_x\left[\frac{\Pi}{\sqrt{2}}C(\Delta\sigma)^3 K_{IC}\right] + \frac{K_{IC}}{\sqrt{\Pi a_i}} \tag{3.10}$$

Now in Eqn.3.1 consider the particular value of the crack length 'a' at which the component fails to be '$a_i$'. Let the particular value of the failure strength at which the component fails be '$\sigma_i$'. Substituting these values in Eqn.3.1 we get

$$\frac{K_{IC}}{\sqrt{\Pi a_i}} = \sigma_i$$

where $\sigma_i$ is considered to be the initial unstable crack propagation strength for a component with crack length $2a_i$.

Substituting the above obtained value of $\sigma_i$ in Eqn.3.10 we get

$$\sigma_x = -N_x\left[\frac{\Pi}{\sqrt{2}}C(\Delta\sigma)^3 K_{IC}\right] + \sigma_i \tag{3.11}$$

Therefore the intercept of the residual strength curve with the Y-axis depends on $\sigma_i$ which in turn depends on $K_{IC}$ and $a_i$. The slope of the residual strength depends on $K_{IC}$ and the stress range $\Delta\sigma$.

The residual strength equation derived is applicable within the limits of the fatigue life of the component. The fatigue life when m=3 is given by Eqn.3.12 using Paris' Equation (Eqn.2.19)

$$N_f = \int_{2a_i}^{2a_f} \frac{da}{C(\Delta K)^3} \qquad (3.12)$$

Substituting Eqn.3.5 we have

$$N_f = \int_{2a_i}^{2a_f} \frac{da}{C(\Delta\sigma\sqrt{\Pi a})^3} \qquad (3.13)$$

Thus the fatigue life depends on the initial crack length, the stress range and the final crack length. The final crack length can be given using Eqn.3.1 as

$$a_f = \frac{K^2{}_{IC}}{\sigma_p{}^2\Pi} \qquad (3.14)$$

where $\sigma_p$ is the peak stress applied. As can be seen from Eqn.3.14 the final crack length therefore depends on the critical stress intensity factor and the peak stress.

### 3.3.2.1 Derivation of asymptotic endurance for structural steel

The discussion in Section 3.3.2 shows that the residual strength equation for structural steel given by Eqn.3.10 is not affected by a variation in the peak stress when the stress range is constant. However, the fatigue life as given by Eqn.3.13 is affected by the peak stress since the final crack length depends on the peak stress (Eqn.3.14). The effect of varying the peak stress with a stress range kept constant is shown in Fig.3.3. It shows that the residual strength curve with a higher peak stress $\sigma_{p1}$ occupies the portion PB of the residual strength curve compared with the portion PC for the smaller peak stress $\sigma_{p2}$. Hence whatever the peak stress, as long as the stress range remains constant the residual strength curve occupies a portion of the envelope P-$N_f$, that initiates from P, (0, $\sigma_i$). P is the initial unstable crack propagation strength and $N_f$, ($N_f$,0) is the asymptotic endurance (Section 2.8.1.1).

Present methods of fatigue design consider endurance to be independent of the peak stress and a constant for a given stress range (Eqn.2.30). For a fundamental component of structural steel it can be shown from Eqn.3.13 and Eqn.3.14 that if the peak stress is varied the endurance of a component will also vary. However, the asymptotic endurance is independent of any variation of the peak stress. Therefore, in contrast to present techniques which use the endurance for fatigue design, the asymptotic endurance has been used in this thesis to develop a fatigue methodology which is independent of the peak stress. The asymptotic endurance $N_f$ is an imaginary value, it has been termed so because it can be approached but never be achieved. It can be visualised as the endurance if the fatigue crack in the element propagated throughout the whole cross section without fracture. However in practise, fracture will occur when the strength of the uncracked region reduces to the peak of the cyclic load.

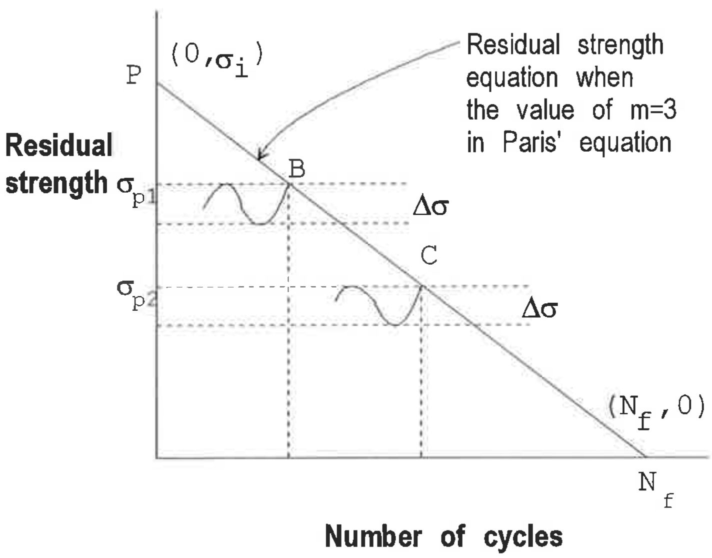



Fig.3.3 Determining the asymptotic endurance

The asymptotic endurance can be calculated mathematically from the residual strength equation, (Eqn.3.9) by substituting $\sigma_X = 0$

$$N_f = \frac{\sqrt{2}}{C(\Delta\sigma\sqrt{\Pi})^3\sqrt{a_i}} \tag{3.15}$$

Substituting $\sigma_i = \frac{K_{IC}}{\sqrt{\Pi a_i}}$ from Eqn.3.1, where $\sigma_i$ is the initial failure strength of the component corresponding to the initial crack length $a_i$, gives

$$N_f = \frac{\sqrt{2}\sigma_i}{C\Pi K_{IC}}(\Delta\sigma)^{-3} \tag{3.16}$$

Thus the asymptotic endurance depends on the inverse of the cube of the stress range and directly on the initial unstable crack propagation strength.

### 3.4 Residual strength variation of an idealised component

The stress intensity factor relation for an infinitely wide plate with a centre crack can be adapted for other geometries by introducing a magnification factor, M Therefore the range of stress intensity when the range of stress acting is $\Delta\sigma$ is given as

$$\Delta K = \Delta\sigma M\sqrt{\Pi a} \tag{3.17}$$

and the equation for failure is given as

$$K_{IC} = \sigma M\sqrt{\Pi a} \tag{3.18}$$

Substituting Eqn.3.17 in Paris' Equation (Eqn.2.19) and integrating one can obtain

$$N_x = \int_{a_i}^{a_x} \frac{da}{C\left(\Delta\sigma M\sqrt{\Pi a}\right)^m} \tag{3.19}$$

For welded components the value of M changes with a change in crack length. If the value of M remains constant and does not vary with 'a' such a component will be called an idealised component. An infinitely wide plate can be considered to be a special case of an idealised component with the value of M equal to 1. The generalised residual strength variation for an idealised component having a constant value of M can be obtained from Eqn.3.19 in a similar manner to the infinitely wide plate. Substituting Eqn.3.17 in Eqn.3.19 gives

$$N_x = -S_1\left(\frac{1}{\sigma_x^2}\right)^{1-m/2} + A_1 \tag{3.20}$$

where $S_1$ and $A_1$ are the following constants

$$A_1 = \frac{(a_i)^{1-m/2}}{CM^m(\Delta\sigma)^m\Pi^{m/2}(m/2-1)} \tag{3.20a}$$

$$S_1 = \frac{\left(\frac{K_c^2}{\Pi}\right)^{1-m/2}}{CM^m\Pi^{m/2}(\Delta\sigma)^m(m/2-1)} \tag{3.20b}$$

For structural steel substituting the value of m=3 in the above equation gives

$$N_x = -G_1\sigma_x + H_1 \tag{3.21}$$

where

$$G_1 = \frac{2}{\Pi \; CM^2(\Delta\sigma)^3 K_{IC}} \qquad (3.21a)$$

and

$$H_1 = \frac{2}{C\left(M\Delta\sigma\sqrt{\Pi}\right)^3 \sqrt{a_i}} \qquad (3.21b)$$

Equation 3.21 shows that when M is constant the relation between residual strength and number of cycles for an idealised component of structural steel is a linear one. Substituting the residual strength $\sigma_x$ in Eqn.3.21 as zero the asymptotic endurance is given as

$$N_f = \frac{2}{CM^3\left(\Delta\sigma\sqrt{\Pi}\right)^3 \sqrt{a_i}} \qquad (3.22)$$

It should be noted here that if the value of M is not constant as in the case of a real component of structural steel and varies with 'a' the residual strength variation will not be linear. The non-linear residual strength of welded components for which the value of M varies with the crack length 'a' is determined later on in Chapter 5. These non-linear curves has been approximated into linear portions in Chapters 7 and 8 with constant values of M to provide an easy and simple method of assessment and design.

## 3.5 Conclusion

It has been discussed in this chapter that the most fundamental aspect of fatigue loading is the propagation of a crack in a component. However, from the point of view of a designer a curve of residual strength verses number of cycles is helpful. The basic principles of determining the residual strength variation for a component subjected to constant amplitude loading has been established here. The residual strength curve has been determined for an idealised component having a constant value of the magnification factor M failing by unstable crack propagation. It is found that the residual strength relation is linear for structural steel.

However the value of M is not constant for most real components of structural steel such as welded components and the residual strength variation of a component would be non-linear in shape. These linear and non-linear relations are for a constant stress range. The reduction in strength of a component when subjected to a series of stress ranges is found for both cases of a linear and a non-linear curve in the next chapter.

# Chapter 4
# Determining the reduction in strength

## 4.1 Introduction

It has earlier been discussed in the literature review (Section 2.5) that when variable amplitude loading is applied to a component, the propagation of a crack depends on the loads applied and also on the sequence in which they are applied. As an example, an overload applied to a component decreases the rate of crack propagation. In general any change in the amplitude of the stresses and the sequence in which they are applied causes a variation in the rate of propagation of a crack as given by Wheeler and by Larsen and Nicholas (Section 2.5).

The fundamental effect of fatigue loading is the propagation of a crack which causes a change in the residual strength as discussed in Section 3.1. Thus the residual strength variation with number of cycles will also depend on the variable loads applied and also on the sequence in which they are applied.

As components in service will be subjected to a large number of variations in the cyclic stresses it would be very difficult, cumbersome and impractical, to find the residual strength curve by taking account of each change in cyclic stresses applied. Furthermore a cycle by cycle account of the stress history is difficult to obtain and load spectrums generally provide only the stress ranges and their frequency acting on a component (Section 2.7.3). Therefore if the method of finding the reduction in strength due to fatigue takes account of the sequence of loading then for a given load spectrum one can determine a large number of possible sequences and for the large number of possible sequences one can obtain a large number of solutions.

A practical method of finding the residual strength variation is required where one does not need to know or to calculate for each change in stress range acting on a component. Hence it would be necessary for the solution to be independent of the sequence of loading so that for a given load spectrum we will get a single result.

In the previous chapter, it was shown that residual strength curves would be linear and non-linear. For example, the residual strength variation for idealised components of structural steel when subjected to a constant stress range is linear as shown in Section 3.4 whereas for other practical components the variation under constant stress range is non-linear. An investigation is carried out to determine the reduction in strength of components by applying blocks of cycles to components that have either a linear or a non-linear residual strength variation under constant amplitude loading. The first stage in the investigation process involves determining the reduction in strength when components are subjected to blocks with equal number of cycles and equal magnitude of stress ranges termed as equal blocks. The next step in the investigation process involves determining the reduction in strength when components are subjected to blocks, where stress range within a block remains the same but the magnitude and number of stresses vary from one block to another.

## 4.2 Reduction in strength due to equal blocks

A linear and a non-linear residual strength variation is shown in Figs.4.1 and 4.2. Consider the case where two blocks having the same number of cycles (N) and equal magnitude of stresses are applied to components whose residual strength variation is given by Figs.4.1 and 4.2. Figure 4.1 shows that in the case of a linear curve the reduction in strength ΔP for the two blocks 1 and 2 are the same. Thus when the curve is linear, a given block of stress ranges would cause the same reduction in strength irrespective (points A and B in Fig.4.1) of when the block is applied. However the reduction in strength caused by the application of a block of cycles on a component

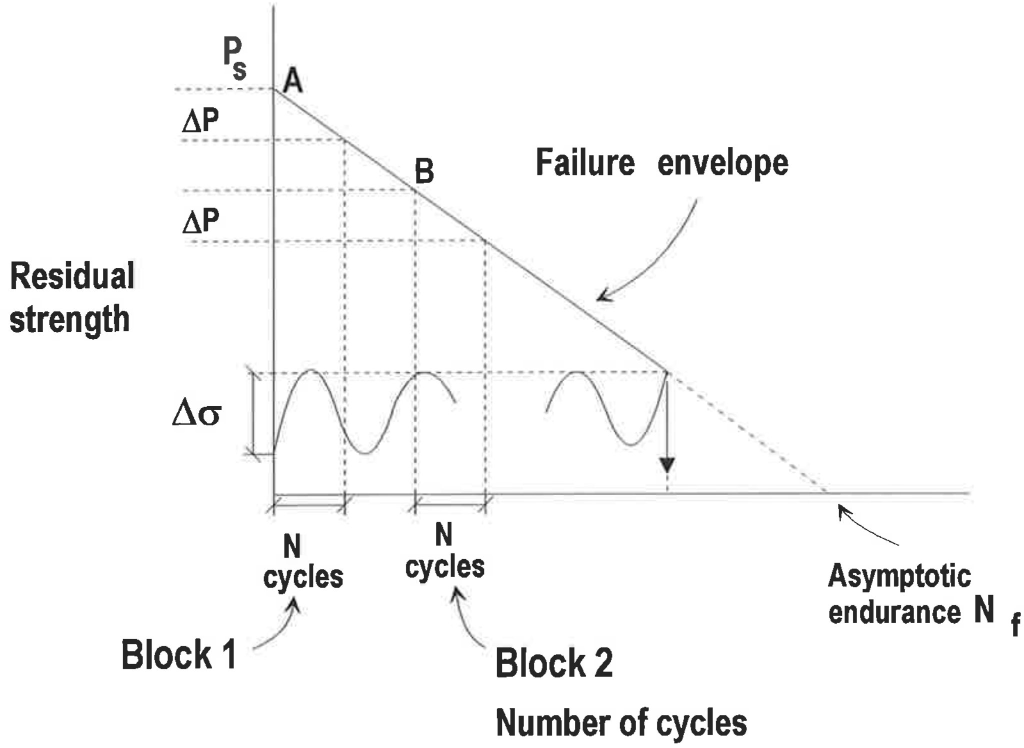



Fig.4.1 Linear residual strength curves

showing a non-linear residual strength variation depends on whether the block is applied early or late in the fatigue life, ie. the period of application (Fig.4.2). The reduction in strength for the second block $\Delta P_b$ depends on the residual strength at the start of the block which in turn depends on the reduction in strength caused by the first block $\Delta P_a$. Hence, the reduction in strength for a block of cycles for a non-linear curve depends on the previous blocks of cycles applied.

It can therefore be seen from the previous paragraph that for a component with a linear variation of residual strength, the reduction in strength is independent of previous blocks and hence does not depend on the sequence of the application of loads. The reduction in strength is the same for equal blocks whenever they are applied within the fatigue life. If the reduction in strength $\Delta P$ is known for one block it can simply be determined for any number of equal blocks by simply multiplying $\Delta P$ by the number of blocks. The reduction in strength for components showing linear residual strength variation are therefore quite easy to find. However for a non-linear curve, the reduction in strength for any one block depends on the previous blocks applied. Hence for non-linear curves the reduction would depend on the sequence of the block and therefore must be found individually for each block.

## 4.3 Reduction in strength due to unequal blocks

The investigation on linear and non-linear residual strength variation curves is carried further by applying two blocks of cycles with different magnitudes of stress range and different number of cycles. The first block consists of $N_1$ cycles of the constant stress range $\Delta\sigma_1$ and the second block consists of $N_2$ cycles of stress range $\Delta\sigma_2$.

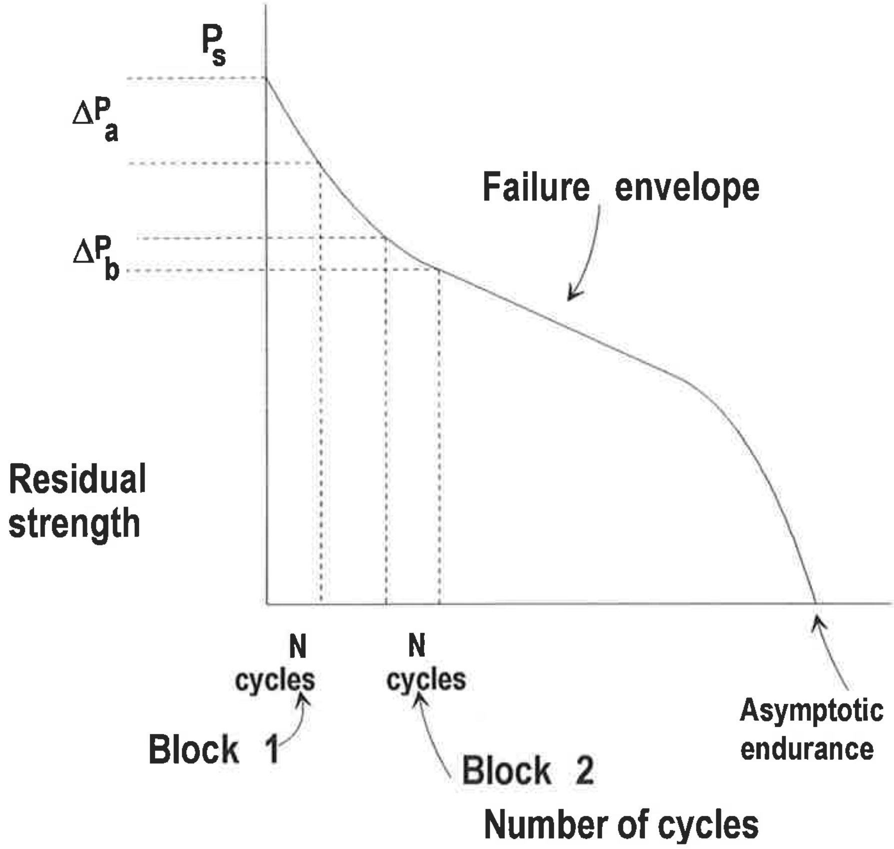



Fig.4.2 Non-linear residual strength curve

### 4.3.1 Non-Linear

The structural component investigated first is considered to have a non-linear residual strength variation, of the form $Y=gX^2+F$, where Y is the dependant variable showing the variation in residual strength and X the independent variable showing the variation in number of cycles. The initial crack length of the component is considered to be $a_i$ and the initial strength considered to be $P_s$. The residual strength variation EJ in Fig.4.3 gives the variation of residual strength when a component is subjected to a stress range $\Delta\sigma_1$ while the residual strength variation EK in Fig.4.3 gives the variation of residual strength when the component is subjected to a stress range $\Delta\sigma_2$. The asymptotic endurance for the curve EJ is taken as $(N_f)_1$ and for the curve EK as $(N_f)_2$.

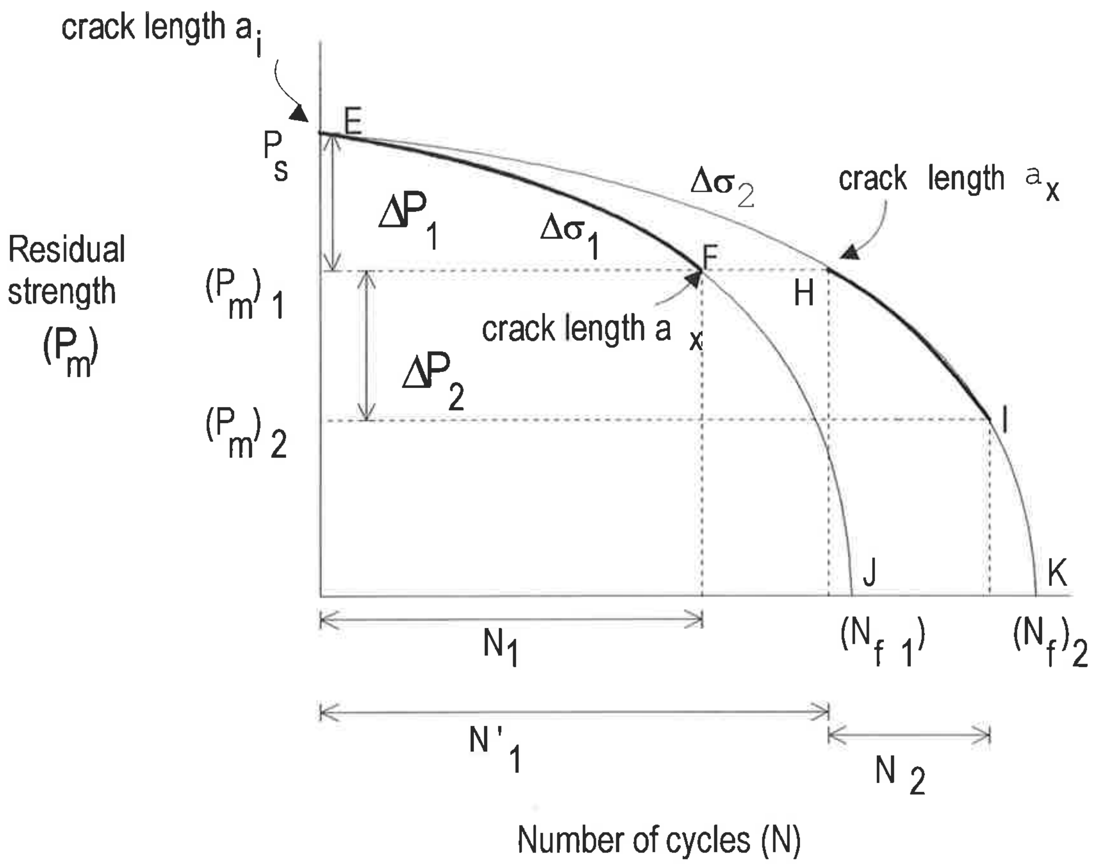



Fig.4.3 Reduction in strength of a component showing non-linear variation

Substituting the boundary conditions of the curve EJ, Y=0, $X=(N_f)_1$, and X=0, $Y=P_s$ in the Eqn. $Y=gX^2+F$, the values of g and F are obtained as $g=-P_s/(N_f)^2{}_1$ and $F=P_s$ to obtain the residual strength equation as

$$Y = -\frac{P_s}{(N_f)^2{}_1}X^2 + P_s \tag{4.1}$$

Similarly, the equation of the curve EK can be obtained by substituting the conditions Y=0, X=$(N_f)_2$ and X=0, Y=$P_s$ and is given as

$$Y = -\frac{P_s}{(N_f)_2^{\ 2}}X^2 + P_s \tag{4.2}$$

Let the first block of fatigue loading applied be of $N_1$ cycles with a stress range $\Delta\sigma_1$. Let the initial crack propagate due to the application of this block to the crack length $a_x$ and let the residual strength at that crack length is denoted by $(P_m)_1$. As the stress range is $\Delta\sigma_1$, the reduction in strength of the component can be derived using the curve EJ in Fig.4.3. Substituting X=$N_1$ and Y=$(P_m)_1$ in Eqn.4.1 one can obtain

$$\frac{P_s - (P_m)_1}{P_s} = \frac{N_1^2}{(N_f)_1^2} \tag{4.3}$$

Denoting the reduction in strength $P_s$-$(P_m)_1$ by $\Delta P_1$, Eqn.4.3 can be written as

$$\frac{\Delta P_1}{P_s} = \frac{N_1^2}{(N_f)_1^2} \tag{4.4}$$

where the relationship between the parameters in Eqn.4.4 is shown in Fig.4.3. Hence after the application of the first block of loading of $N_1$ cycles of stress range $\Delta\sigma_1$ the component has lost $\Delta P_1$ of its strength and the condition of the component is represented by the point F in Fig.4.3.

Let us assume that the component now is subjected to a second block of fatigue loading consisting of $N_2$ cycles at a stress range $\Delta\sigma_2$. The reduction in strength due to this stress range is given using the envelope EK in Fig.4.3. Fig.4.3 shows that the point H in the curve EK corresponds to the same residual strength $(P_m)_1$ as the point F in curve EJ and as the residual strength is the same the crack length must be the same at $a_x$. Thus the application of the stress range $\Delta\sigma_2$ will now cause the strength to reduce along the curve EK starting from the point H. The number of cycles corresponding to the point H in curve EK is denoted as $N'_1$. Substituting the coordinates of the point H which are $(P_m)_1$ and $N'_1$ in Eqn.4.2 gives

$$(P_m)_1 = -\frac{P_s}{(N_f)_2} N_1'^2 + P_s \tag{4.5}$$

Let us assume that after the application of the block of loading of $N_2$ cycles, the component reaches position I in Fig.4.3 that corresponds to the residual strength of $(P_m)_2$. Substituting the coordinates of I which are $(P_m)_2$ and ($N_1'$ +$N_2$) in Eqn.4.2 gives

$$(P_m)_2 = -\frac{P_s}{(N_f)_2{}^2}(N_1' + N_2)^2 + P_s. \tag{4.6}$$

Subtracting Eqn.4.6 from Eqn.4.5 gives the reduction in strength $(P_m)_1$-$(P_m)_2$ as

$$\frac{(P_m)_1 - (P_m)_2}{P_s} = \frac{N_2^2 - 2N'_1N_2}{(N_f)_2^2} \tag{4.7}$$

Denoting in Eqn.4.7 the reduction in strength $(P_m)_1$-$(P_m)_2$ as $\Delta P_2$, Eqn.4.7 can be written as

$$\frac{\Delta P_2}{P_s} = \frac{N_2^2 - 2N'_1N_2}{(N_f)_2^2} \tag{4.8}$$

It can be seen from Fig.4.3 that $N'_1$ in Eqn.4.8 is the number of cycles of a single stress range $\Delta\sigma_2$ that is required by a component with an initial crack length $a_i$ to increase in size to a crack length $a_x$ at which crack size the residual strength is $(P_m)_1$. Application of the first block of the stress range $\Delta\sigma_1$ of $N_1$ cycles also causes the initial crack length $a_i$ to increase in size to a crack length $a_x$ as shown in Fig.4.3. The value of the number of cycles $N_1$ is obtained using Eqn.3.4 as

$$N_1 = \int_{a_i}^{a_x} \frac{da}{C\left(\Delta\sigma_1 M\sqrt{\Pi a}\right)^m} \tag{4.9}$$

and the value of the number of cycles $N_1'$ is obtained from Eqn.3.4 as

$$N_1' = \int_{a_i}^{a_x} \frac{da}{C\left(\Delta\sigma_2 M\sqrt{\Pi a}\right)^m} \tag{4.10}$$

In both cases the crack length propagates from $a_i$ to $a_x$. However the stress range applied in the case of $N'_1$ is $\Delta\sigma_2$ while $\Delta\sigma_1$ is the stress range in the case of $N_1$. The values of the magnification factor M and the constant m in Paris' equation is the same for both cases. Dividing the value of $N'_1$ in Eqn.4.10 by the value of $N_1$ in Eqn.4.9 we get

$$N_1' = \left(\frac{\Delta\sigma_1}{\Delta\sigma_2}\right)^3 \times N_1 \tag{4.11}$$

Substituting $N'_1$ in Eqn.4.11 into Eqn.4.8 gives

$$\frac{\Delta P_2}{P_s} = \frac{N_2^2 - 2\left(\frac{\Delta\sigma_1}{\Delta\sigma_2}\right)^3 N_1 N_2}{(N_f)_2^2} \tag{4.12}$$

It can be seen from Eqn.4.12 that the reduction of strength $\Delta P_2$ due the stress range $\Delta\sigma_2$ depends not only on the $N_2$ cycles applied but also on the $N_1$ cycles applied earlier. Hence for the quadratic residual strength curve, when a number of blocks of different stress range are applied, the reduction in strength for a given block depends on the stress ranges and number of applications of the previous blocks.

### 4.3.2. Linear Curve

An investigation similar to the case of the components that showed non-linear residual strength variations in Section 4.3.1 has been carried out for the idealised welds in Section 3.4 that has a linear residual strength variation. Consider an idealised component with an initial crack length $a_i$ and an initial strength $P_s$ that is subjected to two different stress ranges $\Delta\sigma_1$ and $\Delta\sigma_2$. Fig.4.4 shows the residual strength variations if the component was subjected to only a single stress range $\Delta\sigma_1$ or to only a single stress range $\Delta\sigma_2$ of fatigue loadings throughout their lives. The curve AX gives the variation of the residual strength for the stress range $\Delta\sigma_1$ while the curve AZ gives the variation for the stress range $\Delta\sigma_2$. Let the corresponding asymptotic endurances be $(N_f)_3$ and $(N_f)_4$.

The equation of a linear curve can be given by Y=mX+C where Y is the residual strength and X the number of cycles. Substituting the boundary points Y= 0, X= $(N_f)_3$ and X= 0, Y= $P_s$ the linear residual strength variation of the curve AX is given by the equation

$$Y = -\frac{P_s}{(N_f)_3} X + P_s \tag{4.13}$$

while the equation of the curve AZ can be obtained by substituting Y=0, X=$(N_f)_4$ as

$$Y = -\frac{P_s}{(N_f)_4} X + P_s \tag{4.14}$$

Let the first block of fatigue loading that is applied be $N_3$ cycles at a range $\Delta\sigma_1$ so that the strength of the component reduces by $\Delta P_3$ as shown in Fig.4.4. Substituting $P_s$-$P_m$=$\Delta P_3$ and X=$N_3$ into Eqn.4.13 gives

$$\frac{\Delta P_3}{P_s} = \frac{N_3}{(N_f)_3} \tag{4.15}$$

After losing $\Delta P_3$ of its strength due to the application of the first block of loading the condition of the component is represented by the point B in Fig.4.4. The component at this stage is subjected to a second block of fatigue loading consisting of $N_4$ cycles at the stress range $\Delta\sigma_2$. The reduction in strength due to the second block of loading can be calculated using the residual strength curve AZ. Figure 4.4 shows that the point D in the curve AZ corresponds to the same residual strength and hence crack length as the point B in AX. Thus application of the stress range $\Delta\sigma_2$ causes the strength to reduce along AZ starting from the point D. Hence if the reduction in strength due to the application of stress range $\Delta\sigma_2$ is given as $\Delta P_4$, then substituting $\Delta P_4$ and $N_4$ in Eqn.4.15 gives

$$\frac{\Delta P_4}{P_s} = \frac{N_4}{(N_f)_4} \tag{4.16}$$

Adding Eqns.4.15 and 4.16 gives

$$\frac{\Delta P_3}{P_s} + \frac{\Delta P_4}{P_s} = \frac{N_3}{(N_f)_3} + \frac{N_4}{(N_f)_4} \tag{4.17}$$

Equation 4.17 shows that the reduction in strength for a given stress range is solely dependant on the number of cycles of the stress range applied and the corresponding asymptotic endurance. Therefore, the reduction in strength for a linear residual strength variation is independent of the sequence of loading.

The investigation in the Sections 4.1 and 4.2 on the reduction in strength with linear and non-linear residual strength variations was carried out with blocks of cyclic loads.

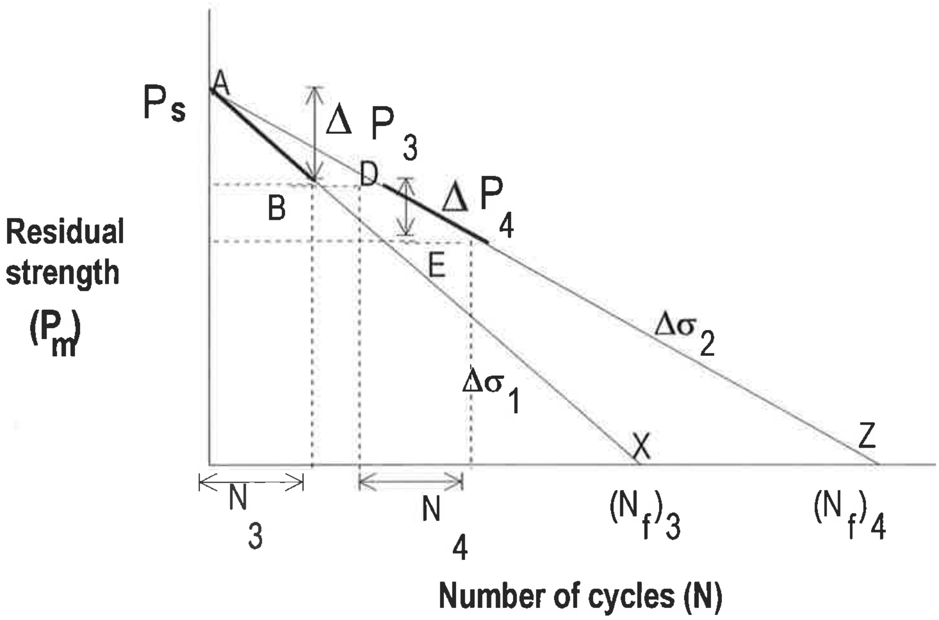



Fig.4.4 Reduction in strength of a component showing a linear variation

In Section 4.2 results were obtained for equal blocks with equal stress ranges while in Section 4.3 results were obtained for unequal blocks with the magnitude of stress ranges varying from one block to another. The smallest block in both cases could consist of a single cycle. The investigation of equal blocks therefore is applicable to individual cycles with unequal stress ranges. Hence for components with a non-linear residual strength variation, the damage caused by a cycle depends on the previous history of stresses applied. Therefore in order to calculate the reduction in strength due to the cycles applied in the end, the reduction in strength due to the cycles applied earlier must be calculated, thus making the procedure increasingly complex. Furthermore if the order of the application of the cycles were altered, this would mean that the residual strength variation would alter considerably and would have to be recalculated, since the reduction in strength due to each cycle would depend on all the previous cycles applied. However for idealised components with linear curves, the change in strength for a given stress range does not depend on the previously applied stress range. The damage due to a stress range is discrete and independent of other load applications. Therefore the total reduction in unstable crack propagation strength is a summation of the reduction in unstable crack propagation strength for each individual stress range. Any change in the order of application of stress range does not alter the total reduction in unstable crack propagation strength.

## 4.4. Fundamental equations of a linearised system

Since linear residual strength curves have been shown to be much easier to deal with in Section 4.3, an assessment procedure is developed in Chapter 7 that is based on linear curves. The basic equations for assessing a single linear residual strength variation are determined here.

The reduction in strength for a component showing a linear residual strength variation is independent of the sequences in which different cycles are applied. Therefore these cycles can be rearranged into a number of sequences for easy calculation. For example if there are X different stresses acting on a component, these stresses can be grouped in X blocks. The reduction in strength due to two different blocks of loading is given by Eqn.4.17. A general damage accumulation equation is formed by extending Eqn.4.17 to accommodate X blocks of ranges. If the reduction in strength due to stress ranges $\Delta\sigma_1$ applied for $N_1$ cycles is $\Delta P_1$, stress ranges $\Delta\sigma_2$ applied for $N_2$ cycles is $\Delta P_2$, stress ranges $\Delta\sigma_3$ applied for $N_3$ cycles is $\Delta P_3$ etc, then the total damage for X stress ranges is given by

$$\frac{\Delta P_1}{P_s}+\frac{\Delta P_2}{P_s}+......+\frac{\Delta P_x}{P_s}=\frac{N_1}{(N_f)_1}+\frac{N_2}{(N_f)_2}+......+\frac{N_x}{(N_f)_x} \tag{4.18}$$

or

$$\frac{\sum_{i=1}^{i=x}\Delta P_i}{P_s}=\sum_{i=1}^{i=x}\frac{N_i}{(N_f)_i} \tag{4.19}$$

Equation 4.19 gives the reduction in total strength of a component by summing up the reduction caused by each stress range. However in actual practice a weld in a bridge will be subjected to an extremely large number of stress ranges and to calculate the reduction in strength of the component for each individual stress range can be cumbersome. An efficient method is to calculate an equivalent single stress range which will cause the same reduction in strength as all the stress ranges put together. Equation 4.19 can be written in terms of the stress ranges applied by substituting the value of the asymptotic endurance $(N_f)_i$ from Eqn.3.22 and rearranging to give

$$\sum_{i=1}^{i=x} \Delta P_i = \frac{P_s C M^3 \Pi^{3/2} \sqrt{a_i}}{2} \sum_{i=1}^{i=x} N_i (\Delta\sigma_i)^3 \qquad (4.20)$$

where $P_s$, C, M and $a_i$ are constants for the given component. Hence the reduction in strength for a given component depends on the summation of the number of cycles times the cube of the stress range. This reduction in strength can also be written in terms of a single equivalent stress range $\Delta\sigma_e$ as

$$\sum_{i=1}^{i=x} \Delta P_i = \frac{P_s C M^3 \Pi^{3/2} \sqrt{a_i}}{2} (\Delta\sigma_e)^3 \sum_{i=1}^{i=x} N_i \qquad (4.21)$$

Equating Eqns. 4.20 and 4.21 gives $\Delta\sigma_e$ as

$$(\Delta\sigma_e)^3 = \frac{\sum_{i=1}^{i=x} N_i (\Delta\sigma_i)^3}{N_e} \qquad (4.22)$$

where $N_e$ is equal to the summation $\sum_{i=1}^{i=x} N_i$, the total number of cycles applied. The equivalent stress range $\Delta\sigma_e$ can now be used to find the residual strength variation of the welded component having a linear residual strength variation. For a linear residual strength curve as shown in Fig.4.1, the residual strength $P_m$ after the application of $N_e$ number of cycles can be given in terms of the static strength $P_s$ and the asymptotic endurance $N_f$ for the equivalent stress range $\Delta\sigma_e$ as follows

$$\left(1 - \frac{P_m}{P_s}\right) = \frac{N_e}{N_f} \qquad (4.23)$$

Equation 4.23 can be modified to give a simple equation to find the increase in crack length. For example substituting $P_m$ for $\sigma_f$ in Eqn.3.1 will give the size of the crack $a_m$. Similarly substituting $P_s$ in place of $\sigma_f$ will give the initial crack $a_i$. Therefore Eqn.4.23 can be written as

$$\left(1-\frac{M_i}{M_m}\sqrt{\frac{a_i}{a_m}}\right)=\frac{N_e}{N_f} \qquad (4.24)$$

where $a_i$ is the initial crack length (corresponding to strength $P_s$), $M_i$ the magnification factor at the crack length $a_i$ and $a_m$ is the intermediate crack length (corresponding to the strength $P_m$), $M_m$ the magnification factor at crack length $a_m$, after application of $N_e$ cycles. If we know that the residual strength variation of a component is linear, Eqn.4.24 gives a simple way of calculating the crack length rather then use crack propagation equations.

## 4.5 Conclusions

The reduction in strength of components showing a linear or a non-linear residual strength variation under a single stress range was discussed in this Chapter. It was shown that when the residual strength curve is non-linear, the reduction in strength depends on the previous stress ranges to which a component has been subjected. The reduction in strength, therefore, must be calculated by considering each cycle in the sequence of stress ranges. This makes the calculation for the reduction in strength difficult and cumbersome, particularly for the case when the component is subjected to a large variety of stresses. Also cycle by cycle account of the stress history of the component is often not available. Load spectrums that are generally available only provide the number and frequency of stresses acting on a component rather than a cycle by cycle stress history. Since these stresses can be arranged in a large number of sequences a large number of solutions can be obtained for the case of a non-linear curve.

However when the residual strength curve is linear, the reduction in strength for a individual stress range does not depend on the previous stress ranges applied. The reduction in strength for each individual stress range can therefore be found out separately. The total reduction therefore is a summation of the reductions due to each individual stress range. Any change in the sequence in which stress ranges are applied does not alter the total reduction in strength.

The practical procedure of assessment and design has been developed in this thesis that is based on linear residual strength curves. The residual strength variation of welded components is developed in Chapter 5. These curves are non-linear and hence have been linearised to provide a simple hand assessment method in Chapter 7 and design method in Chapter 8 for which we do not need to know or calculate for each cycle of stress. Thus load spectrums which only give the stress ranges and frequency can now be used for accurate assessment and design of components.

# Chapter 5
# Residual strength of welded components subjected to constant amplitude loading

## 5.1 Introduction

In Chapter 3 the residual strength variation was determined for an infinitely wide plate with a centre crack and an idealised component failing by unstable crack propagation. It was discussed that when a component containing a crack is subjected to fatigue loading, the primary effect of such loading is the propagation of a crack. Crack propagation laws were used to find the crack length of a component during the fatigue life. The crack length of a component being known, the residual strength variation was determined. The residual strength variation was obtained in Chapter 3 as a closed form solution (Section 3.4) for idealised components in which the magnification factor M is constant. The mode of failure considered was unstable crack propagation.

In this chapter, a procedure for determining the residual strength variation of a welded component when subjected to constant amplitude cyclic loading is discussed and the reduction in strength due to variable loads will be discussed later in Chapter 7. The method developed considers welds to have an initial crack as discussed in Section 2.4 and hence the entire life is spent on crack propagation. An assumed value of the initial crack length is used in this chapter which will propagate with the application of fatigue loading as given by the crack propagation laws in Section 2.5.1. The crack length of a component being known the residual strength can be determined.

For the idealised components in Chapter 3, the value of M is constant and therefore the residual strength variation can be given as a closed form solution. However for real welded components, the value of the magnification factor M will vary with the crack length as shown by Figs. 2.16, 2.17 and 2.18. Therefore it is necessary to determine the residual strength of welded components by numerical integration where small increments in crack lengths are considered within which the value of M is taken as constant.

In Chapter 3, the mode of failure considered was unstable crack propagation failure. However as discussed in the literature review (Section 2.6.2), welded components can fail either by unstable crack propagation failure or by plastic deformation failure. The residual strength variation has, therefore, been determined in this chapter for both unstable crack propagation failure and plastic deformation failure, and factors affecting these curves have been discussed. In order to determine the actual constant amplitude residual strength envelope, the unstable crack propagation envelope and the plastic deformation envelope have been superimposed and the lower bound of these two envelopes gives the strength of the component. The variation of the actual constant amplitude residual strength curve with the material of the component, the loads applied, the rate of application of loads and with the temperature is also discussed.

## 5.2 Determining the Residual Strength Curve

In order to determine the residual strength variation of an idealised component in Chapter 3, the variation of crack lengths was first found using crack propagation equations (Eqn.3.19). The residual strength of a component was then determined from the known variations of the crack lengths. A closed form solution of the variation of residual strength with number of cycles could then be obtained in Eqn 3.20, unlike welded components the magnification factor M did not depend on the variation of the crack length 'a'. However, in the case of real welded components the variation of M with the crack length 'a' is quite complicated as shown by Eqn.2.8. Therefore, a closed form solution does not exist for the integral formed by substituting M as a function of 'a' in Eqn.3.19 and hence a method of numerical integration has to be adopted. In the numerical analysis, the number of cycles is determined for small increments in crack length and within each increment the values of M is assumed to be constant. It is worth noting that this technique is a method of approximating the non-linear curve into small linear portions, a technique earlier discussed in Section 3.4, and developed further in Chapter 7.

The number of cycles $N_{in}$ required by a component to propagate through a crack increment can be derived from Paris' equation (Section 2.5). In Chapter 3 it was shown that by integrating Paris equation we can get Eqn. 3.19 which gave the number of cycles required for a crack to propagate from a initial crack length $a_i$ to any intermediate crack length $a_x$. Here, the number of cycles required by a crack to propagate through an increment, from an intermediate crack $a_x$ to a crack length $a_{x+1}$ is given by

$$N_{in} = \int_{a_x}^{a_{x+1}} \frac{da}{C(M_{x-(x+1)}\Delta\sigma\sqrt{\Pi})^3} \qquad (5.1)$$

where $a_x$ and $a_{x+1}$ are the crack lengths at the start and end of the increment and $M_{X-(X+1)}$ is the average value of M for the increment and any range $\Delta\sigma$ is the constant amplitude stress range applied. If we consider a component with an initial crack length $a_i$ (Fig.5.1) then the crack length after the first increment will be $a_1=a_i+\Delta a$ where $\Delta a$ is the increment of crack length considered. The crack length after the second increment will be $a_2=a_i+2\Delta a$ and so on. The number of cycles $N_1$ required by the crack to propagate through the first increment can be determined by substituting $a_x=a_i$ and $a_{x+1}=a_1$ in Eqn.5.1. The number of cycles $N_2$ required for the crack to propagate through the second increment is obtained similarly by substituting $a_x=a_1$ and $a_{x+1}=a_2$. The strength of the component is found after each increment in crack length using Eqns.2.24, 2.26 and 2.27. The strength of the component at crack length $a_i$ is given by the point A in Fig.5.1 and the strength of the component at crack lengths $a_1$ and $a_2$ are shown as points B and C. The procedure can be repeated to determine the number of cycles required by cracks to propagate through increments are calculated till the crack propagates through the thickness 't' of the component. At this stage the strength of the component is zero as shown by the point D in Fig.5.1. All the strengths obtained due to the different crack lengths are joined by straight lines to get the residual strength variation as shown in Fig.5.1.

In the example considered in this chapter, the components are considered to have an initial crack length of 0.25 mm and subjected to a stress range of 20 N/mm$^2$. The stress range applied is however not important for assessment purposes since residual strength curves for this stress range can be non-dimensionalised as shown in the next section and a non-dimensionalised curve can be used to determine the residual strength variation for any other stress range.

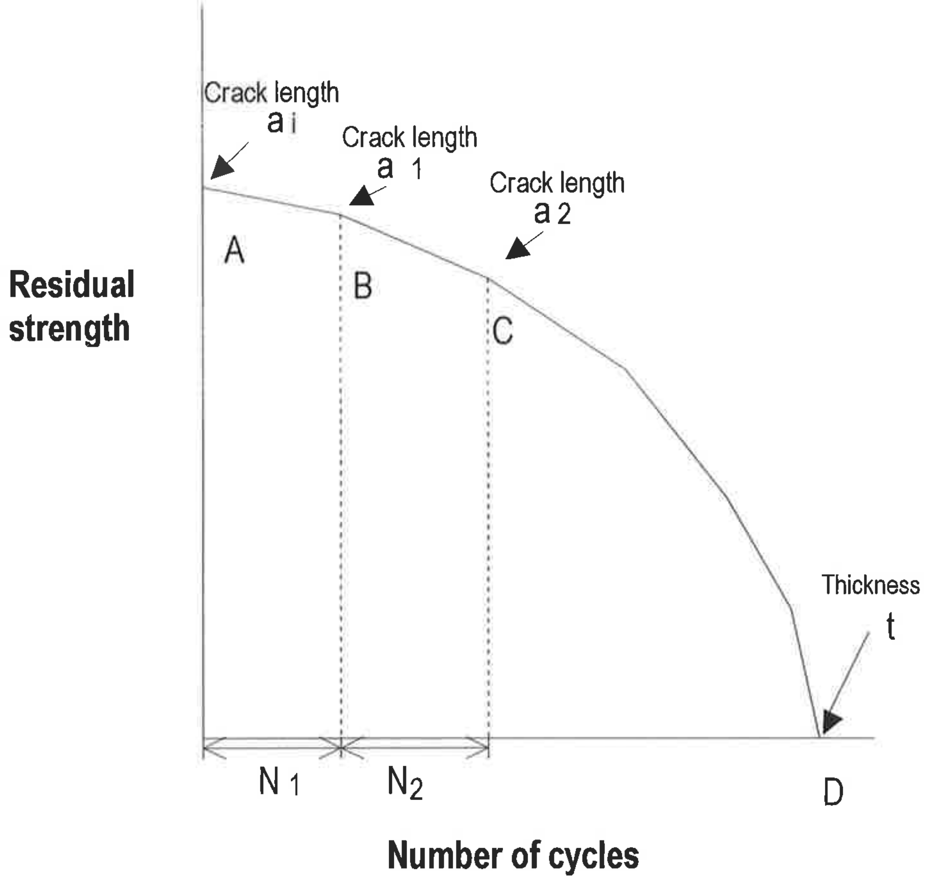



Fig.5.1 Determining the residual strength of welded components

## 5.3 Non-dimensionalised residual strength curve

The equations used to determine the residual strength curve are Eqn.5.1, which gives the number of cycles required by a crack to propagate through an increment and Eqns. 2.24, 2.26 and 2.27 which gives the unstable crack propagation and plastic deformation strength of a component at a given crack length. It can be seen from Eqns. 2.24, 2.26 and 2.27 that the strength of a component depends on the crack size. Furthermore, it can be seen from Eqn.5.1 that the number of cycles required by a crack to propagate by an increment depends on the cube of the stress range. Hence if a different stress range is applied to the same component, the ordinates of the residual strength curve such as those in Fig.5.1 will be unchanged. As the number of cycles required by the crack to propagate through each increment depends on the cube of the stress range then the total number of cycles required by an initial crack to propagate through the thickness of the component will depend on the inverse of the cube of the stress range. Hence the asymptotic endurance (Section 2.8.1.1) will also depend on the inverse of the cube of the stress range.

The variation of the residual strength curve with stress range was discussed in the previous paragraph. The discussion is carried further by non-dimensionsalising the residual strength curve shown in Fig.5.1. In order to determine the ordinates of a non-dimensional residual strength curve, Eqn.5.1 can be written below in the following form

$$N_{in} = \frac{D}{(\Delta\sigma)^3} \tag{5.2}$$

where

$$D = \int_{a_x}^{a_{x+1}} \frac{da}{C\left(M_{x-(x+1)}\sqrt{\Pi}\right)^3} \tag{5.3}$$

The value of D is independent of the stress range applied and for a given component depends only on the initial and final crack length of an increment. If we denote the value of D as $D_1$ for the first increment, $D_2$ for the second increment and so on, then the value of D required for a crack to propagate from the initial crack length $a_i$ to the crack length $a_1$ can be given as $D_1$, and for the initial crack length to propagate to $a_2$ as $D_1$+$D_2$ and so on. Thus if n increments are required by the initial crack to propagate through the plate thickness, then the value of D required can be given as $D_1+D_2+..+D_n$.

These values of D can be substituted in Eqn.5.2 to give the number of cycles required by the initial crack to propagate through various increments. Thus the number of cycles required by the initial crack to propagate to the crack length $a_1$ can be given as

$$N_1 = \frac{D_1}{(\Delta\sigma)^3} \tag{5.4}$$

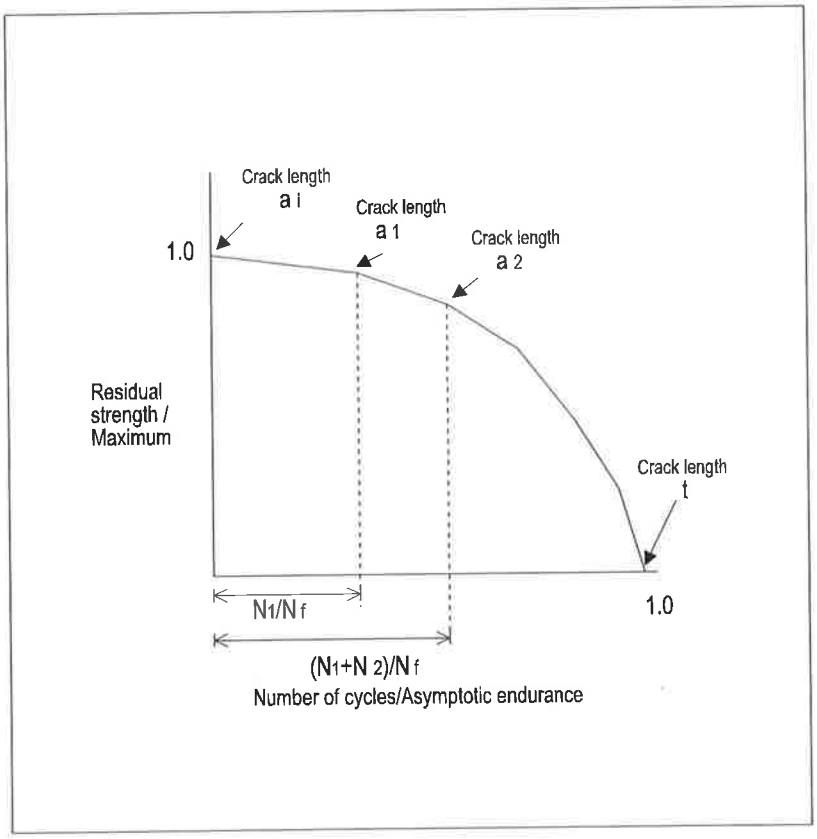



Fig.5.2 Non-dimensional residual strength curve

and the number of cycles required by the initial crack to propagate to the crack length $a_2$ can be given as

$$N_1 + N_2 = \frac{D_1 + D_2}{(\Delta\sigma)^3} \tag{5.5}$$

while the number of cycles required by the initial crack to propagate through the thickness of the component can be given as

$$N_f = \frac{D_1 + D_2 + .. + D_n}{(\Delta\sigma)^m} \tag{5.6}$$

where $N_f$ is the asymptotic endurance. If the residual strength curve due to the stress range $\Delta\sigma$ in Fig.5.1 is non-dimensionalised as in Fig.5.2, then the abscissae corresponding to crack length $a_1$ can be derived from Eqns.5.4 and 5.5 as

$$\frac{N_1}{N_f} = \frac{D_1}{D_1 + D_2 + .. + D_n} \tag{5.7}$$

Similarly the abscissae corresponding to the crack length $a_2$ can be derived from Eqns.5.5 and 5.6 as

$$\frac{N_1 + N_2}{N_f} = \frac{D_1 + D_2}{D_1 + D_2 + .. D_n} \tag{5.8}$$

and so on.

The values of $N_1/N_f$ and $(N_1+N_2)/N_f$ in Eqns.5.7 and 5.8 depends only on the values of D and since values of D is independent of the stress range, the values in the abscissae of a non-dimensional residual strength curve is independent of its stress range. The ordinates of a residual strength curves are given by Eqns. 2.24, 2.26 and 2.27 which does not depend on the stress ranges. The ordinates has been non-dimensionalised in

Fig.5.2 by dividing the values of residual strength on the Y-axis by the maximum residual strength. The non-dimensionalised residual strength curve has been used to develop a design procedure in Chapter 8.

## 5.4 Unstable crack propagation envelope

The method of determining the general residual strength variation of a component that is described in Section 5.2, has been used here to specifically determine the residual strength variation of a component for unstable crack propagation failure. The component is considered to have an initial crack length of 0.25 mm. Increments of 0.25 mm are then taken and the number of cycles required by a crack to propagate through each increment is calculated using Eqn.5.1. Then the strength of the component at the start and end of each increment is found out using Eqns.2.24, 2.26 and 2.27 to determine the residual strength variation. This is carried out till the crack reaches the thickness of the component.

The variation of the crack length with number of cycles and the variation of residual strength with number of cycles is found for a stiffener weld and cover plate of A 36 steel and 4340 steel. A 36 steel is a steel of the ferrite-pearlite type and has a tensile strength of 412 $N/mm^2$, a fracture toughness of 1400 $N/mm^{3/2}$ (Section 2.6.2.2.1) the value of the constant m in Paris' Equation (2.19) of 3 and a value of C=2.3 $x10^{-13}$ (Section 2.5.1). The 4340 steel falls in the martensitic class and has a tensile strength of 1450 $N/mm^2$ and a fracture toughness of 1470 $N/mm^{3/2}$ (Section 2.6.2.2.1), a value of m of 2.25 and a value of C of 2 x $10^{-10}$ (Section 2.5.1). The dimensions of the stiffener weld and a cover plate and the corresponding values of M used has been provided by Albrecht and Yamada (Fig.2.16 and 2.17). Figures 5.3a gives the variation of crack length of a stiffener weld with number of cycles while Fig.5.3b gives the variation of residual strength with number of cycles for a stiffener weld. Figure 5.4 gives the residual strength variation of a cover plate of A 36 steel. Figure 5.5 gives the residual strength variation of a stiffener weld for 4340 steel. It is to be noted that the value of the residual strength of a component in Figs.5.3 to 5.5 are based on unstable crack propagation. The residual strength also depends on the plastic deformation strength, and it is lower of these two strengths ie. the unstable crack propagation strength and the plastic deformation strength, which is the actual strength of the component.

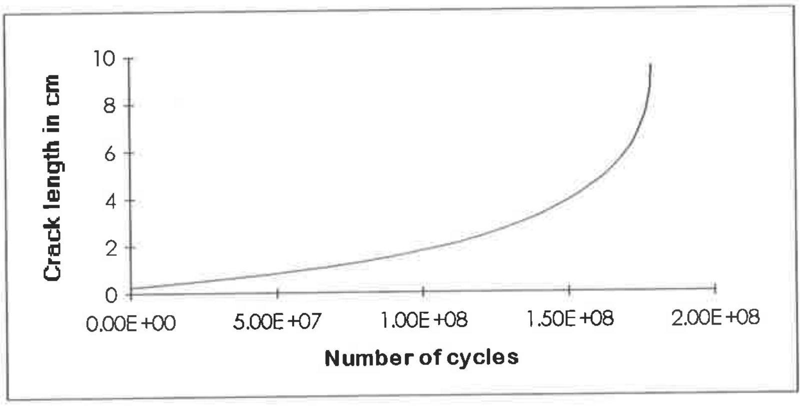



Fig.5.3a Variation of crack length with number of cycles for a stiffener weld of A 36 steel

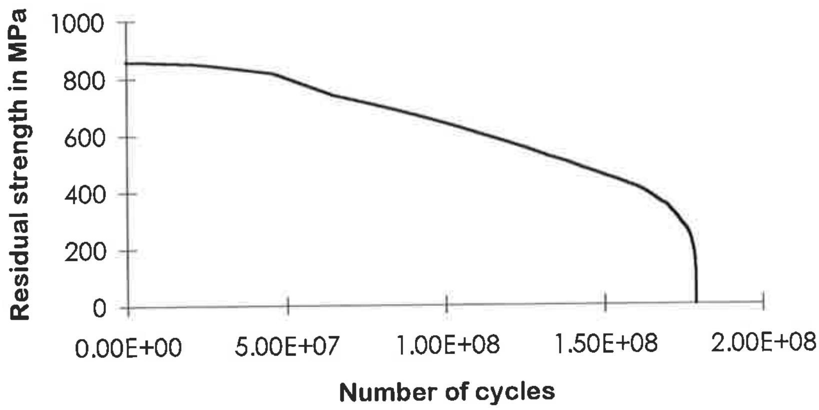



Fig.5.3b Unstable crack propagation envelope of a stiffener weld of A 36 steel

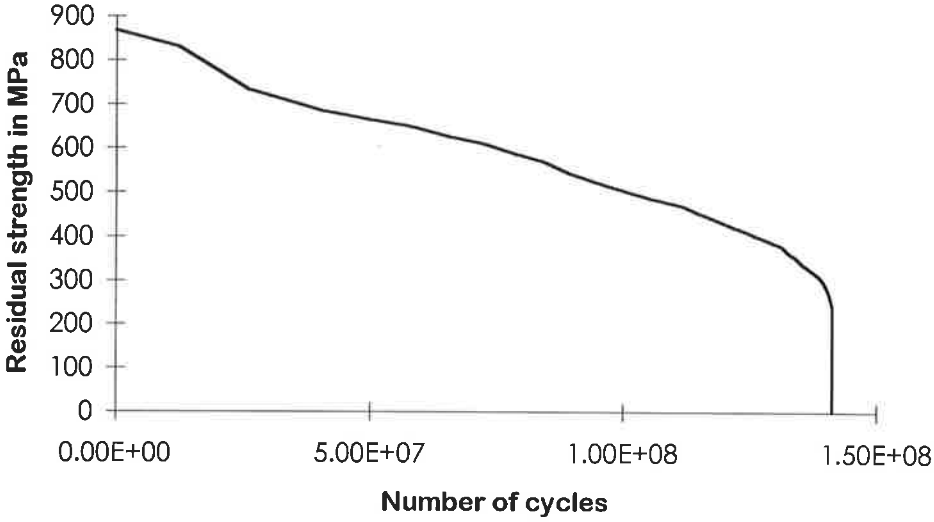



Fig.5.4 Unstable crack propagation envelope of cover plate of A 36 steel (see App.2 for crack length variation).

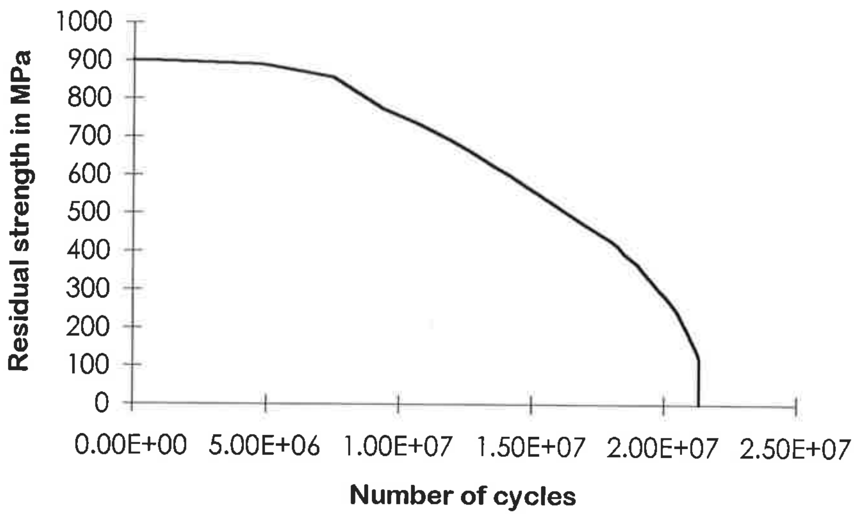



Fig.5.5 Unstable crack propagation envelope of stiffener weld of 4340 steel (see App.2 for crack length variation)

### 5.4.1 Factors affecting the Unstable Crack Propagation Envelope

The failure envelope due to unstable crack propagation is obtained by using Eqns 5.1 and 2.24. Hence the residual strength variation would depend on the parameters given by these equations which are the stress range $\Delta\sigma$, the magnification factor M, the fracture toughness $K_{IC}$ and the constants m and C. The effect of the stress range $\Delta\sigma$ on the residual strength curve has already been discussed in Section 5.3. The effect of the magnification factor M and fracture toughness will be discussed later in Section 5.4.1.1 and 5.4.1.2.

#### 5.4.1.1 Variation of the unstable crack propagation curve with the constants m and C.

The value of m and C will vary only if the type of steel is martensitic instead of ferrite-pearlite. Fig 5.3 shows the residual strength variation of the stiffener weld of ferrite-pearlite steel (A-36) while Fig 5.5 shows the residual strength variation of the same stiffener weld composed of martensitic steel (4340 steel). It can be seen that the asymptotic endurance of a component of martensitic steel is much smaller compared to the asymptotic endurance of a component of ferrite pearlite steel. Since the value of C of a component of martensitic steel is much larger compared to that of ferrite pearlite steel, there is a corresponding variation in the value of the asymptotic endurance.

#### 5.4.1.2 Variation of the unstable crack propagation curve with the magnification factor M

The magnification factor M of a component has been given by (Eqn.2.8) Albrecht and Yamada by the equation

$$M = F_W F_S F_E F_G \quad (5.5)$$

It was discussed in the literature review (Section 2.3.3.2) that the factors $F_W$, $F_S$ and $F_E$, i.e. the width correction factor, the surface correction factor and the elliptical crack front correction factor only depend on the crack length 'a' as shown by Fig.2.16a and is unchanged when there is any variation in the shape or type of component. The only remaining factor is the geometry correction factor $F_G$.

The geometry correction factor is given by the Eqn.2.13. As can be seen from the equation, the correction factor depends on the variation of stress $\sigma_{bi}$. This is the stress acting along the uncracked portion of the component where the crack is expected to form. This distribution of stress would depend on the geometry of the component. Hence for different geometries, the distribution of stress along the section where the crack is expected to form varies causing a change in the geometry correction factor which causes the magnification factor M to change which in turn causes the residual strength to vary. Fig 2.16b gives the variation of stress concentration acting on a section A-A of the stiffener weld and Fig. 2.16a gives the corresponding values of M for the stress distribution along A-A.

In order to investigate the variation of the geometry correction factor $F_G$, with the variation of stresses acting on the section A-A of a stiffener weld, several different stress distributions have been applied. The variation in stress concentration for different stress distributions along the thickness of the component is shown in Fig.5.6. Series 1 does not consider any stress concentration at all while Series 2 and 3 consider higher stress concentration at the edges. A change in stress distributions causes the geometry correction factor to change which results in a corresponding variation in the value of M and hence in the residual strength variation.

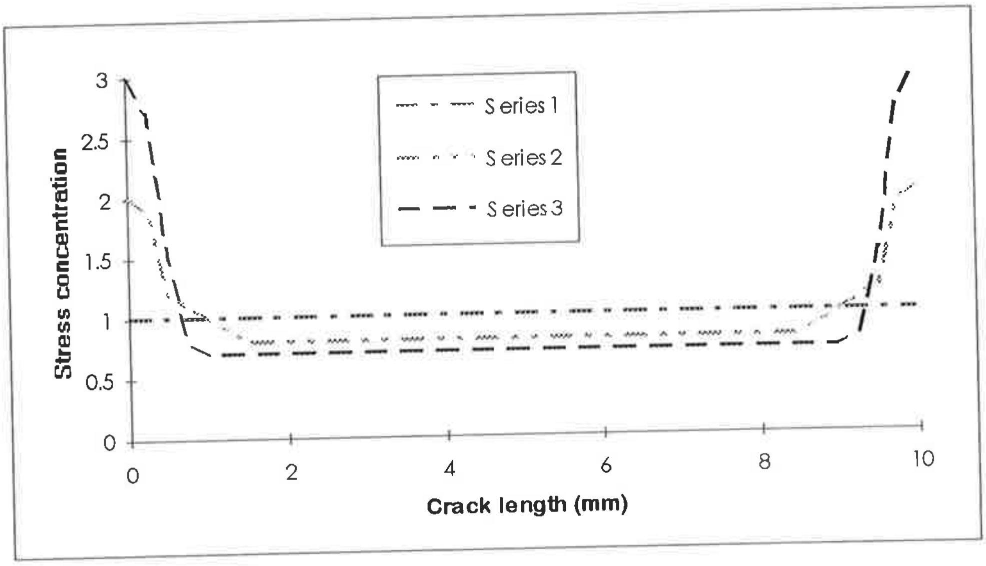



Fig.5.6 Stress concentration

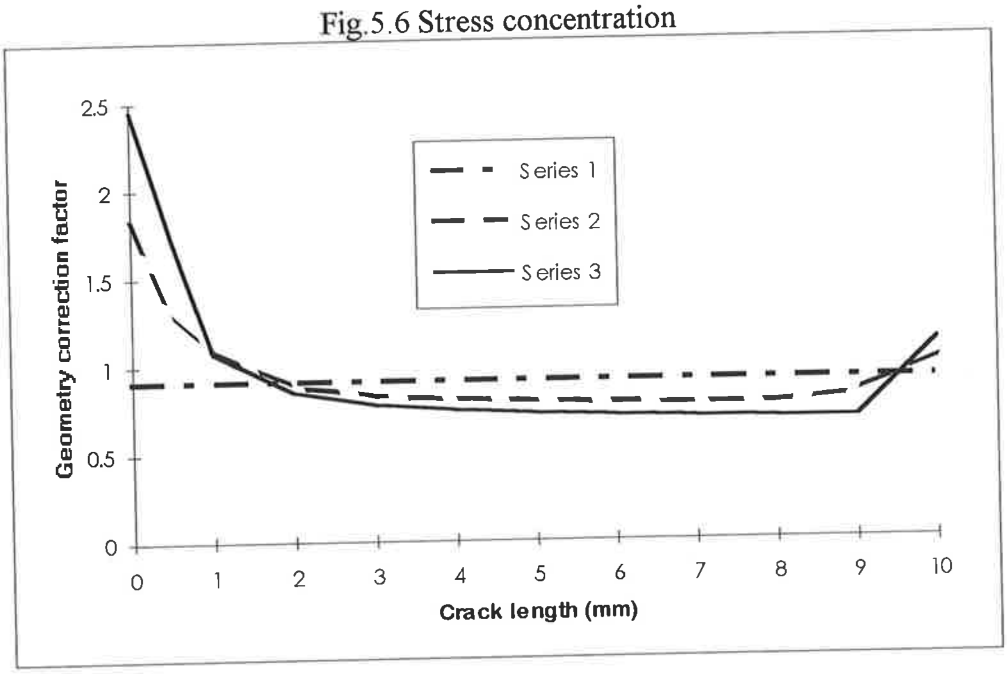



Fig.5.7 Variation of geometry correction factor

Figure 5.7 shows the values of the geometry correction factor for the three series considered. It can be seen that when there is no stress concentration (Series 1) along the section where the crack is expected to form then the value of $F_G$ remains at 0.91. It can also be seen that a higher stress concentration near the surface of the component (Series 2 and 3) causes $F_G$ to be higher in the initial stages of crack growth while there is no significant change for the remaining period of the fatigue life. When the values of $F_G$ are high the corresponding values of M will also be high since the other correction factors do not change for a given shape or type of component. Equation 2.24 shows that large values of M will cause the residual strength due to unstable crack propagation to be lower in the initial part of the fatigue life. Equation 3.19 shows that the crack will propagate faster during the initial stages of the fatigue life when the value of M is high.

#### 5.4.1.3 Variation of the unstable crack propagation curve with fracture toughness

The strength of a component due to unstable crack propagation depends on its fracture toughness as shown by Eqn.2.24. It was earlier discussed in the literature review (Section 2.6.2.2.1 and Section 2.6.2.2.2) that fracture toughness varies with thickness, temperature and rate of loading. Hence the residual strength due to unstable crack propagation will also depend on the thickness, temperature and rate of loading which are discussed here in detail.

##### 5.4.1.3.1 *Variation of unstable crack propagation curve with plate thickness*

As discussed in Section 2.6.2.2.1 of the literature review, the fracture toughness of a component reduces as the component becomes thicker. A smaller value of fracture toughness in Eqn.2.24 causes the residual strength to reduce. Hence the thicker the component the lower the value of fracture toughness and the corresponding residual strength. As the thickness of the component increases a value of the thickness is reached for which the fracture toughness called the plane strain fracture toughness is the lowest value of fracture toughness for any thickness. The plane strain fracture toughness when used in Eqn.2.24 gives the lowest value of the unstable crack propagation strength of a component. For calculation purposes, the plane strain fracture toughness which is readily available, has been considered throughout the thesis and any variation of fracture toughness with thickness has not been taken into account. Hence the analysis procedure followed here will accurately analyse thick plates while the analysis will be conservative for thin plates.

#### 5.4.1.3.2 *Variation of unstable propagation curve with temperature*

The fracture toughness of a component decreases with a fall in temperature as discussed in Section 2.6.2.2.2. Figures 2.25, 2.26 and 2.27 show the variation of the fracture toughness of A 36, A572 and 517 F steel with temperature. The values of the fracture toughness of A 36 steel as given in Fig.2.25 have been used to determine the variation of residual strength of a stiffener weld with temperature in Fig.5.8. It can be seen that the strength of the component at -184° C is almost half its strength at 16° C. Thus a reduction in temperature causes a considerable change in the unstable crack propagation strength of a component. The effect of this change in temperature on the actual residual strength curve will be discussed later in Section 5.6.1.2.

#### 5.4.1.3.3 *Variation of unstable crack propagation curve with rate of loading*

The fracture toughness of a component decreases with an increase in rate of loading as discussed in Section 2.6.2.2.2. Figs.2.25, 2.26 and 2.27 give the variation of fracture toughness of A 36, A 572 and 517 F steel with rate of loading. The residual strength variation of a stiffener weld of A 36 steel with rate of loading is shown in

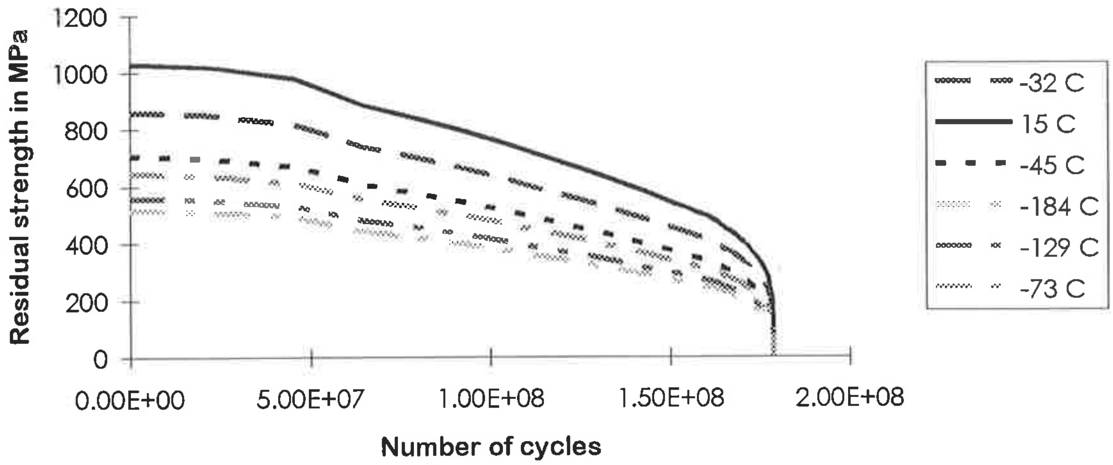



Fig.5.8 Temperature effect on unstable crack propagation envelope of a stiffener weld of A 36 steel

Fig.5.9. It can be seen that the strength of a component decreases by almost one third with an increase in strain rate from $10^{-5}$/s to a strain rate 10/s. Hence there is a considerable variation in the unstable crack propagation strength of a component with a change in the rate of loading. The effect of this change in rate of loading on the actual residual strength curve has been discussed later in Section 5.6.1.2.

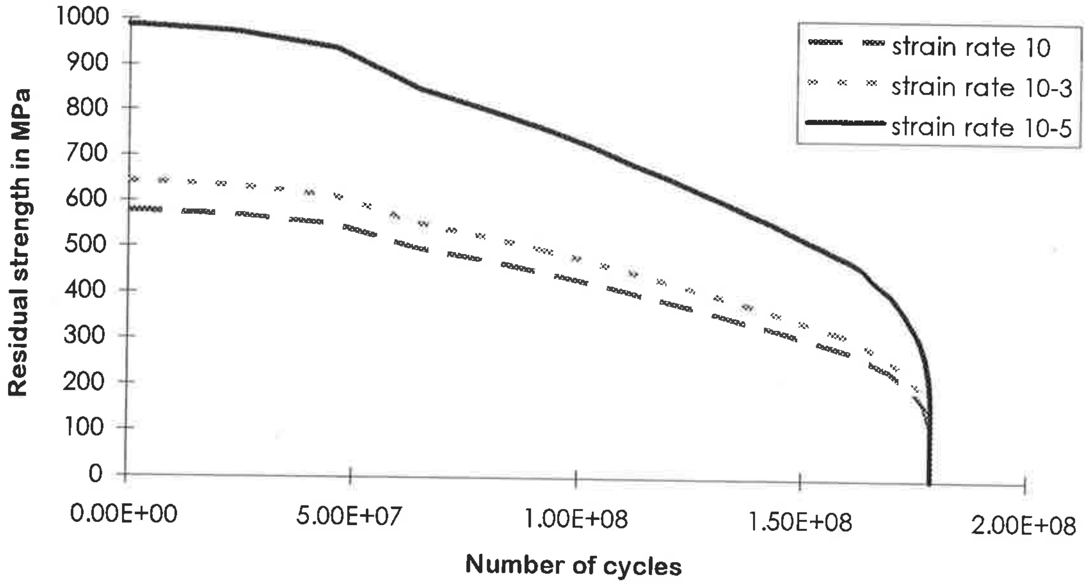



Fig.5.9 Effect of rate of loading on unstable crack propagation curve of stiffener weld at -37 C.

## 5.5 Plastic deformation failure

As discussed in the literature review, that even though fracture mechanics theory gives the unstable crack propagation strength of a component, failure of such a component will often occur by plastic deformation (Section 2.6.3). For example, Feddersen (Section 2.8.2) tested centrally cracked thin sheet panels and found that components with small crack lengths failed due to plasticity while components with intermediate crack lengths fail by unstable crack propagation. Furthermore, Cox (Section 2.8.3) conducted experiments with incomplete penetration butt welds and found that components of all crack sizes failed by plastic deformation. He derived a theoretical model for predicting the plastic deformation strength against crack length which agreed well with the experimental data. The model used the weakest section of the base metal to determine the residual strength for components with smaller crack lengths (Fig.2.37). At this stage the crack length was considered to be small and any reduction in strength due to the presence of a crack length was neglected. For larger crack lengths the model used the material properties of the weld metal to determine the residual strength. The reduction in strength of the component due to presence of the crack was taken into account at this stage.

The model proposed by Cox to determine the plastic deformation strength of a component uses the material properties of both the weld metal and the base metal. The material properties of the weld metal of the component is not commonly known and hence the plastic deformation strengths have been determined in the thesis using the properties of the base metal in Eqns.2.26 or 2.27. Thus either Eqn.2.26, which gives the strength of the component according to Von-Mises criteria, or Eqn.2.27, which gives the residual strength according to Tresca's criteria have been used throughout the fatigue life. The method proposed is simple as the plastic deformation strength can be given as a single equation throughout the fatigue life, instead of using different equations for smaller and larger crack lengths as proposed by Cox. The proposed method is conservative for smaller crack lengths as compared to Cox's model as it takes into account the reduction in strength of a component due to the presence of a crack right from the start of the fatigue life which Cox's model does not. For larger crack lengths the method used is again conservative compared to Cox's model as it takes into account the material properties of the base metal which has a lower tensile strength than the weld metal considered in Cox's model. Hence, the method used here gives more conservative values compared to Cox's model and therefore can be considered to be safe.

The proposed method for determining the plastic deformation strength of a component has been used to determine the residual strength envelope in a similar manner to that of a unstable crack propagation curve earlier described in Section 5.4. Thus the residual strength curve is determined by numerical integration in which small increments of crack length are considered. The number of cycles required by a crack to propagate through an increment is given by Eqn.5.1. The plastic deformation strength of a component is found at the start and end of each increment and is given by Eqns.2.26 and 2.27. The number of cycles required by cracks to propagate between increments and the strengths at the start and end of each increment being known the residual strength variation due to plastic deformation has been determined The plastic deformation strength of a stiffener weld and a cover plate of A 36 steel is shown in Figs.5.10 and 5.11. Fig.5.12 shows the plastic deformation failure envelope for a stiffener weld of 4340 steel.

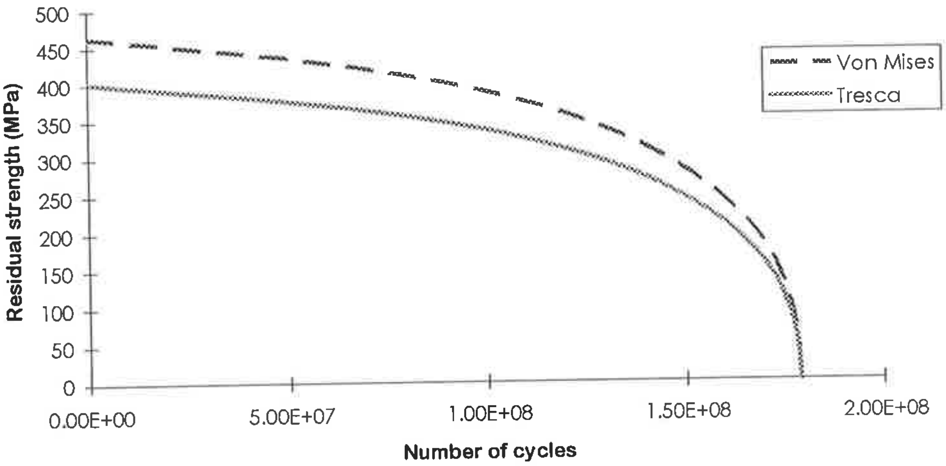



Fig.5.10 Plastic deformation failure envelope of stiffener weld of A 36 steel

### 5.5.1 Factors affecting the Plastic Deformation Envelope

The factors which affect the plastic deformation envelope of a component can be obtained from the equations used to determine the envelope. The envelope is determined by using Eqn.5.1 which gives the number of cycles and Eqns.2.26 and 2.27 which gives

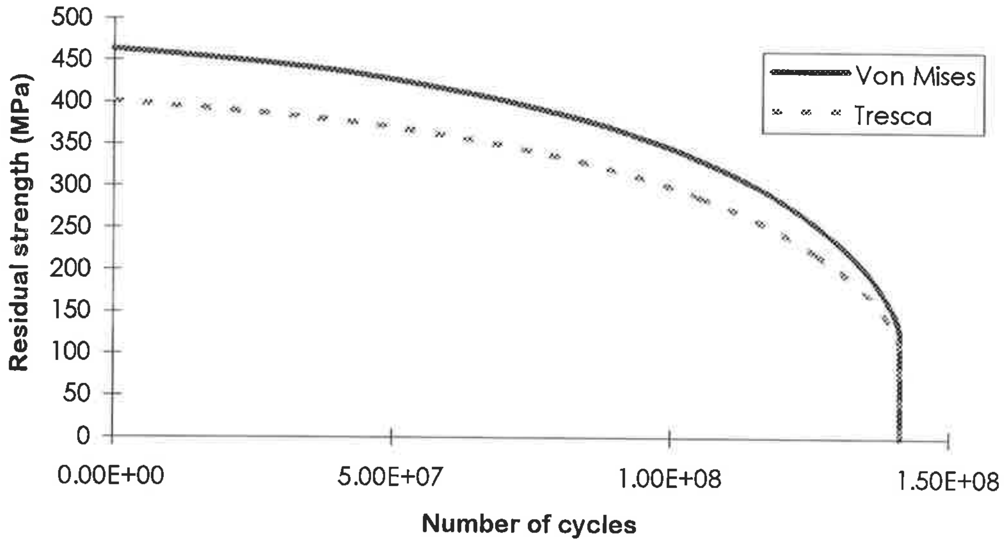



Fig.5.11 Plastic deformation failure envelope for cover plate of A36 steel

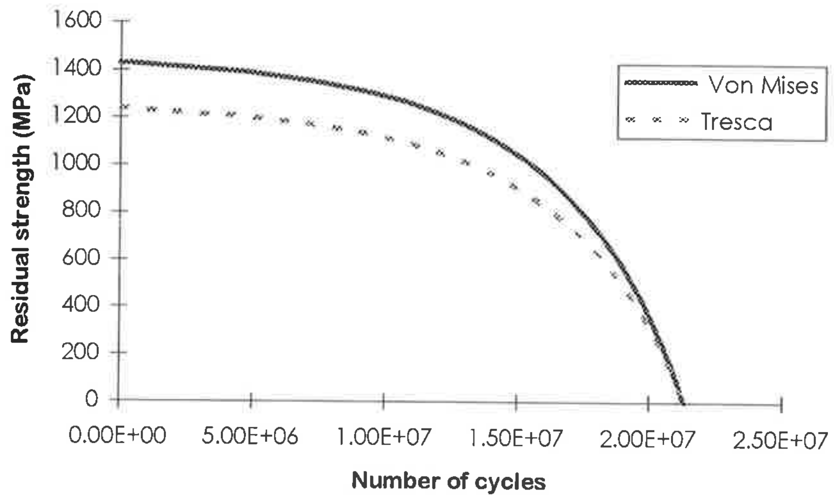



Fig.5.12 Plastic deformation failure envelope for stiffener weld of 4340 steel

the plastic deformation strength. As can be seen from Eqns.5.1, 2.26 and 2.27 the plastic deformation curve depends on the stress range $\Delta\sigma$, the magnification factor M, the material properties C and m and the tensile strength $f_u$. The variation of residual strength with stress range was discussed in Section 5.2. The values of M vary with the crack length (Fig.2.16, 2.17 and 2.18) and when these values are high the number of cycles required by the component to propagate between increments obtained using Eqn.5.1 is low while when these values of M are low the number of cycles required by cracks to propagate between increments are large. The change in the plastic deformation curve due to variation in the material properties C and m can be found by comparing Fig.5.10 which gives the plastic deformation variation of a curve of ferrite-pearlite steel with Fig.5.12 which shows the plastic deformation variation of a curve of martensitic steel. It can easily be seen that components of martensitic steel have a shorter life compared to components of ferrite pearlite steel. A variation in the tensile strength of the component causes the plastic deformation strength to change. A component with a high tensile strength will also have a high plastic deformation strength.

The residual strength of a component failing by plastic deformation increases with a decrease in temperature. The variation of the plastic deformation strength with rate of loading is not known (Section 2.6.3.2). Here it has been assumed that the plastic deformation strength of a component does not change with a change in temperature and rate of loading.

## 5.6 Residual strength envelope

In Sections 5.4 and 5.5, the methods of determining the residual strength variation for two different modes of failure, namely unstable crack propagation and plastic deformation were discussed. A component at any period of its fatigue life will fail by the mode which gives the lesser value of strength. In order to determine the mode by which the component fails at a given period of the fatigue life, the residual strengths due to these two modes must be compared. The lower bound of these two curves gives the mode by which a component fails.

Since the residual strength variation of a component depends on the comparative residual strengths of the unstable crack propagation curve and the plastic deformation curves, residual strength curves in general can be classified into three groups that are based on a comparison of the two modes. The first group includes residual strength curves whose plastic deformation strength is lower than the unstable crack propagation strength and controls failure throughout the fatigue life. The second group would include residual strength curves whose unstable crack propagation failure strength is lower than the plastic deformation strength and controls failure throughout the fatigue life. The third group includes residual strength curves with a mixed mode of failure through both plastic deformation and unstable crack propagation at different portions of the fatigue life.

The residual strength envelope for a stiffener weld and a cover plate earlier described in Section 5.4, has been determined here by superimposing the residual strength curve due to unstable crack propagation (Section 5.4) and the residual strength due to plastic deformation (Section 5.5). Hence superimposing Fig.5.3 with Fig.5.10 we get the residual strength envelope of a stiffener weld of A36 steel as in Fig.5.13. As can be seen from Fig.5.13, the plastic deformation envelope gives the lower value of strength for a stiffener weld of A 36 steel and therefore controls failure. Similarly superimposing Fig.5.4 with Fig.5.11, we get the residual strength envelope in Fig.5.14 of a cover plate of A36 steel which again is controlled by plastic deformation failure. The residual strength variation for a stiffener weld and a cover plate of A 36 steel therefore belongs to the first group of residual strength curves described in the earlier paragraph.

The residual strength envelope of a stiffener weld of 4340 steel is given in Fig.5.15 which is obtained by superimposing Fig.5.5 with Fig.5.12. As can be seen from Fig.5.15 residual strength of such a component is controlled by unstable crack propagation for the major part of the fatigue. As the residual strength curve of a stiffener weld of 4340 steel is also partly controlled by plastic deformation such a residual strength curve belongs to the third group which includes components having a mixed mode of failure.

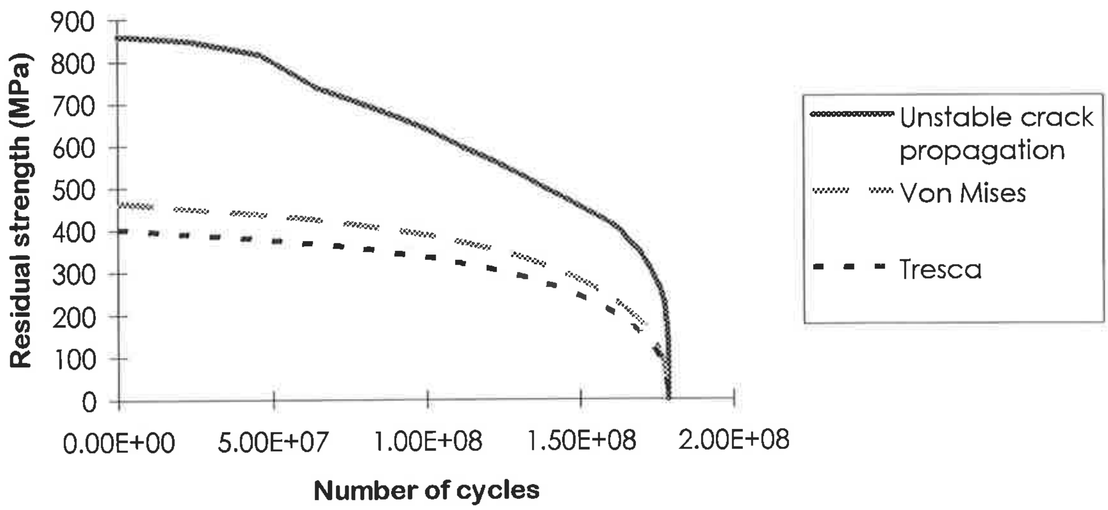



Fig.5.13 Residual strength envelope for stiffener weld of A36 steel

### 5.6.1 Factors affecting the Residual Strength Envelope

The residual strength envelope depends on all the factors that affect the plastic deformation envelope and the unstable crack propagation envelope. These factors have earlier been discussed in Section 5.4.1 and Section 5.5.1. They can be broadly divided into three categories. The first category includes factors relating to the stresses applied to

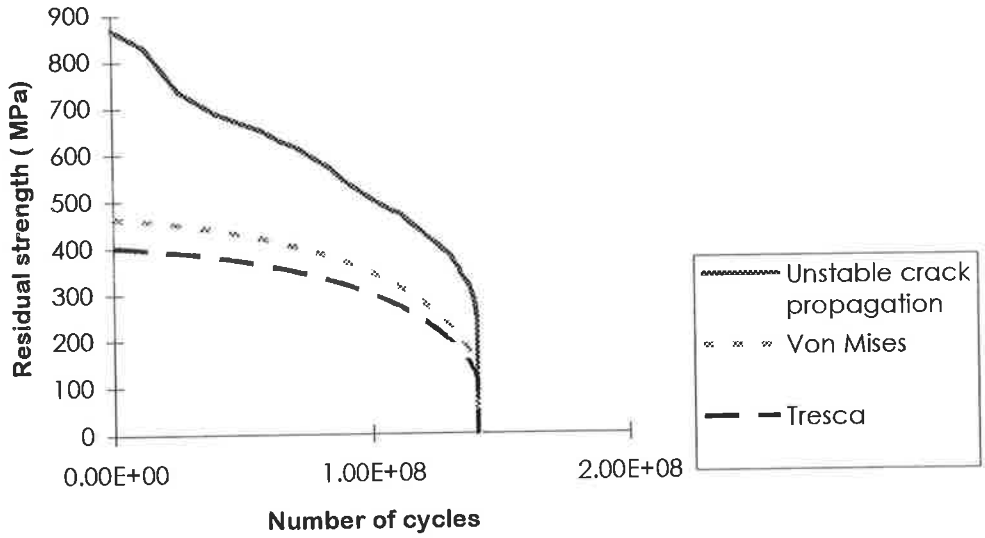



Fig.5.14 Residual strength envelope for cover plate of A36 steel

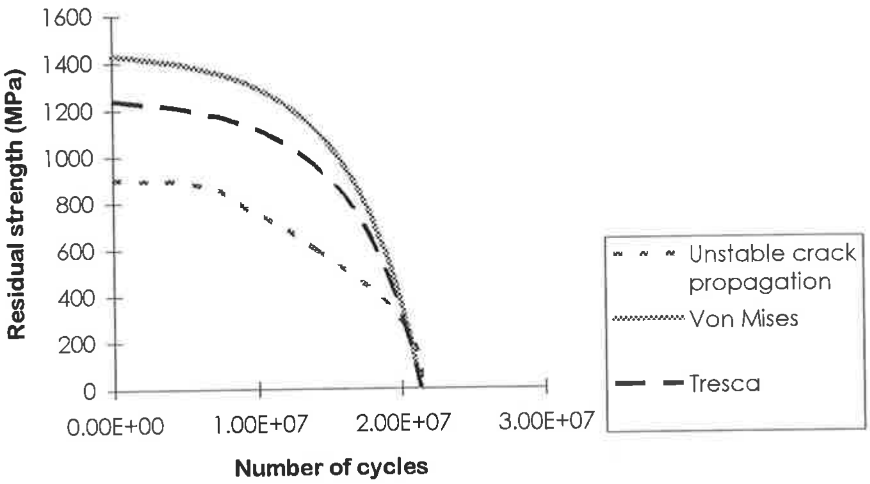



Fig.5.15 Residual strength envelope of stiffener weld of 4340 steel

the component, the second category includes factors relating to the material properties of the component while the third category includes other factors such as temperature and rate of loading. The effect of the stress range, which corresponds to the first category, has earlier been discussed in Section 5.3. The effect of the factors corresponding to the second and third category is discussed here.

### 5.6.1.1 Variation of residual strength curve with material properties

It can be seen from Eqns.5.1, 2.24, 2.26 and 2.27, which were used to determine the residual strength envelopes that the residual strength depends on four material properties: C, m, the tensile strength $f_u$ and the fracture toughness $K_c$. A variation in C and m changes only the rate of crack propagation which affects the total life of the structure as discussed in Section 5.4.1.1 and Section 5.5.1. The fracture toughness and the tensile strength of a component on the other hand are related to the strength of the component. A component of a given crack length would fail by plastic deformation or by unstable crack propagation failure depending upon the comparative values of the tensile strength and the fracture toughness. If the value of the fracture toughness is high and tensile strength is low the component is likely to fail by plastic deformation as in the case of Fig.5.13 and 5.14 and belongs to the first case described in Section 5.6. While if the tensile strength is high and the fracture toughness is low the component is likely to fail by unstable crack propagation and belongs to the second case described in Section 5.6.

### 5.6.1.2 Variation of residual strength curve due to temperature and rate of loading

It has been discussed in Section 2.6.2.2.2 that the temperature and rate of loading affect the fracture toughness of a component. Variation in the fracture toughness causes the residual strength due to unstable crack propagation to change. The variation of residual strength due to unstable crack propagation with temperature and rate of loading was discussed in Section 5.6. A change in temperature and loading rate also affects the actual residual strength envelope. A discussion on the variation of residual strength with temperature and rate of loading is carried out here considering the three groups of residual strength curves described in Section 5.6.

The first group includes components whose plastic deformation strength is lower than the unstable crack propagation strength. If the temperature of a component belonging to this group is decreased or the rate of loading is increased then the fracture toughness will decrease. A decrease in the fracture toughness will cause the residual strength of the unstable crack propagation curve to decrease and at a certain stage the unstable crack propagation envelope will touch the plastic deformation envelope and failure will no longer be entirely controlled by plastic deformation as shown in Fig.5.16a. However, until this stage is reached, the failure envelope strengths will be not affected with a change in temperature and rate of loading. If the fracture toughness decreases further it will cause the unstable crack propagation envelope to occupy a large portion of the fatigue life and eventually may end up occupying the entire fatigue life.

The second group includes components whose unstable crack propagation envelope gives the lower value of strength and controls failure. A decrease in temperature or increase in the rate of loading will cause the residual strength envelope in Fig.5.16b to reduce further.

The third group includes components whose unstable crack propagation envelope controls part of the fatigue envelope. A decrease in temperature or increasing in the rate of loading for such envelope will cause a larger portion of the fatigue life to be occupied by unstable crack propagation (Fig.5.16c). A further decrease may cause the entire residual strength curve to be controlled by unstable crack propagation.

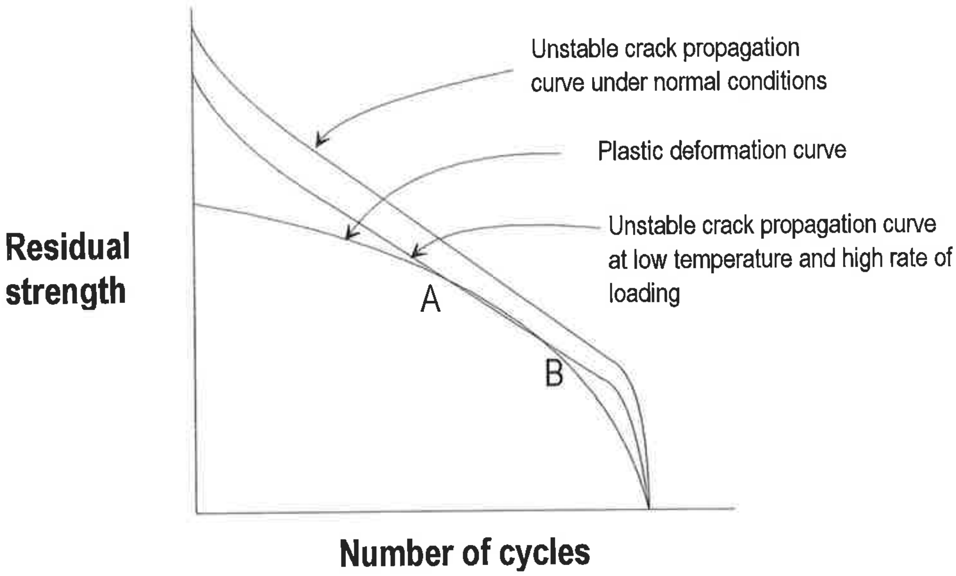



Fig.5.16a Variation of first group of residual strength curve with change in temperature and rate of loading

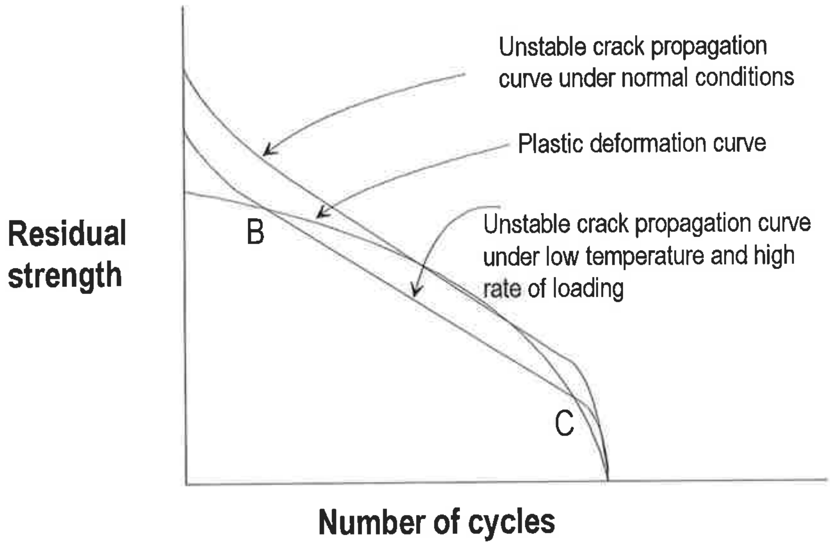



Fig.5.16b Variation of residual strength curve of the second group with change in temperature and rate of loading

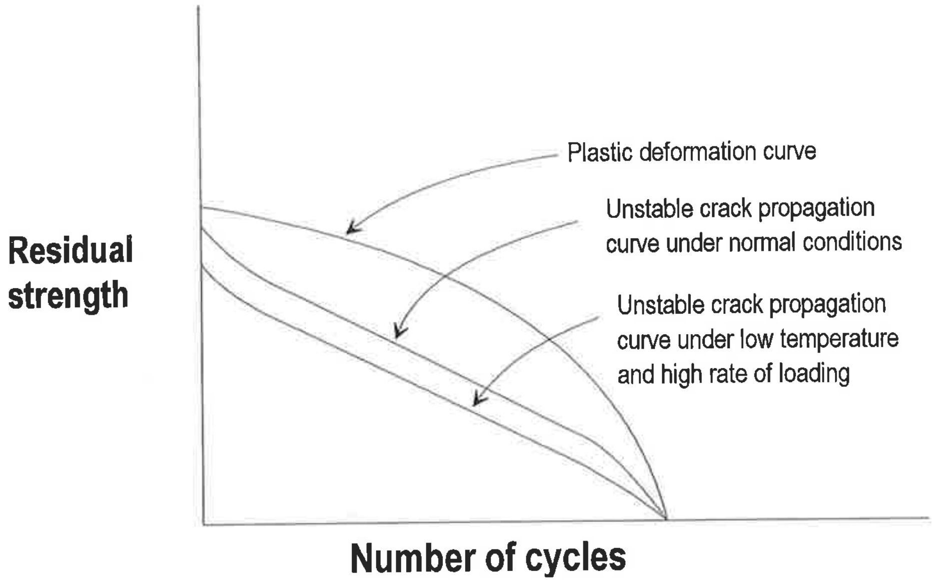



Fig.5.16c Variation of residual strength curve of third group with change in temperature and rate of loading

The variation of residual strength with temperature and with rate of loading is of extreme practical significance as it causes the strength of a component to change. For components included in the first group, if a change in temperature or rate of loading causes a portion of the unstable crack propagation curve to go below the plastic deformation curve as shown in Fig.5.16a then for a component at the stage AB of its fatigue life, the strength decreases and the component is able to sustain a lesser maximum load. An example is shown in Fig.5.18. It can be seen that at a temperature of -45° C and a strain rate of 20 strains/s the unstable crack propagation curve of a stiffener weld of A 517 F steel falls below the plastic deformation curve. In the case of components belonging to the second group, a change in temperature and rate of loading will cause the unstable propagation envelope to reduce (Fig.5.16b) so that the component is able to sustain a reduced maximum load for the entire period of the fatigue life. In the case of components belonging to the third group shown in Fig.5.16c a decrease in the unstable crack propagation failure will cause a larger portion of the fatigue life to be controlled by unstable crack propagation failure and the strength of the portion controlled by unstable crack propagation will reduce (BC in Fig 5.16b)

Thus for components subjected to fatigue loading the strength of the component during the fatigue life may reduce with lower temperature and higher rate of loading. However often the maximum loads that can be applied to a component are calculated based on its strength at room temperature. If one keeps on applying these loads to the component at significantly lower temperatures and higher rates of loading than the actual strength of the component will have reduced compared to its strength at room temperature, and the component might fail.

Failure of welded components at low temperatures and high rates of loading especially in ships (Section 2.6.1) is well documented. It is suggested that such failure occurs as the loads that were applied to such a component were based on its strength at room temperature, but the actual strength of a component at low temperature and high rates of loading is much lower. The above procedure, which determines the strength of a welded component at a given temperature and loading rate at any period of the fatigue life can be used to predict the strength of a component and prevent such failure.

The lowest temperatures ever recorded is around -65° C while the maximum strain rate for bridges is around $10^{-3}$/s (Section 2.6.2.2.2). Figure 5.17 and Fig.5.18 show the residual strength variation of a stiffener weld of A 36 steel and A517 F steel at a temperature of -73° C and a strain rate of $10^{-1}$/s. The curves can therefore be considered as the most conservative. It can also be seen from Fig.5.17 that the plastic deformation strength of a stiffener weld of A 36 steel will give lesser values of strength compared to the unstable crack propagation curve even under the most conservative conditions. Hence a stiffener weld of A 36 steel is a type of component whose plastic deformation envelope will always control failure. Figure 5.15 shows that the residual strength variation of a stiffener weld of 4340 steel is controlled entirely by unstable crack propagation failure. The design of a stiffener weld of 4340 steel in a bridge, therefore can be carried out based on the variation of its strength with temperature and rate of loading. Its most conservative strengths in summer and winter can be determined and loading allowed on the bridge during summer and winter can be based on these strengths.

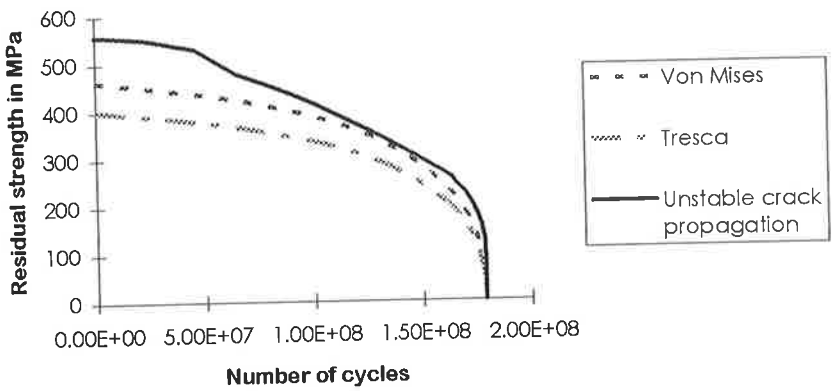



Fig.5.17 Figure showing residual strength curve for stiffener weld of A 36 steel (Temp -73 C, strain rate $10^{-1}$ s)

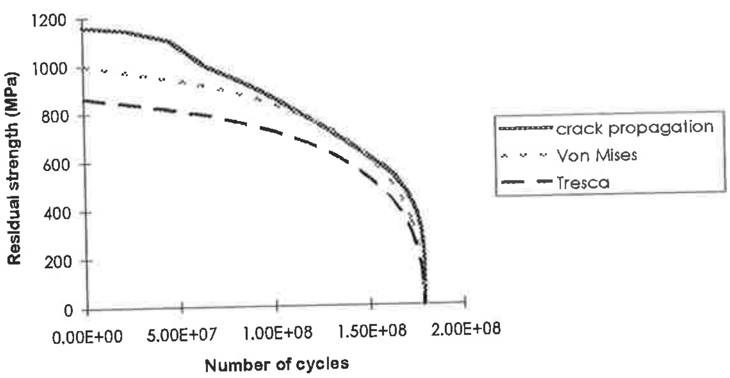



Fig.5.18 Residual strength variation of component of A 517 F steel.
(Temp-73 C, Strain rate $10^{-1}$/s) or (Temp-45 C and Strain rate 20/s)

#### 5.6.1.3 Crack lengths at which mode changes

Among the three groups of residual strength curves discussed in Section 5.6 there is no change in the mode of failure during the fatigue life for the first and second group since failure is controlled by one mode. However as the third group includes curves with a mixed mode of failure, there will be a change in the mode of failure at an intermediate stage of the fatigue life for components corresponding to this curve. The residual strength curve due to unstable crack propagation and due to plastic deformation when superimposed will show the change in the mode of failure of the component (Fig 5.16c). The crack length at which the mode changes can be determined by equating the failure equation for both modes. Thus Eqn.2.24 can be equated with Eqn.2.26 and 2.27 to give the crack lengths at which the mode changes.

## 5.7 Conclusions

A procedure of developing the residual strength variation of welded components under constant amplitude loading has been discussed in this chapter. The method involved determining the residual strength variation due to unstable crack propagation and due to plastic deformation using numerical integration. These curves were superimposed and the lower bound gave the mode of failure and actual residual strength of a component. The residual strength is also found to depend on such material properties as the tensile strength and the fracture toughness of the component. It was also found that the residual strength variation of welded components can decrease sharply with low temperatures and with high rates of loading.

Components in practise are subjected to variable amplitude loading. The constant amplitude curves developed here has been used to find the reduction in strength of components subjected to variable amplitude loading in Chapter 7 and to design components subjected to variable loading in Chapter 8. The procedure developed involves linearising the unstable crack propagation curve to find the unstable crack propagation strength and crack length of a component subjected to variable loads. The plastic deformation strength is found from the crack length and is compared with the unstable crack propagation strength, the lower value giving the residual strength of a component. A discussion on how the factors such as materials, temperature and rate of loading will affect the residual strength of a component subjected to variable amplitude loading is carried out in Section 7.5.2.

# Chapter 6
# Adapting S/N curves for determining the initial crack length

## 6.1 Introduction

In the previous chapter, the procedure for determining the residual strength variation of welded components using fracture mechanics was discussed. Examples of the variations of residual strength were shown for components with an assumed initial crack length of 0.25 mm. However, the variation of the residual strength of a component during the fatigue life can only be determined if one knows the actual initial crack length of the component. Such data on the initial crack lengths of components is not available at present. However, a large amount of data is readily available on the fatigue life of components. S/N curves are based on such data and have been used to determine the initial crack lengths of components.

Once these initial crack lengths are determined, they can be used to find the residual strength variation of components for constant amplitude loading as was earlier described in Chapter 5. These residual strength curves have been linearised and adapted for variable amplitude loading to assess and design components as described in Chapters 7 and 8. A method of design and assessment has, therefore, been developed that uses S/N curves to define both the start and end of the fatigue life and uses fracture mechanics to define the condition of a component within these boundaries.

The procedure that has been developed can also be seen as an advancement over present methods of design that solely use S/N curves. As procedures using S/N curves give only the start and end of fatigue lives, they are ineffective in determining the condition of a structure during the fatigue life. Thus a technique of design that combines S/N curves, which gives the limits of the fatigue life, with fracture mechanics, which provide the analysis within the fatigue life, is more effective than present methods. It integrates fracture mechanics with S/N curves so that they complement each other and makes the best possible use of the current information to develop a fatigue design and analysis procedure.

In this chapter, it will be shown how S/N curves can be used to determine the initial crack lengths of components. The procedure developed here is applicable only for components of structural (ferrite-pearlite) steel for which the value of m in Paris' equation is 3. The initial crack lengths have been determined by a analytical and a graphical method from the fatigue life obtained from S/N curves with various probabilities of failure. It has been shown here how the technique easily copes with these various probabilities of failure by converting them into initial crack sizes. Also since the fatigue life depends on the thickness of a component (Section 2.7.1), the variation of the initial crack length with thickness for different components is discussed.

## 6.2 Fatigue life from codes

The fatigue life of components can generally be obtained from codes as discussed in Section 2.7.1. The British code (BS 5400, 1980) categorises the components into nine groups. The number of cycles a component of a given group can endure when subjected to a constant stress range, is given by an S/N curve for that particular group by the formula

$$N_{en}(\Delta\sigma)^{m'} = K_o \Delta^d \left(\frac{22}{t}\right)^{0.25} \qquad (6.1)$$

as given by Eqn.2.30, where $N_{en}$ is the number of cycles, $\Delta\sigma$ is the stress range applied, t is the thickness and $K_O$, m' and $\Delta$ are constants for a given group and d depends upon the probability of failure for which the fatigue life is being calculated. The constants are given in Table 2.1 and the probability factor corresponding to different probabilities of failure in Table 2.2. Thus for a given component the code gives different fatigue lives corresponding to different probabilities of failure. Using Eqn.6.1, Table 2.1 and Table 2.2 (Section 2.7.1), the fatigue lives of structural components can be determined for different probabilities of failure. The code also gives different fatigue lives for different plate thicknesses and this is accounted for by the plate thickness correction factor $\left(\frac{22}{t}\right)^{0.25}$ in Eqn.6.1.

## 6.3 Determining the initial crack length

S/N curves give the number of cycles a component can sustain before it fails. According to crack propagation theory (Eqn.3.19), these cause the initial crack length to propagate to a final crack length at which the component fails by any given mode. Here it has been assumed that this final crack length is equal to the thickness of the component. In other words it is being assumed that the fatigue life given by the codes is equal to the number of cycles required by an initial crack to propagate through the thickness of the component or the asymptotic endurance (Section 2.8.1.1). In practice, the component will fail at a crack length smaller than the thickness of the component. However as shown by Fig. 2.19 a crack propagates extremely fast as it nears the end of the fatigue life and this assumption should cause only a marginal error. Also it can be seen from Eqn.3.19 that for a given component subjected to a fixed loading if the fatigue life is constant then for a larger final crack length the initial crack obtained will be larger. A larger initial crack in a component would result in smaller value of residual strengths (Eqns.2.24, 2.26 and 2.27) thus making the analysis safer and conservative.

The procedure for calculating the initial crack length as discussed above has been formulated mathematically in Section 6.3.1. A graphical method of determining the initial crack length has also been developed in Section 6.3.2.

### 6.3.1 Analytical Procedure

An analytical procedure for determining the initial crack lengths for components of ferrite-pearlite steel has been developed here for a component whose initial crack length has been denoted by $a_i$. The endurance of such a component can be given using crack propagation theory as

$$N(\Delta\sigma)^3 = \sum_{m=i}^{m=t-1} \frac{2}{CM^3{}_{m\ldots(m+1)}\Pi^{3/2}}\left[\frac{1}{\sqrt{a_m}} - \frac{1}{\sqrt{a_{m+1}}}\right] \qquad (6.2)$$

where $a_m$ and $a_{m+1}$ gives an incremental increase in crack length, $M_{m..(m+1)}$ is the average value of M for a increment and t gives the thickness of the component. The standard endurance obtained from S/N curves is given by Eqn.6.1. For the classes D, E, F, F1, G and W (Table 2.1) the value of $m'$ is 3 and the endurance is given as

$$N_{en}(\Delta\sigma)^3 = K_o \Delta^d \left(\frac{22}{t}\right)^{0.25} \tag{6.3}$$

From Eqns.6.2 and 6.3 we get

$$\sum_{m=1}^{m=t-1} \frac{2}{CM^3_{m...(m+1)}\Pi^{3/2}}\left[\frac{1}{\sqrt{a_m}} - \frac{1}{\sqrt{a_{m+1}}}\right] = K_o \times \Delta^d \times \left(\frac{22}{t}\right)^{0.25} \tag{6.4}$$

Equation 6.4 can be used to determine the initial crack length of the component belonging to the group D, E, F, F1, G and W. For other groups of components B, C and S, the value of m is different from 3. Equating Eqns.6.1 and Eqn.6.2 a relation for the groups B, C and S where the initial crack length depends upon the stress range can be obtained. The initial crack length is a property of the component and hence cannot depend on the stress range. Hence, using this method we cannot determine the initial crack length for components belonging to the groups B, C, and S. The underlying reason for this inability is the presence of a large crack initiation period for these components which is not considered in the analysis, Eqn.6.1 being derived solely from crack propagation theories.

### 6.3.2 Graphical Method

The initial crack length of a component can also be determined using a graphical method. The procedure for calculating the initial crack length is shown in Fig.6.1. First a very small crack of length $a_S$ is assumed to exist in the structure and the unstable crack propagation variation is determined for any given stress range as was earlier discussed in Section 5.4. The fatigue life of the component $N_{en}$ that can be obtained from standard codes for the same stress range is then measured backwards from the asymptotic endurance $N_f$ to find the point L on the X-axis. The corresponding residual strength P at endurance L gives the initial crack propagation strength of the component and Eqn.2.24 can be used to calculate the initial crack length, $a_i$.

The graphical procedure also readily gives the portion of the unstable propagation variation to be used for assessment and design. Hence, in Fig.6.1 the portion given by the S/N endurance $N_{en}$ can be used for assessment and design.

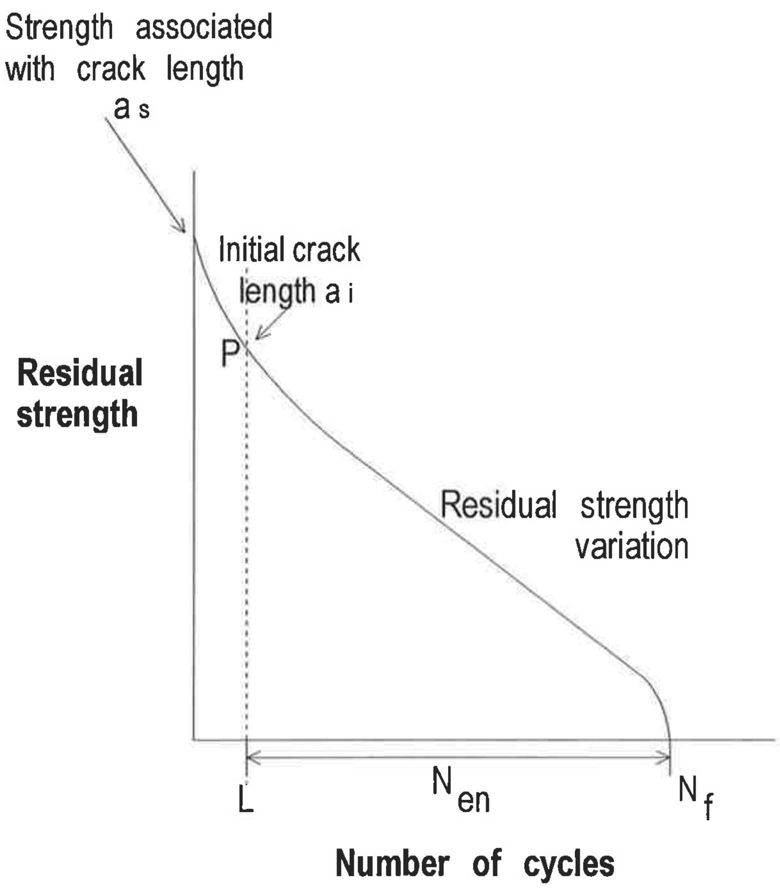



Fig.6.1 Determining the initial crack from codes

## 6.4 Variation of initial crack

It was discussed in Section 6.2 that for a given class of components the code gives different fatigue lives for different probabilities of failure and different thicknesses. The fatigue life of a component being known, Paris' law has been used in Section 6.3 to find the initial crack length. If a constant stress range is being applied to a component and the fatigue life of components changes then the initial crack length also changes.

It should be noted that although here the initial crack length is being back calculated from the fatigue life, actually it is the initial crack length which is the real parameter as it is the value of the initial crack length that determines the fatigue life. It is only because the initial crack length varies that we get components with different fatigue lives. This is illustrated by using an example of a component with a small initial crack $a_1$ that is subjected to constant amplitude loading. The residual strength variation of this component is assumed to be parabolic as shown in Fig.6.2. The fatigue life of this component is given as $F_1$. As the crack propagates it attains a crack size of $a_2$, $a_3$ and $a_4$ at different stages of its fatigue life. When the crack size of this component is $a_2$, the remaining life is $F_2$, when the crack size is $a_3$ the remaining life is $F_3$ and when the crack size is $a_4$ the remaining life is $F_4$. However if the component instead of having an initial crack length $a_1$ had an initial crack length $a_2$, then its endurance would have been $F_2$, or if a component instead of having an initial crack length $a_1$ had an initial crack length $a_3$ then its endurance would have been $F_3$ and if the initial crack length was $a_4$ then its endurance would have been $F_4$. Thus Fig.6.2 not only gives the residual strength variation of components having an initial crack length $a_1$ but can be used to give the residual strength variation of components with initial crack lengths $a_2$, $a_3$, and $a_4$ or any component with an initial crack length greater than $a_1$. This concept has later been used for designing components in Chapter 8.

### 6.4.1 Variation of initial crack length with different probability of failure

The example described in Section 6.4 has been used in this section to explain the variation of the initial crack length with different probabilities of failure. In Fig.6.3, the curves for the different components with initial crack lengths $a_1$, $a_2$, $a_3$ and $a_4$ have been shown individually. The curves 1 to 4 form a space or band in Fig.6.3. Each of these curves is part of the curve in Fig.6.2. If the probabilities of failure of curves 4 and 1 are the minimum and maximum, then these curves gives the extreme limits for all residual

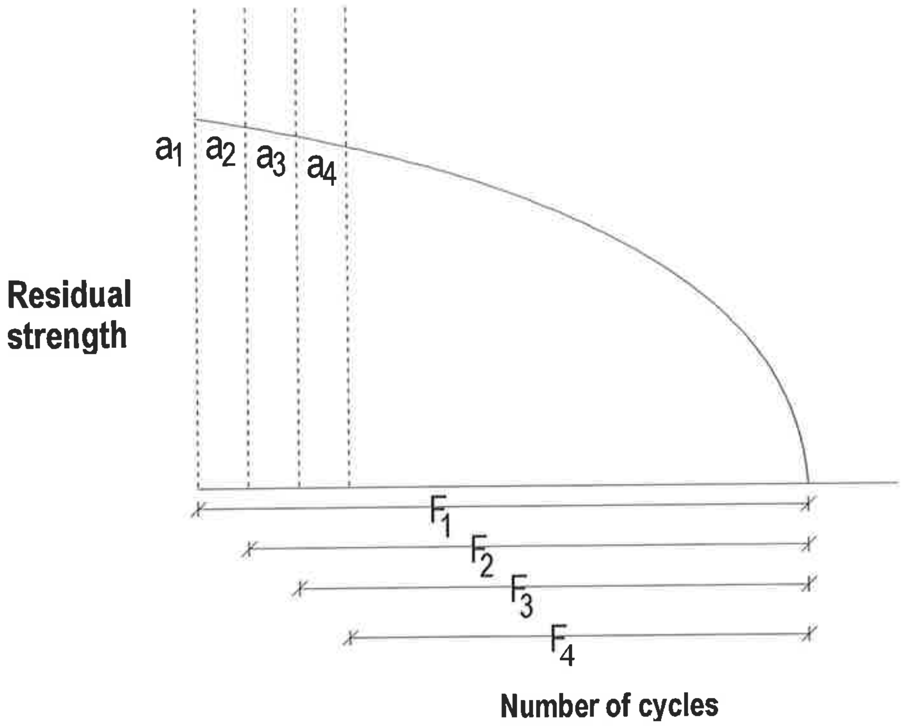



Fig.6.2 Residual strength variation of a component with a small crack

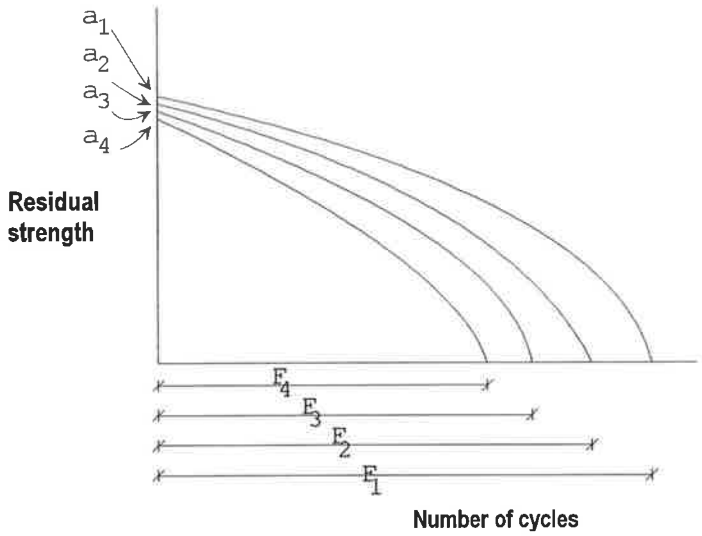



Fig.6.3 Residual strength variations for components with different initial crack lengths

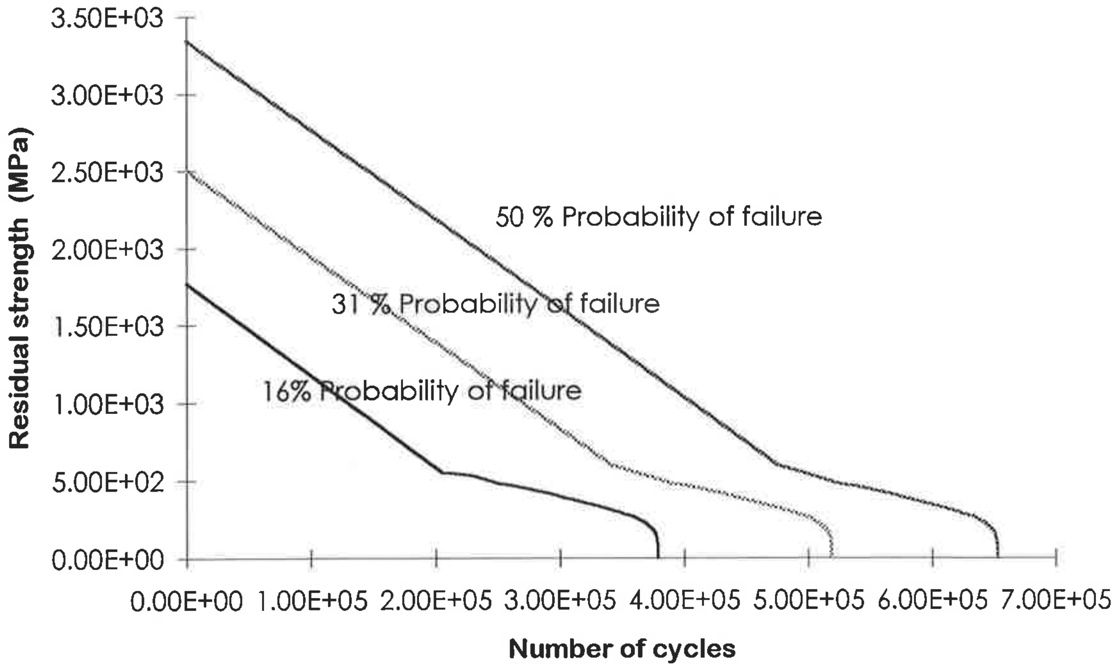



Fig.6.4 Residual strength variation of a stiffener weld for different probabilities of failure

strength curves for this component. Hence curves 1 and 4 encompass a space or band within which the residual strength of the component at any stage of its fatigue life will lie.

Thus for a given probability of failure the corresponding fatigue life, the initial crack length and the residual strength curve (which will lie between curves 1 and 4) can be given. Hence S/N data from codes can be used not only to determine the initial crack lengths and their variations but also the variations of their residual strengths corresponding to different probabilities of failure.

The variation of the residual strengths for unstable crack propagation failure has been found for a 22 mm stiffener weld of A-36 steel and is shown in Fig.6.4. The component was subjected to a constant stress range of 40 $N/mm^2$ and the probabilities of failure considered were 16, 31 and 50 %. It can be seen that the curves are similar in shape and form a space or band within which there is a probability between 16 to 50 % for the stiffener weld to fail. The initial cracks for these stiffener welds were calculated as 0.611 mm for a probability of 0.14% at a standard deviation of 3, 0.278 mm for a 2.3% characteristic probability of failure at a standard deviation of 2, 0.05 mm with a probability of 16% and a standard deviation of 1, 0.025 mm with a probability of 31% and a standard deviation of 0.5, and 0.014mm for a component with a mean probability of 50 %. Thus it can be seen that the initial crack length of a component having a probability of failure of 0.14 % is about 50 times the initial crack length determined for a probability of failure of 50%. This would mean that the corresponding variation of residual strength will be quite substantial.

### 6.4.2 Variation of Initial Crack Length with Thickness

The endurance of a component given by the codes depends on the thickness of components as shown by Eqn.6.1. It can be seen from Eqn.6.4 that the initial crack length will vary with the thickness of the component. Hence for a constant value of M the left hand side of Eqn.6.4 can be regarded as a single increment with $a_m$ as the initial crack length $a_i$ and $a_{m+1}$ as the thickness t. Substituting these values and rearranging

Eqn.6.4 the initial crack length $a_i$ can be given as

$$a_i = \left( \frac{1}{\dfrac{2t^{0.25}}{K_o \Delta^d C M^3 \Pi^{3/2} (22)^{0.25}} + \dfrac{1}{t^{0.5}}} \right)^2 \qquad (6.5)$$

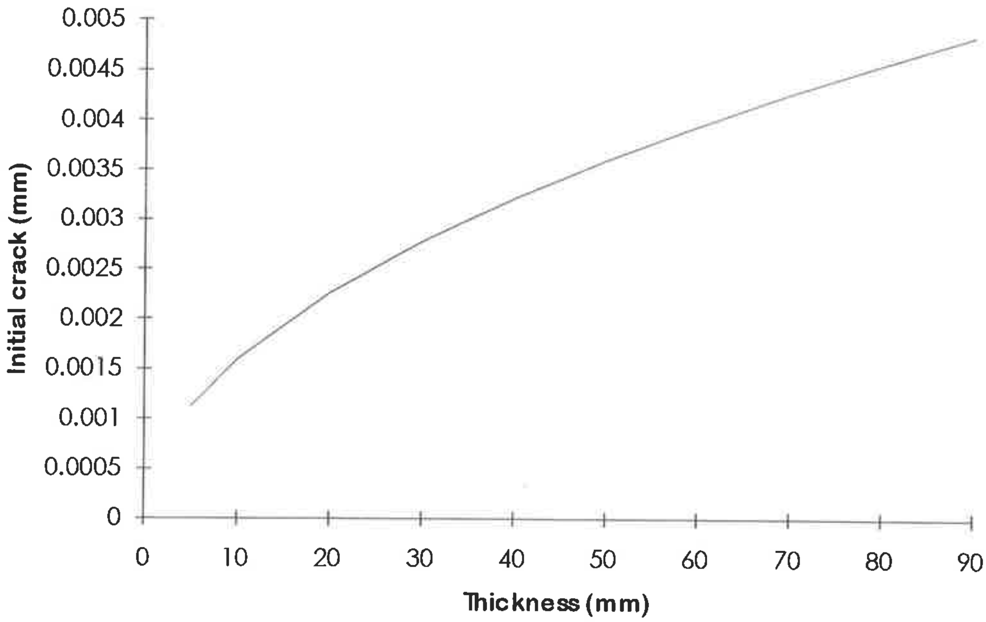



Fig.6.5 Variation of initial crack length with thickness for an idealised weld

The variation of initial crack length has been determined for an idealised component of mild steel in Fig.6.5 for a characteristic value of the fatigue life. It has been assumed that the component belongs to the category D of the British code, BS 5400, 1980. The variation of the initial crack length of a cover plate and a stiffener weld with thickness has been shown in Figs.6.6 and 6.7. The cover plate was categorised as belonging to the category G while a stiffener is categorised as belonging to the category D. The initial crack lengths have been found for the mean value of fatigue life for the cover plate and the characteristic value of fatigue life for a cover plate. The thicknesses chosen

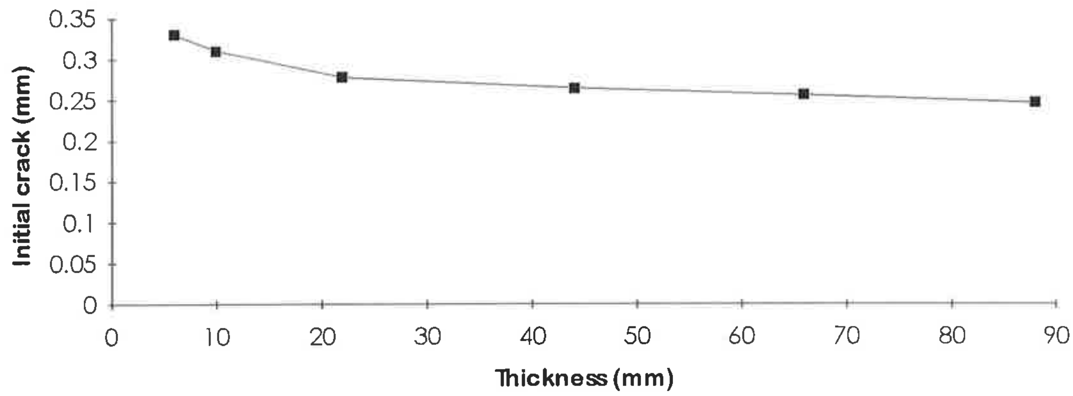



Fig.6.6 Variation of initial crack length with thickness for a stiffener weld

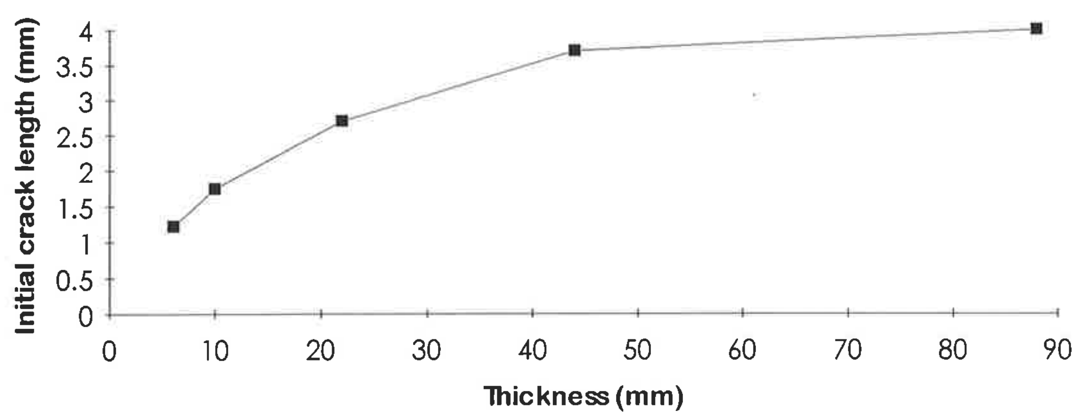



Fig.6.7 Variation of the initial crack length with thickness for a cover plate

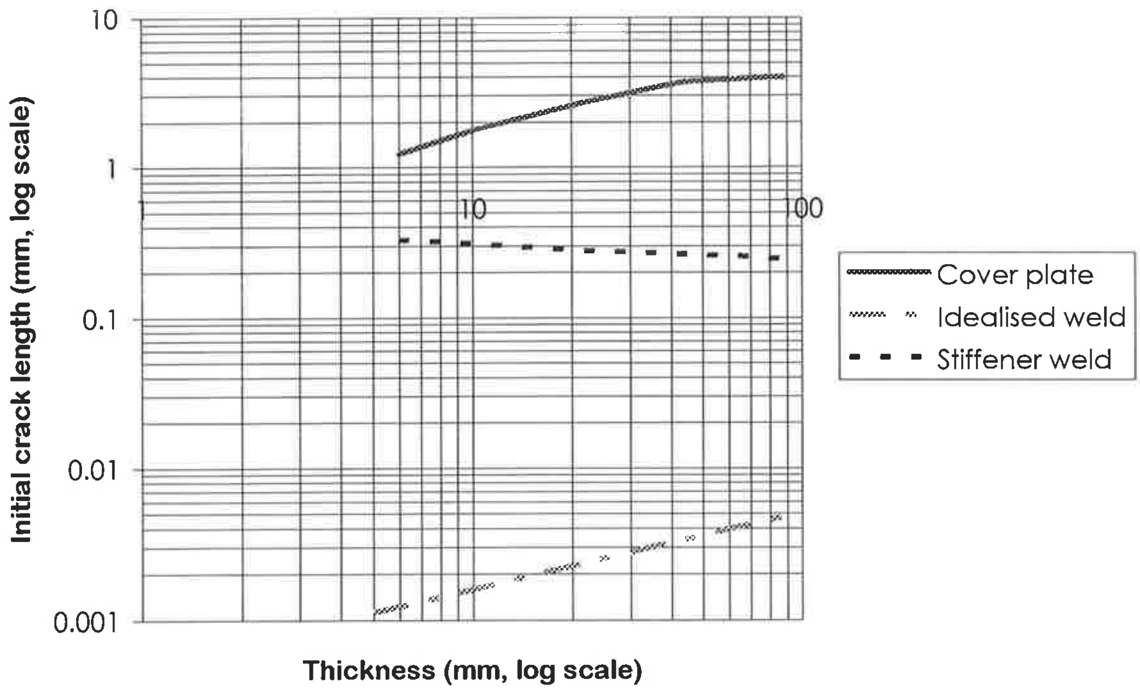



Fig.6.8 Variation of initial crack length with thickness in log scale

were 6, 10, 22, 44 and 88 mm for the cover plate and 6, 10, 22, 44, 66 and 88 mm for the stiffener weld. As the cover plate is weak under fatigue (G being the second weakest category), the initial crack lengths determined are quite large. The large initial crack lengths can also be attributed to the fact that the method adopted in the analysis discussed in Section 6.3 is conservative. In the cases of the idealised weld and the cover plate, the figures show that the initial crack length increases with thickness while in the case of the stiffener weld the initial crack length decreases with thickness.

An investigation was carried out to see if the variation of the initial crack length with thickness for these components could be given by a simple method. A log-log curve was determined for all three components in Fig.6.8 and it can be seen that the variation of the initial crack lengths is linear for a idealised weld and a stiffener weld. In the case of a cover plate the curve was found to be bi linear with a change in direction at a thickness of 44mm. Thus a simple relationship between the initial crack length and the thickness of a component can be developed for these three components using log-log curves. Such a simple relationship will make the task of designing or assessing a welded component much easier as will be shown in Chapter 9.

## 6.5 Conclusions

In order to determine the actual residual strength variation of a component during its fatigue life, its initial crack length must be known. Data on such initial crack lengths are not commonly available but a large amount of data is available on the fatigue life of components in the form of S/N curves. In this chapter, S/N curves are adapted to determine the initial crack lengths for components of ferrite-pearlite steel. The variation of the initial crack length with a change in probability of failure and the thickness of the component was discussed. It was shown that for an idealised component and a stiffener weld, the variation of the initial crack length with thickness is linear on a log-log curve while for cover plates it is bi-linear.

# Chapter 7
# Assessment of welded components

## 7.1 Introduction

In the previous chapter, a procedure for determining the initial crack lengths of components was described. With the application of fatigue loading, this initial crack propagates causing a reduction in strength. In Chapter 3, it was shown how the reduction in strength can be determined using basic fracture mechanics, for components subjected to constant amplitude loading and the residual strength variation was obtained for fundamental components. In Chapter 4, a general procedure was derived from constant amplitude residual strength curves for determining the reduction in strength of components that are subjected to variable amplitude loading. In Chapter 5, the constant amplitude residual strength variation of welded components was determined. In this chapter, the reduction in strength of real welded components is determined when they are subjected to variable loading. The constant amplitude residual strength curves of welded components have been determined based on a procedure developed in Chapter 5 and using the initial crack length in Chapter 6 with the procedure for determining the reduction in strength under variable amplitude loading developed in Chapter 4, the reduction in strength of real welded components is determined.

Welded components are often used in structures where they are regularly subjected to variable loads, such as in bridges. It was discussed in Section 2.10 that most of the structures that were built after the second world war and have past a large portion of their fatigue lives. Such structures, therefore, urgently require assessment and repair. In the U.S.A., it is estimated that about 40% of the 578,000 bridges are to be repaired at a total cost of 92 billion dollars (Section 2.10). About 11,000 bridges are being added to this list each year (Section 2.10). When these bridges are assessed, welded components would form an important part of the assessment procedure being one of the weakest components under fatigue (Section 2.7). Welds have low fatigue life and fatigue strength and are susceptible to failure (Section 2.7). Failure of bridges such as King's bridge in Melbourne and the Point Pleasant bridge in the U.S.A. (Section 2.6.1) have occurred due to welds failing under fatigue loading. Hence the procedure of assessing welded components that is developed in this chapter is of practical importance for the safe operation of bridges and other structures.

Presently welded components, as discussed in Section 2.10, are assessed by using Palmgren-Miners's model (Section 2.7.2) or by carrying out on-site checks. Codes such as BS 5400 (Section 2.7.1) uses Palmgren-Miner's linear damage model to carry out a preliminary assessment of a component subjected to fatigue damage loading while codes such as the BS PD 6493 (2.10.3) help on-site checks by giving guidelines on acceptable crack lengths and by giving methods for calculating the propagation of cracks to find the remaining life. Palmgren-Miner's method for determining the damage caused to a component is a empirical method. It gives the damage caused to the structure as a proportion of the total damage required to cause failure, and is of not much help when making a decision as to whether to carry out on-site checks or not. In this chapter, a theoretical method of assessment is described that is more advanced than Palmgren-Miner's. It is founded on basic mechanics principles and gives the condition of the structure at any intermediate period of its fatigue life in terms of strength, crack length and remaining life. It is also helpful for on-site checks as it can give estimations of the intervals after which checks should be carried out and also what crack lengths one should expect. It is a method that leads to on-site checking thus making the process of theoretical and on-site assessments a continuous procedure.

A discussion is first carried out here on the Palmgren-Miner's method showing its limitations. Facts that must be kept in mind while deciding on a new method of assessment are pointed out and basic procedures that are adapted to develop a new method are discussed. Next, it is shown how a welded component can be assessed, first using a general method which is then adapted for assessment of welds in bridges. Finally, it is shown how the method can be extended to help inspection.

## 7.2 Present methods of assessment

A preliminary assessment of the condition of welded components is generally carried out by using Palmgren-Miner's summation. This is a procedure used by most codes to find the fatigue damage. A more rigorous investigation can be carried out by inspecting a component for cracks and using crack propagation laws to determine the remaining life. Inspection manuals are available which provides techniques used for inspection (Section 2.10.3). The assessment of components using empirically based Miner's solution can be considered to be very simplistic in contrast to on-site inspection of a component that may be considered to be the most rigorous and fool-proof method of assessment. These two methods are often considered as separate or discrete methods of assessments that are not related to each other. A method of assessment that is more advanced in comparison to Palmgren-Miner's and which has a mechanistic basis has been developed in this chapter. The procedure also helps to decide inspection methods, so that the process of assessment is a continuous one as the theoretical assessment leads to a on-site check. In this section, Miner's summation has been analysed form a residual point of view and its limitations are pointed out in order to provide an improved method of assessment later.

Miners summation is given by the equation (Section 2.7.2 )

$$\frac{N_1}{E_1}+\frac{N_2}{E_2}+\ldots+\frac{N_x}{E_x}=A \qquad (7.1)$$

where $N_1$ is the number of cycles of a particular applied stress range $\Delta\sigma_1$ and $E_1$ is the endurance of the component when subjected to the constant stress range $\Delta\sigma_1$. Similarly, $N_2$ is the number of cycles of a particular stress range $\Delta\sigma_2$ and $E_2$ is the endurance of the component corresponding to the constant stress range $\Delta\sigma_2$ and so on. The term $N_1/E_1$ in the previous equation is referred to as the damage caused to the component by the stress range $\Delta\sigma_1$, while the term $N_2/E_2$ is the damage caused by the stress range $\Delta\sigma_2$. There are x such damage terms for the x magnitudes of stress ranges in Eqn.7.1. The total damage is the summation of the damages caused by each magnitude of stress ranges and is given by the term 'A'. The parameter 'A' is considered to give the fatigue life that has been expended as a proportion of the total fatigue life. Also according to Miner's law the total fatigue life is reached when the value of A in Eqn.7.1 equals unity.

Miner's law gives the total damage as a summation of the damage caused by individual stress ranges. It was earlier discussed in Chapter 4 (Section 4.3.2) that the reduction in strength due to individual stress ranges can only be summed up if the residual strength variation is linear. This provides the clue that the residual strength variation of a component that obeys Miner's solution must be linear. In fact it is possible to develop the residual strength model of a component that obeys Miner's law completely. The salient features of such a model as shown in Fig.7.1 are the linear variation of the residual strength and failure at a constant maximum strength as discussed in the following paragraphs.

Let us consider a component that obeys Miner's law and this is subjected to a constant stress range $\Delta\sigma_1$ throughout the fatigue life. Let the corresponding variation of residual strength be along AB as shown in Fig.7.1 with the component failing at an endurance $E_1$ when the strength reduces to $P_{max}$. Similarly for the constant stress ranges $\Delta\sigma_2$, $\Delta\sigma_3$ ..$\Delta\sigma_x$, the corresponding residual strength variations are along AC, AD etc, and the corresponding endurance are $E_2$, $E_3$...$E_x$ etc.

If the component represented by Fig.7.1 is subjected to a series of stress ranges, ie. $N_1$ number of cycles of $\Delta\sigma_1$ so that the strength reduces by $\Delta P_1$, $N_2$ number of cycles of $\Delta\sigma_2$ so that the strength reduces by $\Delta P_2$ and so on, then after the application of 'x' stress ranges the reduction in strength will be given by the following equation that was derived from Fig.7.1 as

$$\frac{N_1}{E_1}+\frac{N_2}{E_2}+\ldots+\frac{N_x}{E_x}=\frac{\Delta P_1+\Delta P_2+\ldots+\Delta P_x}{P_s-P_{max}}=\frac{\Delta P}{P_s-P_{max}} \qquad (7.2)$$

where $P_s$ is the initial static strength and $\Delta P$ is the total reduction in strength. The total reduction $\Delta P$ will be given by the summation of the reductions due to all the stress ranges $\Delta P_1 + \Delta P_2 + \Delta P_3 + \ldots + \Delta P_x$.

The left hand side of Eqn.7.2 is the same as the L.H.S of Eqn.7.1. The right hand side of Eqn.7.2 is given in terms of a reduction in strength, but can be written in terms of endurance if we consider the reduction to be caused by a single stress range. For example let us consider that instead of ‘x’ different stress ranges in Fig.7.1 if the component was subjected to only the stress range $\Delta\sigma_i$ and the reduction $\Delta P$ occurs after $N_i$ number of cycles. Hence the ratio $\Delta P/P_s\text{-}P_{max}$ in Eqn.7.2 can be obtained from curve AF in Fig.7.1 as $N_i/E_i$. Thus Eqn.7.2 can be written as

$$\frac{N_1}{E_1}+\frac{N_2}{E_2}+\ldots+\frac{N_x}{E_x}=\frac{N_i}{E_i} \qquad (7.3)$$

where $N_i/E_i$ is a ratio which gives the portion of the fatigue life compared to the total fatigue life as given by Miner's solution.

Miner's solution not only gives the ratio of damage caused but predicts that failure will occur when this ratio is equal to unity. Let us next consider failure of the component discussed above whose residual strength variation is given by Fig.7.1. Such failure will occur when the strength of the component reduces to $P_{max}$. If such failure occurs after the application of y stress ranges (Fig.7.1), then by carrying out the summation for y stress ranges we can write Eqn.7.2 as

$$\frac{N_1}{E_1}+\frac{N_2}{E_2}+\ldots+\frac{N_y}{E_y}=\frac{\Delta P}{P_s-P_{\max}}=\frac{P_s-P_{\max}}{P_s-P_{\max}}=1 \qquad (7.4)$$

which is the same as Miner’s prediction. Hence the above model which considers the residual strength variation to be linear and the component to fail at a maximum strength obeys Miner’s law completely. However as discussed in chapter 5, the residual strength variation of components will generally be non-linear. Even if components have a linear variation (example stud shear connectors, Section 2.8.1) they will generally fail at different maximum strengths. A solution for such components was given by Eqn.4.19 in Chapter 4 using the asymptotic endurance. Miner's damage law can be considered to be a particular case of Eqn. 4.19 when the component fails at a constant maximum strength.

## 7.3 Proposing a new method of assessment

It was discussed in the Section 2.7.2 that present standard methods of assessing components have an empirical basis, are an oversimplification of the actual condition of the structure, and does not provide much information about when inspection can be carried out. Therefore, there is much scope to improve on existing

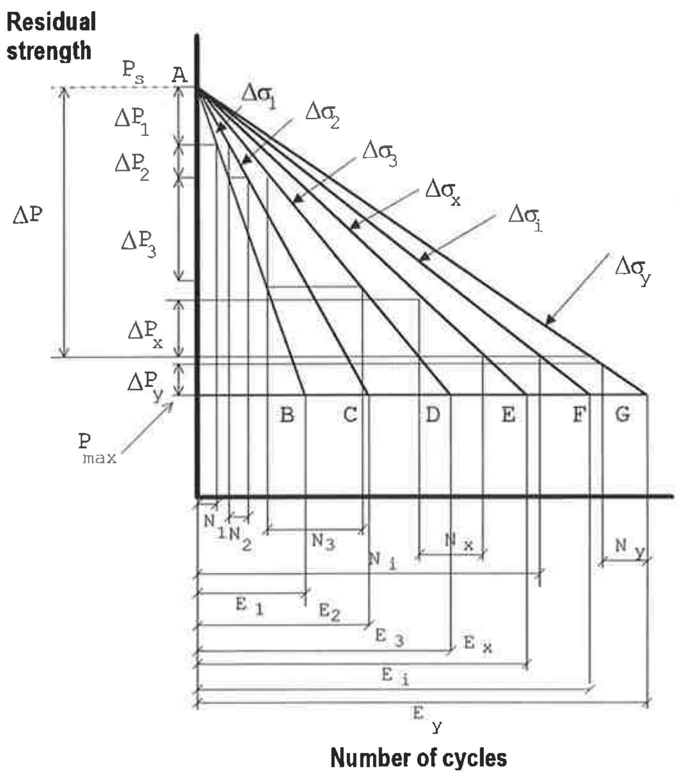



Fig.7.1 Residual strength model obeying Miner's law

methods. However, improvement needs to be made in such a manner that the procedure though more sophisticated as compared to present procedures, still remains a hand method which can be used by practising engineers with comfort. The procedure, therefore, must use commonly available information and should not require complex calculations.

The method of assessment developed in this thesis is based on the residual strength that is obtained from basic fracture mechanics as discussed in Section 3.2. In Section 5.4 and 5.5, residual strength curves were determined for welded components and it was found that their variations were generally non-linear. However as earlier discussed in Chapter 4, Section 4.3 in order to find the reduction in strength of a component whose residual strength does not vary linearly, information is needed on the sequence in which loads are applied and calculations need to be carried out considering each cycle. A cycle by cycle account of loads applied is not generally available and a calculation which involves each cycle is too cumbersome and difficult to carry out by hand.

Thus on one hand we have determined in Chapter 5 the constant amplitude residual strength variation of welded components which are generally non-linear, while on the other hand we need to use a linear solution for providing a practical method for assessing components. How do we solve this apparent paradox? The answer is given by the fact that the residual strength variation for idealised components that has a constant value of M and which fails by unstable crack propagation is linear for structural steel (Section 3.3.2). Also it is to be noted that M for real components is nearly constant for a major portion of the fatigue life as shown in Fig.2.16, 2.17 and 2.18 and hence the corresponding residual strength curve due to unstable crack propagation is also linear for this portion.

A procedure of assessment is, therefore, developed where the unstable crack propagation curve determined for real components is approximated into linear portions symbolising an idealised component. This linearised curve is then been used to determine the damage and variation in crack length. The variation in crack length, once determined, is then used to determine the plastic deformation strength.

## 7.4 Discussion on linearisation

The singularly important concept proposed in Section 7.3 is the linearisation of the residual strength curve. A discussion on the concept of linearisation is carried out here using Fig.7.2 which shows that the residual strength variation AD, due to constant amplitude loading of a welded component failing by unstable crack propagation can be approximated by the three linear portions, AB, BC and CD.

### 7.4.1. Determining the reduction in Strength

The reduction in strength of a component due to a number of stress ranges can be determined using the constant amplitude residual strength curve ABCD in Fig.7.2. Let us consider the component when its crack length is between crack sizes $a_1$ and $a_2$ and hence its residual strength lies between A and B. Let the component when subjected to $N_1$ cycles of stress range $\Delta\sigma_1$ undergo a reduction in strength of $\Delta P_1$, when subjected to $N_2$ cycles of stress ranges $\Delta\sigma_2$ undergo a reduction in strength of $\Delta P_2$ and when subjected to $N_3$ cycles of stress range $\Delta\sigma_3$ undergo a reduction in strength of $\Delta P_3$. If $N_1$ cycles of the stress range $\Delta\sigma_1$, followed by $N_2$ cycles of the stress range $\Delta\sigma_2$ and $N_3$ cycles of the stress range $\Delta\sigma_3$ is applied to the component then the strength of the component will reduce along AE, EF and FG in Fig.7.3.

If the order in which stress ranges applied are reversed so that $N_3$ cycles of $\Delta\sigma_3$ is applied first followed by $N_2$ cycles of $\Delta\sigma_2$ and $N_1$ cycles of $\Delta\sigma_1$ , then the reduction in strength will vary along AH, HK and KG in Fig.7.3 and the final reduction in strength will be the same as in the previous case. However if the non-linear curve between A and B was used to give the reduction in strength, then the reduction would depend on the sequence.

The residual strength variation of a component when subjected to a number of stress ranges is independent of the sequence of loading only when the residual strength curve is linear and varies along either AB, BC or CD. However while the curves AB, BC and CD are linear curves the curve ABC is not linear. Therefore for a component whose strength reduces along ABC when subjected to a given stress range, the residual strength variation, when subjected to a number of stress ranges will not be independent of sequence of loading. Hence, the reduction in strength when the crack propagates from $a_1$ to $a_2$ in Fig.7.2 as the residual strength reduces from A to B must be dealt separately to when the crack propagates between $a_2$ and $a_3$ and the residual strength reduces from B to C.

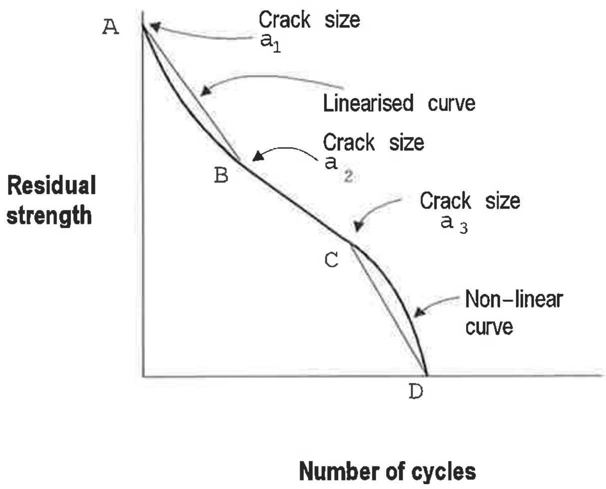



Fig.7.2 A linearised residual strength curve

### 7.4.2 Using an Effective Stress Range

Figure 7.3 shows the residual strength variation of a component that is subjected to the stress ranges $\Delta\sigma_1$, $\Delta\sigma_2$ and $\Delta\sigma_3$ when the component follows the linear curve AB in Fig.7.2 that was derived for constant amplitude loading. The actual residual strength variation therefore is non-linear when subjected to different stress ranges. In actual practise since there will be a large number of cycles acting, it will be easier to find the reduction in strength if these different stresses can be replaced by a single stress range. This problem has been solved in Chapter 4 (Section 4.4) where an effective stress range that cause the same reduction in strength as all the different magnitudes of stress ranges put together has been derived. Thus while AEFG in Fig.7.3 gives the real variation of the strength of a component subjected to stress ranges $\Delta\sigma_1$, $\Delta\sigma_2$ and $\Delta\sigma_3$ respectively, AG gives the variation if the component was subjected to an effective stress range. The effective stress range has been used here in the analysis procedure.

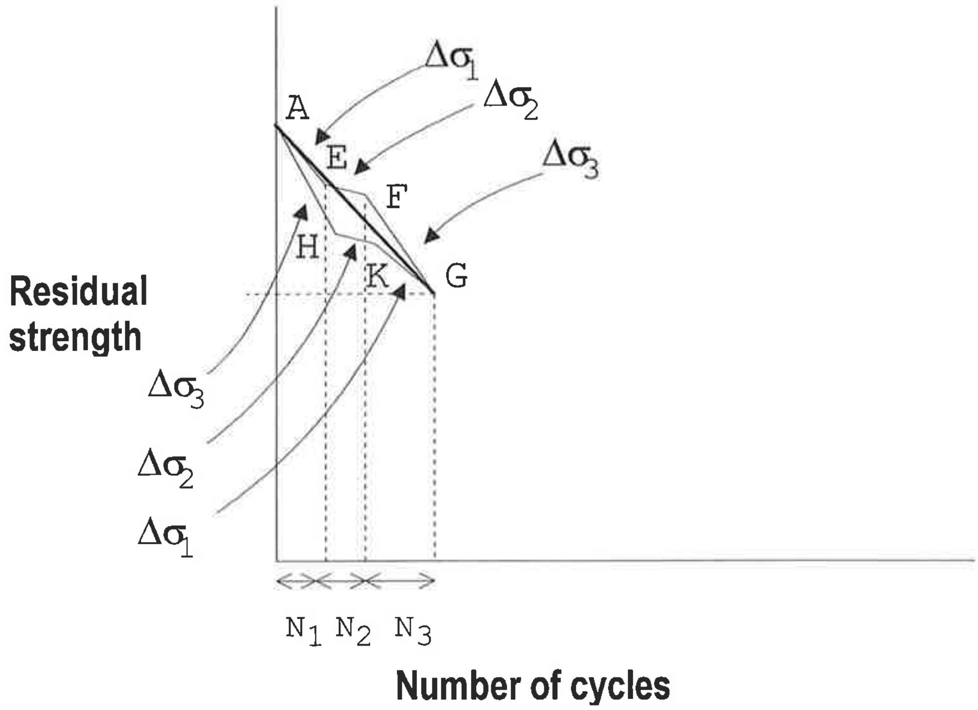



Fig.7.3 Reduction in strength due to varying stress ranges

### 7.4.3 Length of Linear Curves

The constant amplitude residual strength curve can be linearised into short or long linear curves for assessment. Variation in the length of these curves would cause the results of assessment to vary. The use of shorter linear curves means more accurate approximation of the non-linear curve. However, in order to analyse a component using shorter linear portions, the stresses that are applied within these short portions must be known. For example, if it is considered that the entire fatigue life of a component can be divided into linear segments each of five years duration, then the stresses during each segment of the five year terms must be known. Thus, while using shorter linear linear curves would mean more accurate assessment it would also mean that more information regarding the stresses applied to the component must be available. On the other hand use of long linear curves means less information would be required about the sequence of stresses acting on the component. Longer linear curves also means fewer number of them so that the assessment procedure is simpler. As the aim is to develop an easy hand method of assessment, the component has been linearised into long linear curves but exactly the same procedure would be applied to shorter linear simulations.

## 7.5 Assessing welded components

Most commonly, information regarding stresses acting on a component and their corresponding frequencies are available as load spectrums (Section 2.7.3). The procedures of assessment adopted here involves determining the non-linear residual strength envelope for constant amplitude loading. Even though stress ranges of any magnitude could be used the constant amplitude residual strength curve has been determined for a unit stress range. The non-linear residual strength curve due to unit stress range is then linearised into as few linear curves as possible. It is assumed that the loads acting on the component are uniformly distributed throughout the fatigue life so that the pattern of loading and hence the effective stress range is the same for each linear portion. The idealised linear variation of the residual strength depends on the shape of the constant amplitude non-linear variation which in turn depends only on the component and not on the loads applied and not on the range of the constant amplitude from which it was derived (Section 5.3).

Within a linear portion of a linearised curve, the sequence in which stress ranges are applied are not important and the assessment procedure is simple. The procedure can further be simplified by considering the linearised portion eg. AB in Fig.7.4, to be a part of the linear residual strength variation of an idealised weld such as AE. Thus, if the non-linear curve is linearised into three linear portions AB, BC and CD in Fig.7.4, then these three linear portions can be considered to be part of the résidual strength variation of the three different idealised welds that are represented by the failure envelopes AE, HK and GD. The properties of these idealised welds, such as its initial strength and magnification factor needs to be determined for use in assessment procedure.

The initial crack length $a_i$ in Fig.7.4 is determined for a component using S/N curves as discussed in Chapter 6. The non-linear curve for a unit stress range has been determined in the same manner as in Chapter 5 by taking small increments in crack length and numerically integrating. A unit stress range of 1 N/mm$^2$ is used. The number of linear portions into which the non-linear curve is broken depends on the shape of the residual strength curve. In the example of a cover plate, the M-curve is shown in Fig.2.17 and can be separated into three distinct regions. The value of M decreases in the first region, remains constant in the second region and increases in the third region. A typical residual strength curve for the cover plate would also have three distinct regions based on these distinct regions of the M curve as shown in Fig.7.4. $M_1$ in Fig.7.4 gives a constant value of M which will cause the same reduction in strength as the entire first region, $M_2$ gives a constant value of M which will cause the same reduction in strength as the entire second region and $M_3$ gives a constant value of M which will cause the same reduction in strength as the entire third region. The crack lengths $a_1$ and $a_2$ at which the shape of the residual strength curve changes shape are the same crack lengths at which the M curve in Fig.2.17 changes shape. The three different portions of the residual strength curve in Fig.7.4 have been linearised as AB, BC and CD. These three different residual strength curves are considered to be a part of the residual strength variations of three different idealised welds. Here the properties of the three idealised welds are determined which are then used for assessment.

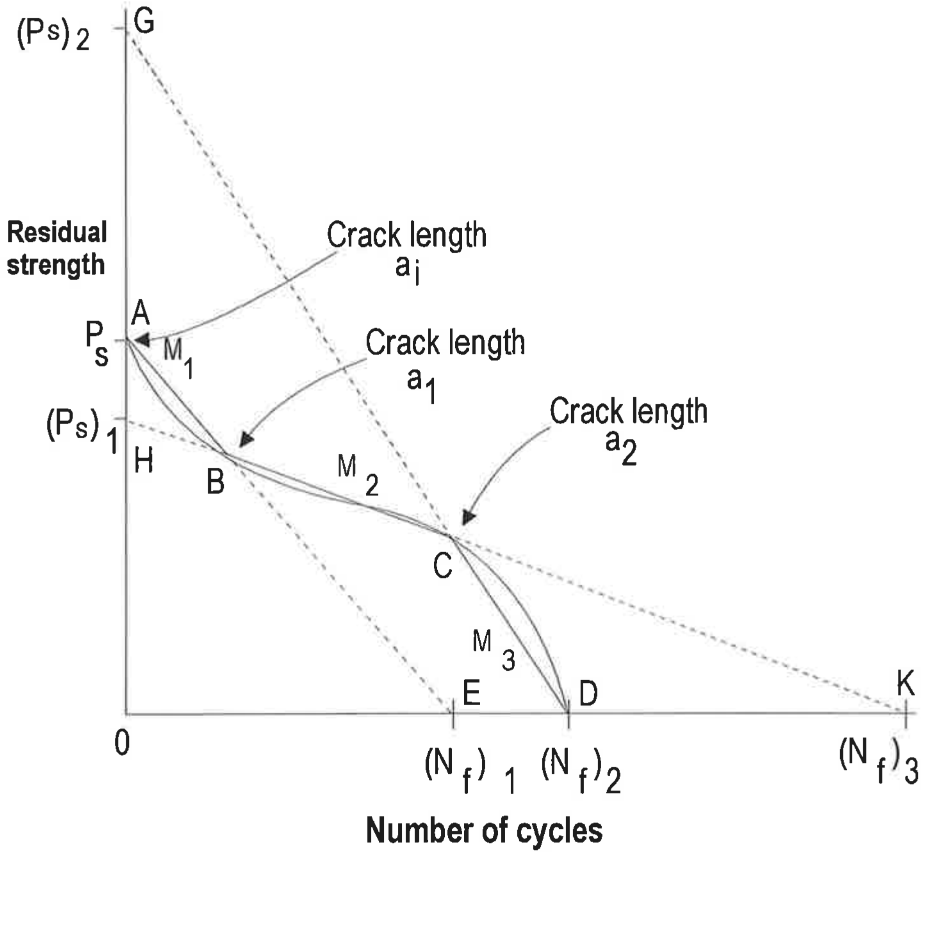



Fig.7.4 Determining properties of idealised welds

Consider the linear portion AB in Fig.7.4. The initial crack length of the component $a_i$ being known the intial crack propagation strength $P_s$ of the component can be determined using Eqn.2.24. The number of cycles required by the crack to propagate from A to B can be given by the non-linear curve for a unit stress range. If the reduction in strength between A and B is divided by the number of cycles between A and B we can get the slope S of the linear portion AB. The slope of an idealised component of ferrite-pearlite steel with a constant value of M and having a linear variation has been determined in Section 3.4. The slope is given by the equation $G = 2/\Pi(\Delta\sigma)^3 CM^2 K_{IC}$ (Eqn.3.21a) for a component subjected to a given stress range $\Delta\sigma$. For a unit stress range substituting $\Delta\sigma = 1$ N/mm$^2$ the above equation becomes $G = 2/\Pi \; CM^2 K_{IC}$. If we equate the slope G with the slope S a constant value of M for the linear portion AB, denoted as $M_1$ in Fig.7.4 can be calculated. The value of the initial strength $P_s$ at A, and the magnification factor $M_1$ are the properties of the first idealised weld whose residual strength variation due to unit stress range is given by AE of which the linear portion AB forms a part.

The magnification factor $M_2$ for the portion BC in Fig.7.4 is calculated in a similar manner to that for AB. If the line BC is extended backwards and forwards, the point where it crosses the Y-axis gives the initial strength $P_{s1}$ of of the idealised component and the point where it crosses the x-axis gives the asymptotic endurance $N_{f1}$ for an unit stress range. The slope of the component is given by $S = P_{s1}/N_{f1}$ which is then equated to the slope G calculated for a stress range of 1 N/mm$^2$ to determine $M_2$. The value of the initial strength $P_{s1}$ and the magnification factor $M_2$ are the properties of the second idealised weld whose residual strength variation due to a unit stress range is given by HK of which the linear portion BC forms a part.

The magnification factor $M_3$ for the third portion CD is found in a similar manner to the portions AB and BC. The initial strength $P_{s2}$ is found in a similar manner to $P_{s1}$.

The values of the initial strength $P_{s2}$ and the magnification factor $M_3$ are the properties of the third idealised weld whose residual strength variation due to a unit stress range is given by GD of which the linear portion CD forms a part.

The initial strength $P_s$ and the magnification factor M calculated for each of the three idealised welds can now be used in an assessment procedure. The linear curves $A_1E_1$, $H_1K_1$ and $G_1D_1$ in Fig.7.5 give the residual strength variations of the three idealised components for any given stress range. For any of the idealised welds, if the asymptotic endurance $N_f$ for any given stress range is known and as the residual strength is linear, the strength $P_m$ at any number of cycles $N_e$ at which the assessment is carried out, will be given by the Eqn.4.23 in Section 4.4 which is

$$\left(1-\frac{P_m}{P_s}\right)=\frac{N_e}{N_f} \tag{7.5}$$

where $P_s$ is the initial strength.

The welded component will generally be subjected to a number of magnitudes of stress range rather than a single stress range. However as discussed in Section 7.4.3, a single effective stress range can be determined which will give the same reduction in strength as the large variety of ranges of stress acting. Hence, the effective stress range is used here for assessing welds which have been subjected to a large number of stresses defined by a load pattern. Substituting the value of the asymptotic endurance in Eqn.7.5, given by Eqn.3.22, in terms of the effective stress range $\Delta\sigma_e$ we get

$$1-\frac{P_m}{P_s}=\frac{N_e(\Delta\sigma_e)^3 CM^2\Pi K_{IC}}{2P_s} \tag{7.6}$$

The fatigue material properties $K_{IC}$ and C being known, we can calculate the strength $P_m$ of the component after the application of $N_e$ number of cycles for a single idealised weld. Hence the governing equation for each of the linear variation in Fig.7.4 can be determined from Eqn.7.6 by substituting the appropriate values of $P_s$ and M.

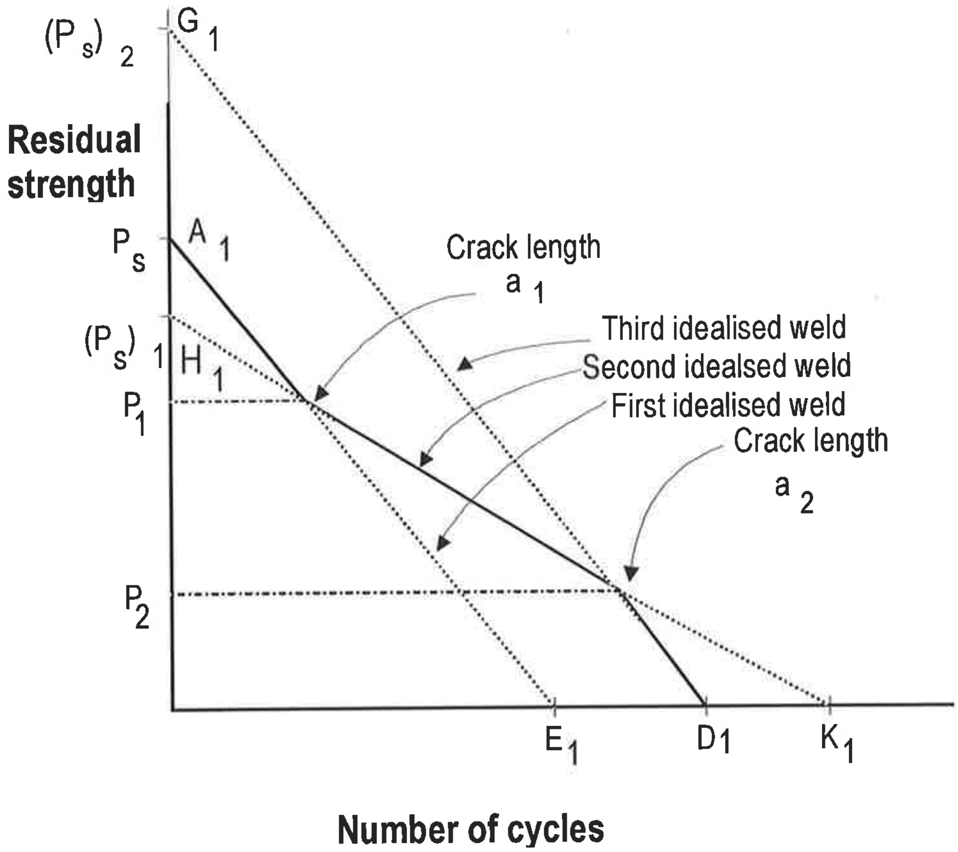



Fig.7.5 Assessment using a linearised curve

The residual strengths $P_1$ and $P_2$ in Fig.7.5 which separate the linear portions and which corresponds to the crack lengths $a_1$ and $a_2$, give information on which of the idealised welds governs the analysis. Thus if the strength of the component is between $P_s$ and $P_1$ the initial crack length and magnification factor of the first idealised weld governs the analysis and the curve followed is $A_1E_1$. However if the strength of the component is between $P_1$ and $P_2$ the initial crack length and magnification factor of the second idealised weld governs the analysis and the curve followed is $H_1K_1$ while if the strength is between $P_2$ and zero the initial crack length and the magnification factor of the third idealised weld needs to be used and the curve followed is $G_1D_1$.

Thus the process of assessing a component that has been subjected to $N_e$ number of cycles needs to begin with the first idealised weld. If the strength $P_m$ calculated is greater than $P_1$ then this gives the correct value of $P_m$. If the strength $P_m$ of the component calculated is less than $P_1$ then we need to calculate the strength using the properties of the second idealised weld. If the strength calculated using the properties of the second idealised weld is greater than $P_2$ then this gives the correct value of $P_m$. If the strength is less than $P_2$ then the properties of the third idealised welds needs to be used to calculate the strength $P_m$.

### 7.5.1 Determining Crack Length and Plastic Deformation Strength

In Eqn.7.6, the remaining strength $P_m$ can be written in terms of its crack length as $P_m = K_{IC} / M_m \sqrt{\Pi a_m}$ by using the stress intensity equation (Eqn.2.24). Substituting this equation into Eqn.7.6 gives the variation of crack length for an idealised weld

$$\left(1 - \frac{K_{IC}}{M_m P_s \sqrt{\Pi a_m}}\right) = \frac{N_e (\Delta\sigma)^3 C M^2 \Pi K_{IC}}{2P_s}. \tag{7.7}$$

where $M_m$ is the magnification factor corresponding to the crack length $a_m$. $P_s$ and M are the initial strength and magnification factor of the idealised weld and the fracture toughness $K_{IC}$ being known, the crack length $a_m$ can be determined after application of $N_e$ number of cycles.

The plastic deformation strength based on Von-mises criteria, $P_{mp}$ of a component having a crack length $a_m$ is given by Eqn.2.27 which is written in the following form

$$P_{mp} = 2f_u / \sqrt{3}\left(1 - a_m / t\right) \tag{7.8}$$

where t is the thickness of a component and $f_u$ the tensile strength of the material. Alternately, when it is based on Tresca's criteria, the plastic deformation strength is given by Eqn.2.26 as

$$P_{mp} = f_u\left(1 - a_m / t\right) \tag{7.9}$$

Substituting $a_m$ in Eqn.7.8 into Eqn.7.7 gives

$$\left(1 - \frac{K_{IC}}{M_m P_s \sqrt{\Pi t\left(1 - \frac{\sqrt{3}P_{mp}}{2f_u}\right)}}\right) = \frac{N_e\left(\Delta\sigma_e\right)^3 CM^2 \Pi K_{IC}}{2P_s} \tag{7.10}$$

according to Von-mises criteria,

and substituting Eqn.7.9 in Eqn.7.7 gives

$$\left(1 - \frac{K_{IC}}{M_m P_s \sqrt{\Pi t\left(1 - \frac{P_{mp}}{2f_u}\right)}}\right) = \frac{N_e\left(\Delta\sigma_e\right)^3 CM^2 \Pi K_{IC}}{2P_s} \tag{7.11}$$

according to Tresca's criteria.

Equations 7.10 and 7.11 can be used to determine the plastic deformation strength $P_{mp}$ of an idealised weld with an initial strength $P_s$, magnification factor M and a fracture toughness $K_{IC}$ after application of $N_e$ number of cycles.

### 7.5.2 Factors affecting the Residual Strength

At any period of the fatigue life when the number of applications $N_e$ is known, the unstable-crack-propagation strength of a component can be determined using Eqn.7.6 and the plastic deformation strength of the component can also be determined from either Eqn.7.10 and 7.11. The lower of these strengths gives the mode of failure and the actual residual strength of the component. This fact was discussed in detail for a component subjected to constant amplitude loading in Chapter 5. Here it has been discussed for a component subjected to variable amplitude loading.

The unstable-crack-propagation strength of the component can be obtained using the linearised constant amplitude residual strength variation (Section 7.4.1) for a component subjected to variable amplitude loading. This constant amplitude curve was developed using the stress intensity equation (Eqn.2.24) which depends on the fracture toughness. Hence, the unstable crack propagation strength depends on the fracture toughness. In contrast, the plastic deformation strength in Section 7.5.1 was obtained from the crack length which was determined from the linearised unstable crack propagation curve. Furthermore, the plastic deformation strength depends on the tensile strength of the component (Section 2.6.3). Thus the residual strength of a component, that is subjected to variable loading, depends on either the fracture toughness when failure occurs by unstable crack propagation or depends on the tensile strength when failure occurs by plastic deformation. Hence as in the case of a component subjected to constant amplitude loading as discussed in Section 5.6.1.1, a component subjected to variable amplitude loading will fail by either unstable crack propagation when the fracture toughness is low or by plastic deformation when the tensile strength is low. The strength of the component may be given by plastic deformation and unstable crack propagation for different periods of the fatigue life.

It was discussed in Section 2.6.2.2 that the fracture toughness of the component depends on the temperature and rate of loading. A decrease in temperature or a increase in the rate of loading causes the fracture toughness to decrease which causes the unstable crack propagation strength to decrease for a component subjected to variable loading. Hence, if the failure of a component is controlled by unstable crack propagation, the strength will decrease as earlier discussed in the case of a component subjected to constant amplitude loading in Chapter 5. Therefore, when assessment is being carried out, the fracture toughness corresponding to the given temperature and rate of loading needs to be used for assessment.

## 7.6 Assessment of welds in bridges

The method of assessment described in Section 7.5 can be used to predict the condition of components of structural steel that are subjected to fatigue loading. Welded components used in bridges are subjected continuously to repeated loads and the technique described in Section 7.5 can be extremely helpful for assessing welds in bridges. It was earlier described in the literature review (Section 2.7.3) that a load model has been developed that can be used to calculate the fatigue damage to components in bridges. In this section, the load model has been combined with the assessment procedure discussed in Section 7.5 to give a easy procedure for evaluating the condition of welds in bridges.

### 7.6.1 A Load Model

The model involves categorising all fatigue vehicles on a bridge in terms of a vehicle of fixed axle configuration and weight that is referred to as a standard fatigue vehicle SFV earlier described in Section 2.7.3. The fatigue vehicles are categorised as a proportion of the weight of the SFV and their frequency of occurrence is determined. The parameter L was defined as

$$L = \sum_{x=1}^{x=j} \left( B_x W_x^{m'} \right) \tag{7.12}$$

where B gives the probability of occurrence of a vehicle, W times the weight of a SFV, a particular type of vehicle is denoted by x and there are j different weights of vehicles. Furthermore the stresses and their frequency at the design point when a SFV crosses the bridge are used to derive the parameter F defined as

$$F = \sum_{k=1}^{k=z} \left( N_k (\Delta\sigma)_k^{m'} \right) \tag{7.13}$$

where N is the frequency of occurrence of the range Δσ when a SFV crosses the bridge. A particular range is denoted as k and there are z types of ranges. If the total number of applications of fatigue vehicles is given by the variable T then

$$TFL = T \sum_{x=1}^{x=j} B_x W_x^{m'} \sum_{k=1}^{k=z} N_k (\Delta\sigma)_k^{m'} \tag{7.14}$$

Eqn.7.14 can be rearranged to give the equation

$$TFL = T\sum_{x=1}^{x=j}(B_x \sum_{x=1}^{x=j} W_x^{m'})(\sum_{k=1}^{k=z} N_k \sum_{k=1}^{k=z} (\Delta\sigma)_k^{m'}) \tag{7.15}$$

Rearranging Eqn.7.15 gives

$$TFL = \sum_{k=1}^{k=z}(T N_k \sum_{x=1}^{x=j} (B)^{m'}{}_x)(\sum_{x=1}^{x=j} W_x \sum_{k=1}^{k=z} (\Delta\sigma)^{m'}{}_k) \tag{7.16}$$

As T is the total number of applications of load, that is the total number of fatigue vehicles, and $B_X$ is the probability of occurrence of a vehicle of proportional weight of type x, $TB_X$ gives the number of applications of vehicles of weight x. When a SFV crosses the bridge, the bridge is subjected to z different ranges of stress $(\Delta\sigma)_k$. Therefore when a vehicle of weight $W_X$ times the SFV crosses the bridge, the stress acting on the bridge will be $W_X$ times that due to a SFV, that is $W_X(\Delta\sigma)_k$. The stress range $W_X(\Delta\sigma)_k$ occurs $N_k$ times, with a single passing of the vehicle. As $TB_X$ is the total number of times this vehicle crosses the bridge, $TB_XN_k$ is the total frequency of the stress range $W_X(\Delta\sigma)_k$. Hence

$$TFL = \sum_{1}^{x=j,k=z}(TN_xB_k)(W_x(\Delta\sigma)_k)^{m'} \tag{7.17}$$

where $TB_XN_k$ is the number of cycles corresponding to stress range $W_X(\Delta\sigma)_k$. With a passing of one vehicle there are a total number of z different types of stress ranges and since there are j different types of vehicles, the total number of stress ranges acting will be jz. Eqn.7.17 can be written as

$$TFL = \sum_{i=1}^{i=jz} N_i(\Delta\sigma_i)^{m'} \qquad (7.18)$$

where the number of cycles $TB_x f_k$ in Eqn.7.17 for a specific range is denoted as $N_i$ in Eqn.7.18 and stress range $W_x R_k$ in Eqn.7.17 is written as $\Delta\sigma_i$ in Eqn.7.18. Therefore, TFL in Eqn.7.18 is the summation of the number of cycles multiplied by $m'$ times all the stress ranges acting. The value of $m'$ is equal to 3 for all components belonging to the classes W, G, F2, F, E and D (Table 2.1) which is the same as the value of m for ferrite-pearlite steel. The summation $\sum_{i=1}^{i=jz} N_i(\Delta\sigma_i)^{m'}$ can be written as $\sum_{i=1}^{i=jz} N_i(\Delta\sigma_i)^{m}$ for these classes of steel.

It was shown in Chapter 4 (Eqn.4.22) that the summation $\sum_{i=1}^{i=jz} N_i(\Delta\sigma_i)^{m}$ can be written in terms of the equivalent stress range as $N_e(\Delta\sigma_e)^m$ where $\sum_{i=1}^{i=jz} N$ is equal to $N_e$. Therefore Eqn.7.18 can be written as

$$TFL = N_e(\Delta\sigma_e)^m \qquad (7.19)$$

The term FL in the above equation remains constant to a given load spectrum and T gives the number of vehicle applications.

### 7.6.2 Assessment Procedure

The assessment procedure for welded components in bridges can be considered to be an adaptation of the general assessment procedures discussed in Section 7.5. The general principles involve determining the residual strength envelope for a unit stress range. The residual strength envelope is then linearised and the initial strength and magnification factor determined for each linear curve. In Section 7.5 it was shown that if the effective stress range is known the residual strength $P_m$ can be obtained after $N_e$ number of cycles using the Eqn.7.6

Substituting TFL for $N_e(\Delta\sigma_e)^m$ where m is equal to 3 in Eqn.7.6 one can obtain

$$TFL = \left(1 - \frac{P_m}{P_s}\right)\frac{2P_s}{CM^2\Pi K_{IC}} \tag{7.20}$$

Equation 7.20 can be used to determine the reduction in strength of a component with the application of vehicle traversals for a given idealised weld. If the weld in Section 7.5 is considered, whose residual strength variation due to unit stress range was given by AD in Fig.7.4, then Eqn.7.20 can be used in a similar manner to Eqn.7.6 to give the variation in the strength of the component. In Eqn.7.20 the stress ranges acting do not need to be known and the reduction in strength can be found out only from the number of vehicle applications. Thus, the initial strength and magnification factor of the first idealised weld in Fig.7.5 is used in Eqn.7.20 to give the reduction in strength for a known number of vehicle traversals and this applies until the strength reduces to $P_1$. When the strength is between $P_1$ and $P_2$ the properties of the second idealised weld is used to give the reduction in strength, while the properties of the third idealised weld needs to be used when the strength reduces below $P_2$ .

### 7.6.3 Determining the Crack Length and Plastic Deformation Strength

In Eqns.7.20, the stress $P_m$ can be written in terms of crack lengths by substituting $P_m = K_{IC}/M_m\sqrt{\Pi a_m}$ from the stress intensity equation (Eqn.2.24). Substituting the stress intensity equations into Eqn.7.20 gives the variation of crack length

$$TFL = \frac{2P_s}{CM^2\Pi K_{IC}}\left(1 - \frac{K_{IC}}{M_m P_s\sqrt{\Pi a_m}}\right) \qquad (7.21)$$

where $M_m$ is the magnification factor corresponding to $a_m$. Thus the properties $P_s$ and M of the idealised weld and the fracture toughness $K_c$ being known the crack length $a_m$ can be determined after application of T number of vehicle traversals.

The plastic deformation strength, $P_{mp}$ of a component having a crack length $a_m$ was given by Eqn.7.8, according to Von-mises criteria, and Eqn.7.9 according to Tresca's criteria. Substituting Eqn.7.8 in Eqn.7.21 gives

$$TFL = \left(1 - \frac{K_{IC}}{M_m P_s\sqrt{\Pi t\left(1 - \frac{\sqrt{3}P_{mp}}{2f_u}\right)}}\right)\frac{2P_s}{CM^2\Pi K_{IC}} \qquad (7.22)$$

which gives the plastic deformation strength according to Von-mises criteria, and substituting Eqn.7.9 in 7.22 gives

$$TFL = \left(1 - \frac{K_{IC}}{M_m P_s \sqrt{\Pi t \left(1 - \frac{P_{mp}}{2 f_u}\right)}}\right) \frac{2P_s}{CM^2 \Pi K_{IC}} \tag{7.23}$$

which gives the plastic deformation strength according to Tresca's criteria.

Eqn.7.22 and 7.23 can be used to determine the plastic deformation strength $P_{mp}$ of an idealised weld with an initial unstable-crack-propagation-strength $P_S$, a magnification factor M and a fracture toughness $K_{IC}$ after application of T number of vehicle traversals.

### 7.6.4 Assessment using Multiple Load Patterns

The assessment procedure developed in Section 7.5 for the general case and in Section 7.6.2 for bridges is only applicable to a single load spectrum with constant values of F and L (Section.7.6.1). However the values of F and L (Section 7.6.1) may not be constant throughout the life of the component. Thus if extra loads pass through the bridge, if the loads passing through the bridge is restricted or if the bridge is damaged or repaired the value of the constants F and L will change. The total reduction in strength is then found out by calculating the reduction for each portion of the fatigue life having constant values of F and L and then adding them up.

In this paragraph we have shown how the constants F and L can change with a change in the loads passing through the bridge and a change in the condition of the bridge. The corresponding variation of residual strength is also discussed. Assume that for the loads acting on the component, the values of F and L is given by Tables 7.1 and 7.2. Consider the case where the component is normally subjected to the load levels (1) to (5) in Table 7.1. The corresponding value of L is given as $L_a$. Fig.7.6 give the variation of residual strength of a component with number of vehicle traversals. Consider that the loads acting on such a component is suddenly reduced when the strength of the component is given by point B in Fig.7.6 after being subjected to $T_a$ traversals. This is done by removing the weight

Table 7.1 Table used to determine the value of L

| Weight Category | Weight ratio W | Probability B | $BW^m$ |
|---|---|---|---|
| 1 | $W_1$ | $B_1$ | $B_1W_1^m$ |
| 2 | $W_2$ | $B_2$ | $B_2W_2^m$ |
| 3 | $W_3$ | $B_3$ | $B_3W_3^m$ |
| 4 | $W_4$ | $B_4$ | $B_4W_4^m$ |
| 5 | $W_5$ | $B_5$ | $B_5W_5^m$ |
| 6 | $W_6$ | $B_6$ | $B_6W_6^m$ |

$$L_a = \sum_{i=1}^{i=5} B_i W_i^m$$

$$L_1 = \sum_{i=2}^{i=5} B_i W_i^m$$

$$L_2 = \sum_{i=1}^{i=6} B_i W_i^m$$

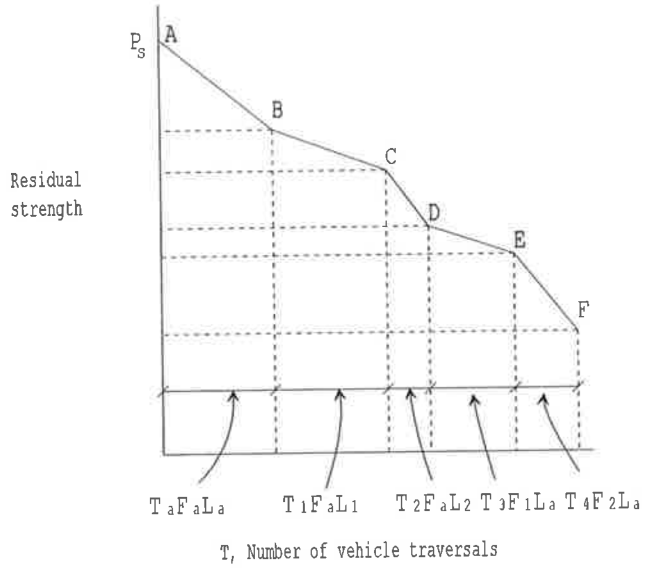



Fig.7.6 Residual strength variation of an idealised weld subjected to multiple load patterns.

category (1) of the weight categories (1) to (5) in Table 7.1 so that the constant L in the load model will change to $L_1$ and is given as a summation of the weight categories (2) to (5). Thus since the loads acting have been reduced the residual strength of the component reduces more gradually. Consider that the component is subjected to this load pattern for $T_1$ traversals and its residual strength variation is given by BC in Fig.7.6. Next consider the case where the loads acting on the bridge increases for the next $T_2$ traversals due to the addition of category (1) which was previously removed and also of load category (6). Then L will increase to $L_2$ being given as a summation of weight categories (1) to (6) and this would result in a sharper decline in residual strength. This is shown by the portion CD in Fig.7.6. If instead of the loads being changed if the stress acting on a component changes due to repair or strengthening of the bridge then this would cause the constant F in Table 7.2 to change. Thus, if there is some damage to the bridge, then this would increase the stresses on a given component acting on the bridge. If the series of stress ranges $(\Delta\sigma)_i$ to $(\Delta\sigma)_v$ given in Table 7.2 represent the stresses after damage and are greater than the series of stress ranges $(\Delta\sigma)_1$ to $(\Delta\sigma)_5$, the stresses before damage, then $F_1$

will be higher than $F_a$ causing the residual strength to decrease more sharply. Assume that the bridge stays in this stage for $T_3$ cycles and its strength reduces from D to E. Next if the bridge is repaired then this may reduce the stresses acting on the bridge and hence reduce the value of F to $F_2$. If the series of stress ranges $(\Delta\sigma)_a$ to $(\Delta\sigma)_e$ gives the stresses after repair and are lower than the series $(\Delta\sigma)_1$ to $(\Delta\sigma)_5$, it will cause the residual strength to decrease more gradually. If it is assumed that the bridge is in this state for $T_4$ cycles then its strength reduces from E to F as shown in Fig.7.6.

Table 7.2 Table used to determine the value of F

| Range of stress $(\Delta\sigma)$ | Range after damage | Range after repair | Frequency (N) | $N(\Delta\sigma)^m$ | $N(\Delta\sigma)^m$ after damage | $N(\Delta\sigma)^m$ after repair |
|---|---|---|---|---|---|---|
| $(\Delta\sigma)_1$ | $(\Delta\sigma)_i$ | $(\Delta\sigma)_a$ | $N_1$ | $N_1(\Delta\sigma)_1{}^m$ | $N_1(\Delta\sigma)_i{}^m$ | $N_1(\Delta\sigma)_a{}^m$ |
| $(\Delta\sigma)_2$ | $(\Delta\sigma)_{ii}$ | $(\Delta\sigma)_b$ | $N_2$ | $N_2(\Delta\sigma)_2{}^m$ | $N_2(\Delta\sigma)_{ii}{}^m$ | $N_2(\Delta\sigma)_b{}^m$ |
| $(\Delta\sigma)_3$ | $(\Delta\sigma)_{iii}$ | $(\Delta\sigma)_c$ | $N_3$ | $N_3(\Delta\sigma)_3{}^m$ | $N_3(\Delta\sigma)_{iii}{}^m$ | $N_3(\Delta\sigma)_c{}^m$ |
| $(\Delta\sigma)_4$ | $(\Delta\sigma)_{iv}$ | $(\Delta\sigma)_d$ | $N_4$ | $N_4(\Delta\sigma)_4{}^m$ | $N_4(\Delta\sigma)_{iv}{}^m$ | $N_4(\Delta\sigma)_d{}^m$ |
| $(\Delta\sigma)_5$ | $(\Delta\sigma)_v$ | $(\Delta\sigma)_e$ | $N_5$ | $N_5(\Delta\sigma)_5{}^m$ | $N_5(\Delta\sigma)_v{}^m$ | $N_5(\Delta\sigma)_e{}^m$ |

$$F = \sum_{i=1}^{i=5} N_i(\Delta\sigma)_i^m$$

$$F_1 = \sum_{i=i}^{i=v} N_i(\Delta\sigma)_i^m$$

$$F_2 = \sum_{i=a}^{i=e} N_i(\Delta\sigma)_i^m$$

The previous paragraph illustrates how the load pattern acting on a component can change and how the strength of the bridge reduces from A to E after the bridge has been subjected to these different load patterns.

The reduction in strength of an idealised component subjected to multiple load patterns illustrated earlier using an example can be derived mathematically using Eqn.7.20 which can be written as

$$P_s - P_m = TFL\left(\frac{CM^2\Pi K_{IC}}{2}\right) \tag{7.24}$$

$P_s$-$P_m$ in the above equation gives the reduction in strength of a component after being subjected to T traversals of the load pattern FL. Denoting the reduction in strength as ΔP the above equation can be written as

$$\Delta P = TFL\left(\frac{CM^2\Pi K_{IC}}{2}\right) \tag{7.25}$$

Consider a component subjected to 'x' different load patterns. Let the load pattern $(TFL)_1$ cause a reduction in strength $\Delta P_1$ of the component, a load pattern $(TFL)_2$ cause a reduction in strength $\Delta P_2$ of the component and so on. Then the total reduction in strength can be given by

$$\Delta P_1 + \Delta P_2 + \ldots + \Delta P_x = \left[(TFL)_1 + (TFL)_2 + \ldots + (TFL)_x\right]\frac{CM^2\Pi K_{IC}}{2} \tag{7.26}$$

or

$$\sum_{i=1}^{i=x} \Delta P = \sum_{i=1}^{i=x} (TFL)_i \frac{CM^2 \Pi K_{IC}}{2} \qquad (7.27)$$

If $P_s$ is the initial strength and $P_{mx}$ is the strength of the structure after being subjected to load patterns $\sum_{i=1}^{i=x} (TFL)_i$ then Eqn.7.27 can be written as

$$P_s - P_{mx} = \sum_{i=1}^{i=x} (TFL)_i \frac{CM^2 \Pi K_{IC}}{2} \qquad (7.28)$$

The term 'x' in the above equation can be any number and $\sum_{i=1}^{i=x} (TFL)_i$ and $P_{mx}$ are the variables. Equation 7.28 shows that the relation between the variables $\sum_{i=1}^{i=x} (TFL)_i$ and $P_{mx}$ is a linear one for a given idealised weld. Therefore for the component whose residual strength variation due to unit stress range was linearised into straight portions in Fig.7.4, each portion being considered a part of a idealised weld, the variation of $\sum_{i=1}^{i=x} (TFL)_i$ and $P_{mx}$ for each idealised weld is shown in Fig.7.7.

Fig.7.7 can be used to assess component subjected to multiple load spectrums. As in the case of single load spectrum when assessing a component subjected to multiple load spectrums each linear portion must be dealt differently. Thus considering the component whose residual strength was given by AD in Fig.7.4 the values of the initial crack length and magnification factor of the first idealised weld in Fig.7.7 needs to be used in Eqn.7.28 till the strength reduces to $P_1$ while the values of the second idealised weld needs to be used till the strength reduces to $P_2$ and the value of the third idealised weld needs to be used when the strength reduces below $P_2$.

The unstable crack propagation strength of a component being known, the crack length can be determined using Eqn.2.24. The crack length being known the Von-mises plastic deformation strength can be determined using Eqn.7.8 and the plastic deformation strength can be determined using Eqn.7.9. If the maximum unstable crack propagation strength at which the component fails is known the remaining life of the component as illustrated by an Example in Section 9.3.2 of Chapter 9 can be found.

## 7.7 Inspection

It has earlier been discussed that assessment procedures that are used at present to find the condition of structural components include a preliminary theoretical assessment using Miner's law and a more rigorous practical assessment which includes periodic site checks. Miner's law and site inspection are generally regarded as separate processes for assessing the structure.

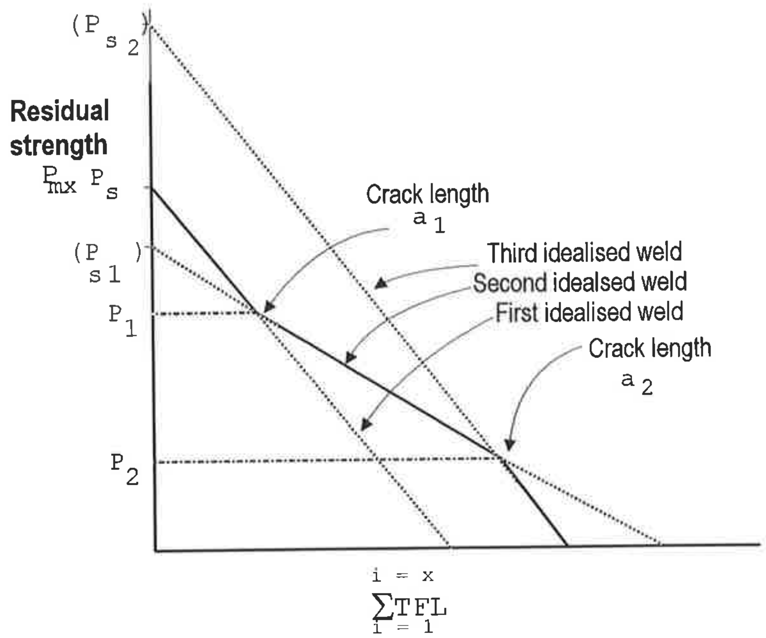



Fig.7.7 Variation of residual strength with $\sum_{i=1}^{i=x}(TFL)_i$

In this section, it is shown how the theoretical procedure of assessing a component using residual strength can be used to determine inspection periods, thus making the process of assessment a continuous process. The procedures discussed here have been developed specifically for welds in bridges, though the general principles can be applied to welded components in other structures subjected to fatigue loading.

Inspection of welds in bridges is generally carried out at a fixed time interval, for example two years. During this time interval the damage caused to a bridge may be small or large. When undertaking the inspection we have little prior idea of the condition by which the component would have deteriorated. The component could deteriorate below one's expectation. If such inspection had been carried out earlier, such deterioration of the component would have been prevented. On the other hand there may be no deterioration of the component and inspection may be waste of time and effort.

It is suggested that it is more reasonable to carry out inspection based on a theoretical knowledge of damage which the component is expected to undergo during the fatigue life. Using fracture mechanics the reduction in strength a component has undergone during the fatigue life has been determined. Inspection can be based on this prior knowledge of the condition of the component and can be carried out each time some constant amount of damage, for example 5 % of the total strength has been theoretically determined.

Assume that the component whose unstable-crack-propagation-residual-strength-variation is given by Fig.7.4, is subjected to a single stress spectrum. One can determine the inspection periods for this component based on the linearised unstable crack propagation curve. Inspection is carried out each time there is a reduction of 5 % of the initial strength of the component.

If $P_s$ is the initial strength of the component, then the number of vehicle applications $T_1$ after which the first inspection is to be carried out can be determined by substituting $P_s$-0.05 $P_s$ for $P_m$ ie. 0.95 $P_s$ for $P_m$ in Eqn.7.20 (Fig.7.8).

$$T_1 = \left(1 - \frac{0.95P_s}{P_s}\right)\frac{2P_s}{CM^2\Pi K_{IC}FL} \tag{7.29}$$

Inspection periods are constant for a given idealised weld and inspection is to be carried out after each $T_1$ vehicle traversals till the strength reduces to $P_1$

The number of vehicle traversals $T_{n1}$ (Fig.7.8) required by a component to reduce from the initial strength $P_s$ to the strength $P_1$ can be calculated by substituting $P_1$ for $P_m$ in Eqn.7.20.

$$T_{n1} = \left(1 - \frac{P_1}{P_s}\right)\frac{2P_s}{CM^2\Pi K_{IC}FL} \tag{7.30}$$

After being subjected to $T_{n1}$ cycles, the component follows the residual strength variation of the second idealised weld. If $P_{s1}$ is the initial strength of the second idealised weld, then by substituting $P_{s1}$-0.05 $P_s$ for $P_m$ in Eqn.7.20 gives $T_2$ the number of vehicle traversals after which inspection is to be carried out as

$$T_2 = \left(1 - \frac{P_{s1} - 0.05P_s}{P_{s1}}\right)\frac{2P_{s1}}{CM^2\Pi K_{IC}FL} \tag{7.31}$$

Inspections are to be carried out at an interval of $T_2$ vehicle traversals till the strength reduces to $P_2$. The number of vehicle traversals $T_{n1}+T_{n2}$ required by a component to reduce from the initial strength $P_s$ to the strength $P_2$ can be calculated by substituting $P_2$ for $P_m$ in Eqn.7.20.

$$T_{n1} + T_{n2} = \left(1 - \frac{P_2}{P_{s1}}\right)\frac{2P_{s1}}{CM^2\Pi K_{IC}FL} \tag{7.32}$$

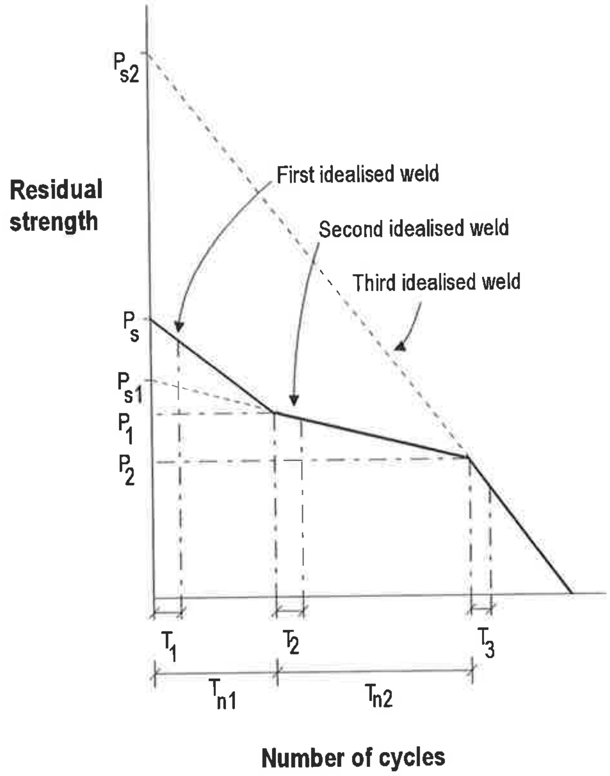



Fig.7.8 Determining inspection periods

After being subjected to $T_{n1}+ T_{n2}$ cycles, the component follows the residual strength variation of the third idealised weld (Fig.7.8). If $P_{s2}$ is the initial strength of the third idealised weld, then by substituting $P_{s2}$-0.05$P_s$ for $P_m$, in Eqn.7.20 gives $T_3$ as

$$T_3 = \left(1 - \frac{P_{s2} - 0.05P_s}{P_{s2}}\right)\frac{2P_{s2}}{CM^2\Pi K_{IC}FL} \qquad (7.33)$$

Inspection is to be carried out after each $T_3$ vehicle traversals till the component fails.

## 7.8 Conclusions

In this chapter, it has been shown how the research presented in the earlier chapters can be used to provide an assessment technique for welded components. The chapter, therefore, integrates all the techniques and procedures developed in the earlier chapters to bring the research to a stage where it is directly applicable.

Existing techniques of assessing welded components such as Miner's law or site inspection have been discussed. Miner's law gives the damage caused to a structure as a proportion of the total damage to cause failure. It gives a rather vague idea of the condition of the structure and does not give the strength, crack length or remaining life of the component. The information obtained after assessing a component using Miner's law cannot be directly used to find out whether site checks need to be carried out or not.

The technique of assessment developed here can be considered to be an improved method of assessment as it is based on mechanics principles, uses information that is commonly available, simulates the actual condition of the structure more closely and gives a definite idea of the strength, crack length and remaining life of the structure. As the method is founded on basic mechanics, it can be used to predict inspection periods thus making the assessment procedure a continuous one where theoretical assessment leads to on-site check. Also, information from on-site checks can be fed back to make the theoretical assessment more accurate.

# Chapter 8
# Design of welded components

## 8.1 Introduction

In the previous chapter, the residual strength curve was used to assess welded components that are already in service. In this chapter, residual strength curves are used to design new components. The procedure for determining the constant amplitude residual strength variation for fundamental components was discussed in Chapter 3 and the procedure for welded components in Chapter 5. The method for determining the reduction in strength under variable amplitude loading was discussed in Chapter 4, the technique for calculating the initial crack length in Chapter 6 and the concept of linearisation was discussed in Chapter 7. All of these techniques have been used in this chapter to develop a new design procedure.

Present standard methods of design of welded components (Section 2.9.1) involve a static design for strength and a fatigue design for endurance. These designs are done separately so that the strength of the component is considered independent of fatigue loading. The method therefore ignores any dependence of strength on fatigue. It is thus implicitly assumed that the strength of a component does not alter during the fatigue life and falls rapidly at the end of the fatigue life. The residual strength variation that is followed by present standard methods of design is shown in a schematic form in Fig.2.38 and can be considered to be a simplistic assumption of the actual variation. As the variation shown in Fig.2.38 does not assume any reduction in strength of a component during fatigue loading, it cannot be used to determine the maximum overload the structure can withstand at any period of its fatigue life.

In Chapters 3 and 5, it was shown that by the application of fracture mechanics the actual variation of the residual strength of a component can be found. A design procedure based on the variation of the residual strength will be more accurate compared to present methods as it simulates the actual condition of a structure more closely. In contrast to present methods, this residual strength variation approach can be used to determine the strength of the component and hence the maximum overload that can be applied to the component at any intermediate period of its fatigue life.

The general concepts used for designing structural components are described here. As in the case of the assessment procedure described earlier in the last chapter, the technique of designing developed in this chapter is based on the variation of residual strength due to unstable crack propagation. The technique is used to design idealised components failing by unstable crack propagation. This is followed by a design procedure that is developed for real welded components.

## 8.2 General design technique

A component subjected to fatigue loads needs to be designed in such a manner that its strength at any intermediate period of its life is greater than the maximum overload applied; this will prevent failure of the component during its fatigue life. Hence we will design a component to determine the least set of dimensions which will enable the component to both withstand the maximum overload at the end of the fatigue life and operate safely during its design period.

It is assumed here that the shape of the component which is to be designed is already known, that is the different dimensions of the geometry of the component are known as a proportion of each other. Design is carried out to find the thickness of the section of the component which is expected to develop a crack and fail. The design thickness is calculated in such a manner as to ensure that the strength of the component at the end of the fatigue life is equal to the maximum overload applied. Once this thickness is known, the dimensions of rest of the geometry can be determined.

A design procedure has been developed here, first for idealised components and then for real components. The thickness has been determined for idealised structural components for failure by unstable crack propagation; a single linear residual strength variation has been used and hence a solution can be derived from a single iteration. For real welded components which may fail either by plastic deformation or by unstable crack propagation, the thickness has been determined using an iterative procedure.

### 8.2.3 Design of Idealised Components

In Chapter 7, the following equation (Eqn.7.6) was used to assess the performance of idealised welds that failed by unstable crack propagation failure and which showed a linear variation of residual strength

$$1-\frac{P_m}{P_s}=N_e\left(\Delta\sigma_e\right)^3\frac{K_{IC}\Pi M^2 C}{2P_s} \qquad (8.1)$$

where $P_s$ is the initial strength (in terms of stress), $P_m$ is the strength of the structure (in terms of stress) after application of N number of cycles of the stress range $\Delta\sigma_e$, $K_{IC}$ and C are the material properties and M is the constant value of the magnification factor of an idealised component.

Eqn.8.1 can be written in terms of shear flow and plate thickness 't'. Since stress=shear flow/thickness, writing Eqn.8.1 in terms of the shear flow and rearranging results in

$$\frac{K_{IC}\Pi M^2 C}{2}\frac{N(\Delta Q_e)^3}{t^3} = \frac{Q_s - Q_m}{t} \tag{8.2}$$

where $\Delta Q_e$, $Q_S$ and $Q_m$ are shear flows. $Q_S$ is the shear flow strength of the structure at the start of the fatigue loading and $Q_m$ is the shear flow strength of the structure after the application of fatigue loading.

Let us denote the initial design strength of the structure required to resist both fatigue loads during the design life and the maximum design overload at the end of the fatigue life as $Q_{of}$ and the strength of the structure required to resist the maximum overload at the end of the design life as $Q_o$. Substituting $Q_S$ and $Q_m$ in Eqn.8.2 as $Q_{of}$ and $Q_o$, Eqn.8.2 can be written in the following form for design purposes.

$$\frac{K_{IC}\Pi M^2 C}{2}\frac{N(\Delta Q_e)}{t^3} = \frac{Q_{of} - Q_o}{t} \tag{8.3}$$

where $Q_{of} - Q_o$ gives the loss in strength of the component due to fatigue loading during the design life

A component will be designed for given values of $Q_O$ and $\Delta Q$. The value of the initial strength $Q_{Of}$ and the value of thickness t are unknowns. Eqn.8.3 gives a relation between $Q_{Of}$ and t. Another relation between $Q_{Of}$ and t can be found by using the stress intensity relation (Eqn.2.24) in the following form

$$K_{IC} = M\frac{Q_{of}}{t}\sqrt{\Pi a_i} \tag{8.4}$$

where $a_i$ is the initial crack length, $K_{IC}$ is the critical stress intensity factor and M the constant value of the magnification factor. A relation between $a_i$ and t has earlier been given by Eqn.6.5 in Section 6.4.3. Substituting Eqn.6.5 into Eqn.8.4 results in

$$K_{IC} = M\frac{Q_{of}\sqrt{\Pi}}{t}\left(\frac{1}{\frac{1}{K_o\Delta^d CM^3\Pi^{3/2}(22)^{0.25}}+\frac{1}{t^{0.25}}}\right) \tag{8.5}$$

which gives a relation between $Q_{of}$ and t. Thus Eqn.8.3 and 8.5 gives two relations between the two unknowns the initial strength $Q_{Of}$ and the thickness t. These equations can be used to determine the design thickness t of the component.

### 8.2.4 Procedure of Welded Components

The method of designing welded components that is described in this chapter, is based on the variation of residual strength. In Chapter 5, the residual strength variation was determined for welded components by taking increments in crack length till the crack propagated through the thickness of the component. Therefore, in order to determine the residual strength variation of a component, as described in Chapter 5, the thickness of the component must be known. However, here we are designing the component and hence the thickness is unknown. Therefore it would appear that a paradoxical situation exists; in order to design a component for its thickness the residual strength of the component is required to be known, however the residual strength cannot be found if the thickness is not known. The problem is solved here by first studying the variation of the residual strength curve of a component with thickness. Components having the same initial-crack-length/thickness are investigated and it is shown that such components have residual strength variations similar in shape. It is also shown that a normalised curve can adequately represent all thicknesses and this concept has been used for designing welded components.

### 8.2.4.1 Residual strength variation and thickness

An investigation is carried out in this section to find a relation between the thickness and residual strength of components whose initial crack lengths are proportional to their plate thickness. Let us denote the proportion of the initial crack $a_i$ to the thickness t as x, that is $a_i/t = x$. The following discussion is carried out by considering two different components one of which is k times the thickness of another. Let us denote the thickness of these components as Y and kY. Thus the initial crack length of the component of thickness Y is xY and the initial crack length of a component of thickness kY is xkY. The number of cycles required for a crack to propagate from an initial crack length $a_i$ to a thickness t for a component of structural steel is given by Paris' equation as

$$N = \int_{a_i}^{t} \frac{da}{C\left(M\Delta\sigma\sqrt{\Pi a}\right)^3} \qquad (8.5)$$

Eqn.8.5 must be integrated numerically as the value of M varies with the crack length of a welded component (Fig.2.16, 2.17 and 2.18). Let the numerical integration be carried out by taking 'n' equal increments in each thickness. These n equal increments causes the crack to propagate from the initial crack length through to the thickness of the component. Integrating Eqn.8.5 gives

$$N = \frac{2}{C(\Delta\sigma)^3}\left[\frac{1}{M_1^3}\left(\frac{1}{\sqrt{a_i}} - \frac{1}{\sqrt{a_1}}\right)\right] + \left[\frac{1}{M_2^3}\left(\frac{1}{\sqrt{a_1}} - \frac{1}{\sqrt{a_2}}\right)\right] +.+ \left[\frac{1}{M_n^3}\left(\frac{1}{\sqrt{a_{n-1}}} - \frac{1}{\sqrt{t}}\right)\right] \quad (8.6)$$

where $a_1$ is the crack length after the crack propagates through the first increment, $a_2$ is the crack length after the crack propagates through the second increment, etc. The average value of M has been denoted as $M_1$ for the first increment, $M_2$ for the second increment and so on. For the situation where the increments are equal, their size can be determined by dividing the difference between the initial crack length (xY or kxY) and the thickness (Y and kY) by the total number of increments 'n'. Thus the size of the increment of crack for a component of thickness Y is given as G in the following equation

$$G = \frac{Y - xY}{n} \quad (8.7)$$

Furthermore for an thickness kY, the increment denoted by $G_1$ is given as

$$G_1 = \frac{kY - xkY}{n} = kG. \quad (8.8)$$

Let us denote the parameter xY/G as ' I '. Therefore the initial crack length xY for a component of thickness Y is iG, and the initial crack length xkY for the component of thickness kY is ikG. Thus for the component of thickness Y, the initial crack length can be given as iG, the crack length after the first increment can be given as (i+1)G, the crack length after the second increment (i+2)G etc as shown in Fig.8.1. For a component of thickness kY, the initial crack length can be given as ikG, the crack length after the first increment as (i+1)kG, the crack length after the second increment as (i+2)kG, etc as also shown in Fig.8.1.

Substituting into Eqn.8.6 the values of the crack lengths in terms of G for a component of thickness Y gives

$$N_y = \frac{2}{C(\Delta\sigma)^3}\left[\frac{1}{(M_1)^3}\left(\frac{1}{\sqrt{iG}} - \frac{1}{\sqrt{(i+1)G}}\right) + \frac{1}{(M_2)^3}\left(\frac{1}{\sqrt{(i+1)G}} - \frac{1}{\sqrt{(i+2)G}}\right)\right] + \ldots \quad (8.9)$$

where $N_y$ is the number of cycles required for the initial crack to propagate through the thickness, Y. The magnification factor of a component is constant for a given crack length by thickness ratio as can be seen from Figs.2.16, 2.17 and 2.18. The crack length/thickness ratio of a component of thickness Y at the start and end of the first increment is given by iG/Y and (i+1)G/Y, at the start and end of the second increment by (i+1)G/Y and (i+2)G/Y and so on as can be seen from Fig.8.1. Also it

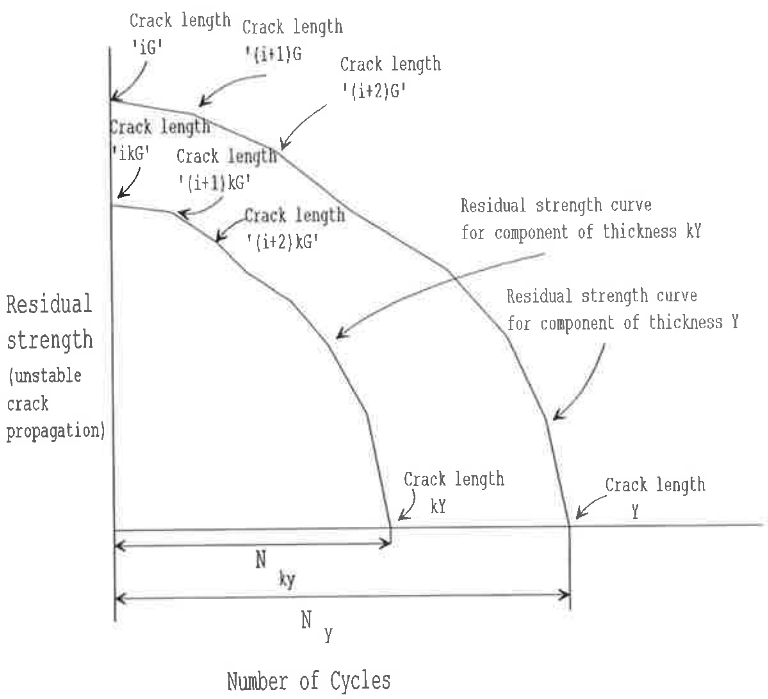



Fig.8.1 Variation of residual strength with thickness

can be seen from Fig.8.1 that the crack length/thickness ratio of a component of thickness kY at the start and end of the first increment is iG/Y and (i+1)G/Y, at the start and end of the second increment is (i+1)G/Y and (i+2)G/Y and so on. Thus the crack length/thickness ratio at the start and end of each increment is the same for both thicknesses. The crack length/thickness ratio at the start and end of increments being the same, the values of M is the same at the start and end of each increment for both thicknesses. The values of M being the same at the start and end of each increment, the average value of M is the same for each increment for both thicknesses. Therefore the number of cycles $N_{ky}$ required for the initial crack to propagate through the thickness kY can be given as

$$N_{ky} = \frac{2}{C(\Delta\sigma)^3\sqrt{k}}\left[\frac{1}{(M_1)^3}\left(\frac{1}{\sqrt{iG}}-\frac{1}{\sqrt{(i+1)G}}\right)+\frac{1}{(M_2)^3}\left(\frac{1}{\sqrt{(i+1)G}}-\frac{1}{\sqrt{(i+2)G}}\right)\right]+\ldots \tag{8.10}$$

Comparing Eqn.8.9 to Eqn.8.10, it can be seen that the number of cycles required for each increment of crack in the case of the component of thickness kY is $1/\sqrt{k}$ times that for the component of thickness Y. As this is true for all the 'n' increments $N_y / N_{ky} = \sqrt{k}$.

It is to be noted that the crack length for the component of thickness kY is k times the crack length of the component of thickness Y for the same increment stage. For a component failing by unstable crack propagation the residual strength is given by the stress intensity equation $\sigma = K_{IC} / M\sqrt{\Pi a}$ (Eqn.2.24). Thus if the crack length of a component is k times the crack length of another component, the residual strength will reduce by $\sqrt{k}$. Furthermore, for a component failing by plastic deformation failure the strength is given by $P_{mp} = 2k(1-a/t)$ (Eqn.2.25); as the function (a/W) is the same for both thicknesses for any given crack increment, the plastic deformation strength of both the components are the same. Hence it can be seen that for components of different thicknesses, the ordinates and abscissae of the residual strength curve for a given thickness can be represented as a multiple of the residual strength curve of other thicknesses as shown in Fig.8.1. The residual strength curves for cover plates of different thicknesses are plotted in Fig.8.2. An initial crack length of thickness of 0.15 is assumed. As shown in Fig.8.2 the residual strength curves of the three thicknesses are similar in shape and the ordinate and abscissae can be considered to be a multiple of each other.

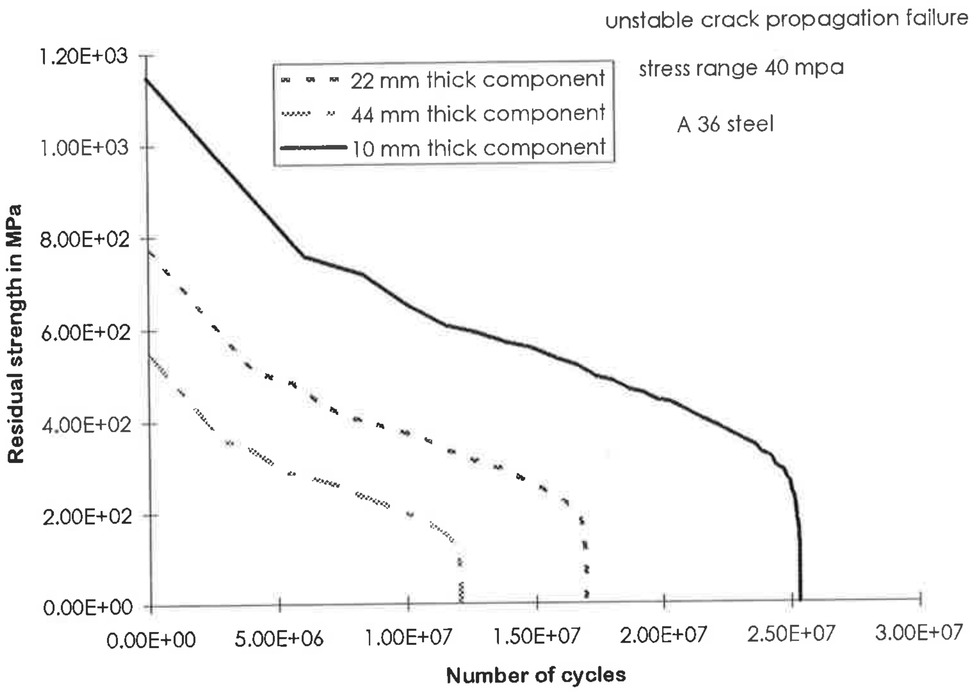



Fig.8.2 Residual strength of components with the same initial crack length/thickness ratio

A single normalised curve with the residual-strength/maximum-strength as the ordinate and the number-of-cycles/asymptotic endurance as the abscissae would therefore adequately represent all thicknesses. It has earlier been discussed in Chapter 5 that this curve will also represent all stress ranges as altering the stress range does not alter the shape of the residual strength curve. In conclusion, it is very important to note that this single normalised curve represents the residual strength variations for all components having a given initial-crack-length/thickness-ratio and for all stress ranges.

### 8.2.4.2 Information from a normalised residual strength curve

Assume that the normalised residual strength curve in Fig.8.3 has been determined for a component with, $a_i/t$ ratio of j. The point A, therefore, corresponds to the crack length/thickness ratio j. With the application of fatigue loading, the initial crack propagates and the crack length increases in size. Thus the crack-length/thickness ratio increases while residual strength decreases along AC until the crack extends through the full thickness. Consider a portion BC of this curve. If the crack length/thickness ratio at B is given as k, then BC gives the shape of the residual strength variation of a component having a initial-crack-length/thickness ratio k. In a similar manner the initial crack length/thickness ratio of any component is known with the same geometry, the corresponding point having the same crack length/thickness ratio in AC can be found. The residual strength variation from this point to C gives the shape of the residual strength variation of the component. Thus the curve AC gives the shape of the residual strength variation for a component of any thickness with a initial crack length/thickness ratio greater than j. It can be seen that the curve is independent of the thickness of the component and has been shown in Section 8.4.1 it is also independent of the stress range applied to the component. Hence the normalised failure envelope depends only on the geometry and the material properties of the component.

### 8.2.4.3 Design technique

The concept of a normalised curve discussed in the earlier section has been used for design in this section. The normalised curve can be used to determine whether a component of a given thickness can sustain the maximum overload at the end of the fatigue life. An iterative design procedure is carried out using different thicknesses to determine the thickness which is able to withstand the maximum overload.

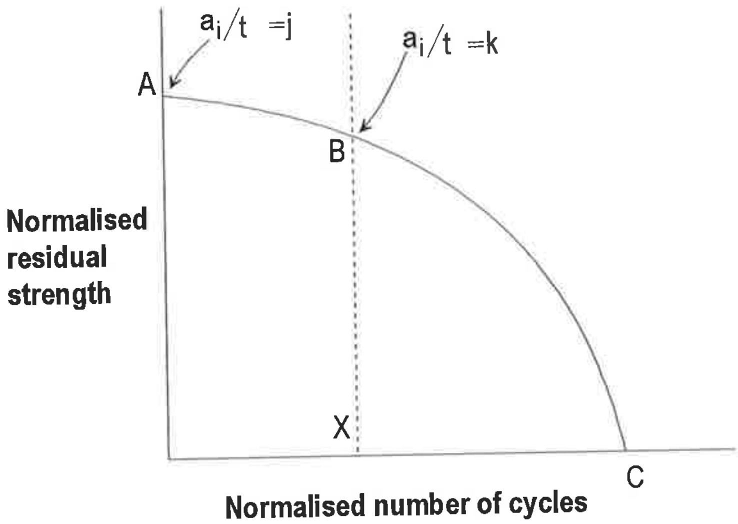



Fig.8.3 A normalised residual strength curve

The curve used for design is developed in Section 8.2.4.3.1. In Section 8.2.4.3.2, it is shown how this curve can be used to determine the reduction in strength that a component has undergone when subjected to fatigue loads. Finally, in Section 8.2.4.3.3, an iterative design procedure is developed, where different thicknesses are tried out before the design thickness is determined.

#### 8.2.4.3.1 *Design Curve*

In order to design components, it is first necessary to develop the curves shown in Figs.8.4a and 8.4b from which the design curves are developed. The curves in Figs. 8.4a and 8.4b are determined for a component of thickness $t_b$ for which the initial crack length $a_i$ can be determined using the procedure developed in Section 6.3 of Chapter 6.

For a component of thickness $t_b$ and initial crack length $a_i$, the crack length grows with the number of cycles of stress as shown in Fig.8.4a which is developed for a unit stress range. As the crack grows, the residual strength reduces and the variation of the residual strength with number of cycles for a unit stress range is shown in Fig.8.4b.

The residual strength curve in Fig.8.4b is next linearised according to the procedure earlier discussed in Sections 7.4 and 7.5 of Chapter 7. The linearisation depends on the variation of the magnification factor M. Figures such as 2.16, 2.17 and 2.18 show the variation of M with crack length/thickness has a typical shape. Such typical variation of crack-length/thickness (a/t) with the magnification factor M is shown in Fig. 8.5 (a). The crack lengths $a_1$ and $a_2$ at which the variation of magnification factor with crack length changes shape can be used to separate the variation of crack length verses number of cycles (Fig.8.5 (b)) and the variation of residual strength against number of cycles (Fig.8.5 (c)) into three different regions. In Fig.8.5 (d) these three regions are linearised as shown.

Fig. 8.4b shows the residual strengths $P_1$ at point K and $P_2$ at point F corresponding to crack lengths $a_1$ and $a_2$. The residual strengths at points A, K, F and C are joined by straight lines. The slope of each of the linear portions, AK, KF and FC is derived by the procedure discussed in Section 7.5 and each of these three straight portions is considered to be a part of the residual strength variation of an idealised component as also discussed in Section 7.5. The values of the constant magnification factors $M_1$, $M_2$ and $M_3$ for each of the three idealised components of which these linear curves form a part, can be determined from the equation $G = 2 / \Pi C M^2 K_{IC}$ where G is the slope, M is the constant value of the magnification factor and $K_{IC}$ is the critical stress intensity factor. This completes the linearisation procedure of the residual strength curve as shown in Fig.8.4b.

The two figures 8.4a and 8.4b are placed deliberately, one below the other so that for a given crack-length/thickness value of $a_d/t_b$ in curve 8.4a, one can find the corresponding value of the point D in curve 8.4b. Hence the curve DC in 8.4 (b) gives the residual strength variation of a component of thickness $t_b$ and crack length $a_d$. It has been shown in Section 8.2.4.2 that all components that have the same initial-crack-length/thickness ratio will have the same shape of residual strength curve. Hence whatever the thickness of the component, the curve DC gives the general shape of the residual strength variation of any component that has an initial crack

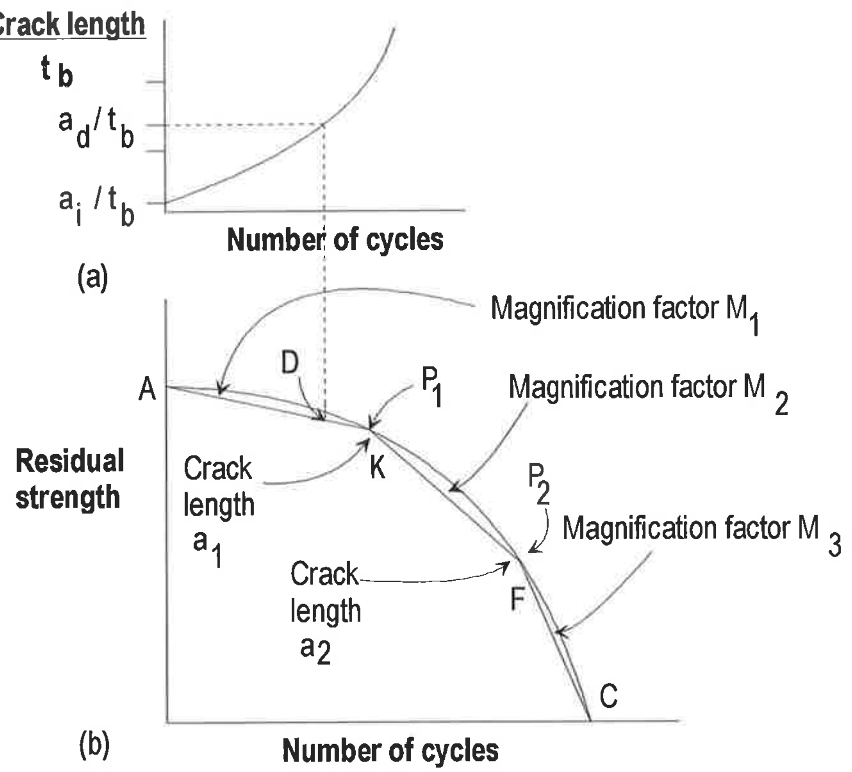



Fig.8.4a and Fig 8.4b showing variation of crack length and residual strength

length/thickness ratio $a_d/t_b$. Therefore, even though Fig.8.4 was drawn for a component of a particular thickness, it can be used to give the shape of the residual strength variation of any component having an initial-crack-length/thickness ratio greater than $a_i/t_b$. This fact has been used to develop the design curve in Fig.8.6.

In contrast to Fig.8.4a which gives the variation in the crack-length to thickness ratio for a particular component of thickness $t_b$, the values of a/t in Fig.8.6a do not pertain to any particular thickness. Also since only the shape of the curve is of interest here and not the actual dimensions (in contrast to 8.4b which gives the

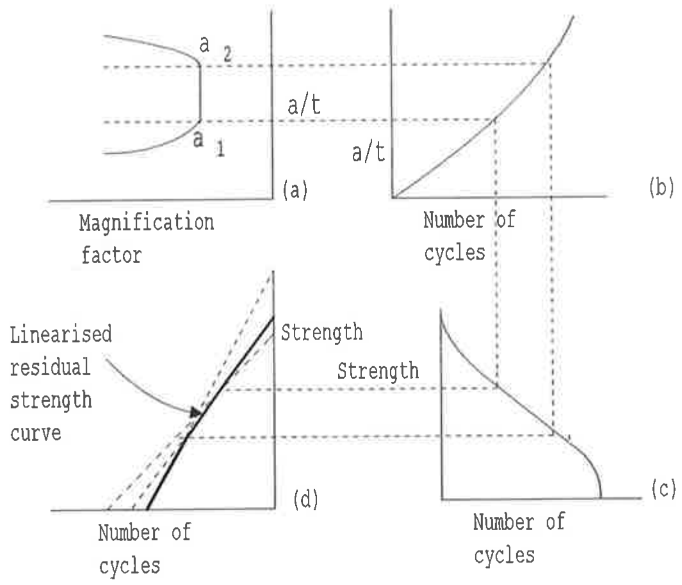



Fig.8.5 Procedure of linearising a curve

linearised residual variation of a component of thickness $t_b$) Fig.8.6b gives the normalised non-dimensional residual strength variation. Thus in Fig.8.6b the residual strength on the Y-axis is divided by the maximum strength and the number of cycles on the Y-axis by the asymptotic endurance.

Thus for a component with a given value of initial-crack-length/thickness m in Fig.8.6a, the corresponding point B on the normalised residual strength curve on Fig.8.6b can be obtained, the portion BD giving the normalised residual strength variation of the component.

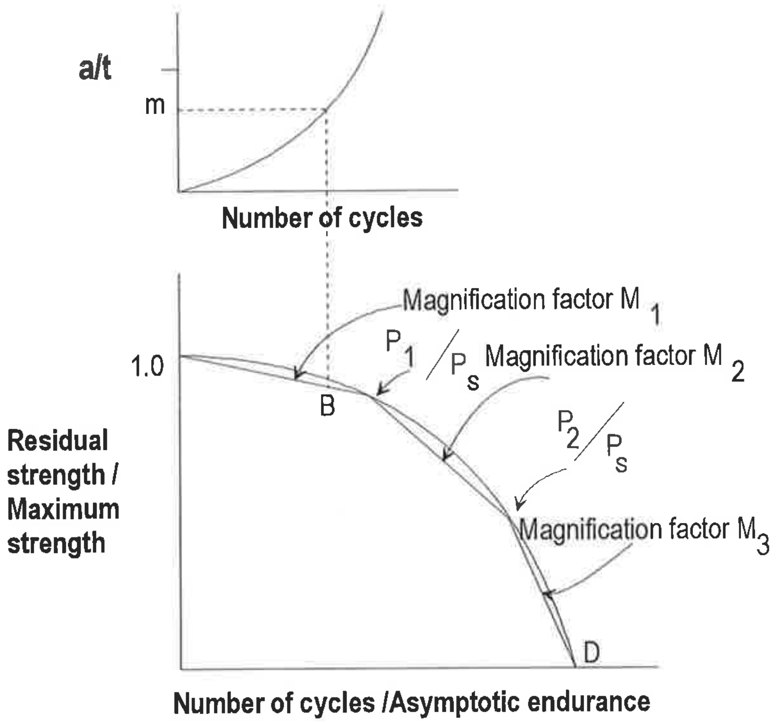



Fig.8.6a (top) and Fig.8.6b (bottom) showing variation of crack length and normalised residual strength.

#### 8.2.4.3.2 *Reduction in strength for a given thickness*

Consider Fig.8.7 as the design curve pertaining to the component to be designed. The figure shows the corresponding normalised and linearised curve for any crack length/thickness ratio greater than any value 'h'. The strength ratios $P_1/P_s$ and $P_2/P_s$, where $P_1$ and $P_2$ are the strengths at which the residual strength curve is linearised and $P_s$ the static strength, the magnification factors M that define the

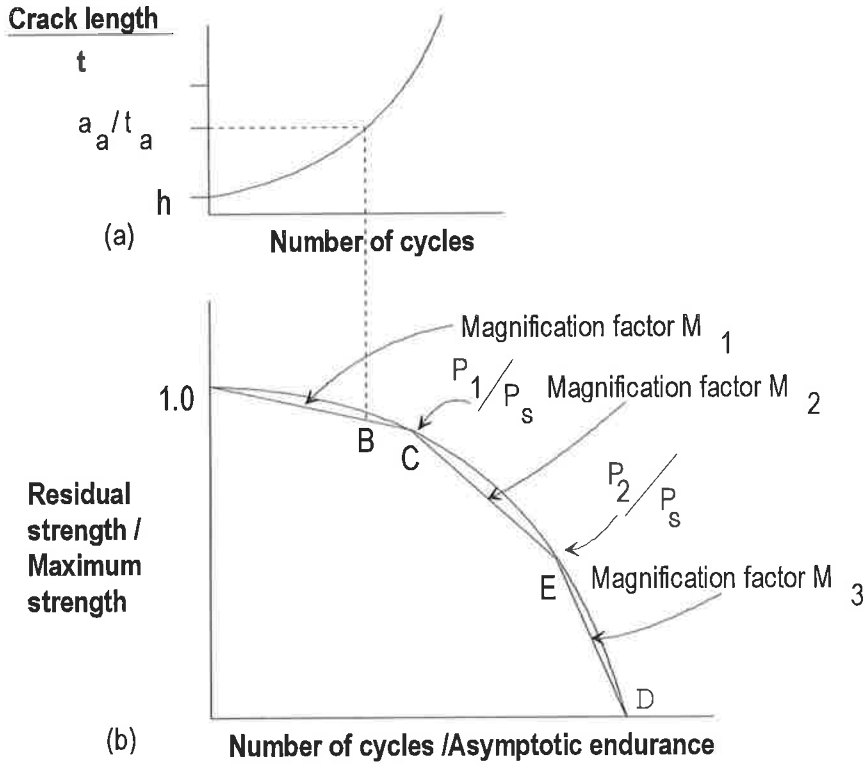



Fig.8.7 Figure showing curve to be used for design

linearised portion of the curves are given in the above figure. A procedure for determining the reduction in strength of a component of a given thickness $t_a$ is described in this section. The initial crack length '$a_a$' corresponding to the given thickness $t_a$ is first derived from S/N curves as discussed in Chapter 6 (Section 6.3). For the initial-crack-length/thickness value $a_a/t_a$, the corresponding point B in the residual strength curve is determined from Fig.8.7. BD then provides the normalised residual strength variation of the component.

The residual strength at B in Fig.8.7b which will be denoted by $R_b$ can be derived from Eqn.2.7, $\sigma = K_{IC} / M \sqrt{\Pi a}$ where $K_{IC}$ is the critical stress intensity factor and M is the corresponding magnification factor. It is to be noted that the value of the magnification factor for the portions BC, CE and ED has been calculated, as discussed in Section 8.2.4.3.1. The value of $R_b$ being known, the value of the initial strength $P_s$ of a component of crack length/thickness 'h' can be found from Fig.8.7. As the values of $P_s$ and from Fig.8.7 the values of $P_1/P_s$ and $P_2/P_s$ are now known the values of $P_1$ and $P_2$ can be determined. Thus the strengths corresponding to B,C and E are known.

For design purposes, the effective stress range and the number of cycles to be applied during the fatigue life is known. In order to find the strength $P_m$ of the given component after the application of an effective stress range $\Delta\sigma_{e1}$ and number of cycles $N_1$, the analysis is started with the Eqn.7.6 which was developed for an idealised weld. After substituting the values of $R_b$, the initial strength of a component of crack length/thickness $a_a/t_a$, for $P_s$, $\Delta\sigma_e$, and $N_1$ for $N_e$ and $M_1$ for the magnification factor M, a value of residual strength $P_m$ is obtained. If this value is less than $P_1$ then the linear portion CE to find $P_m$ is tried. To do this, first the number of cycles of stress range $\Delta\sigma_e$ required by the component so that its strength reduces from B to C is calculated by substituting $R_b$ for $P_s$ and $P_1$ for $P_m$ in Eqn.7.6. If this number is $N_{BC}$, then $P_1$, $N_1$-$N_{BC}$, $M_2$ and $\Delta\sigma_e$ is substituted into Eqn.7.6 to find $P_m$. If this value is less than $P_2$, then the linear portion ED to find $P_m$ is tried. To do this, the number of cycles of stress range $\Delta\sigma_e$ required for the strength to reduce from C to E is calculated by substituting $P_1$ for $P_s$ and $P_2$ for $P_m$ in Eqn.7.6. If this number of cycles is $N_{CE}$, then $P_2$, $N_1$-$N_{BC}$-$N_{CE}$, $M_3$ and $\Delta\sigma_e$ are substituted in Eqn. 7.6 to find $P_m$.

It is to be noted here that $P_m$ is the unstable crack propagation strength of the component. The plastic deformation strength corresponding to the unstable crack propagation strength is calculated using Eqn.7.10., the lower of the two strengths calculated giving the actual strength of the component.

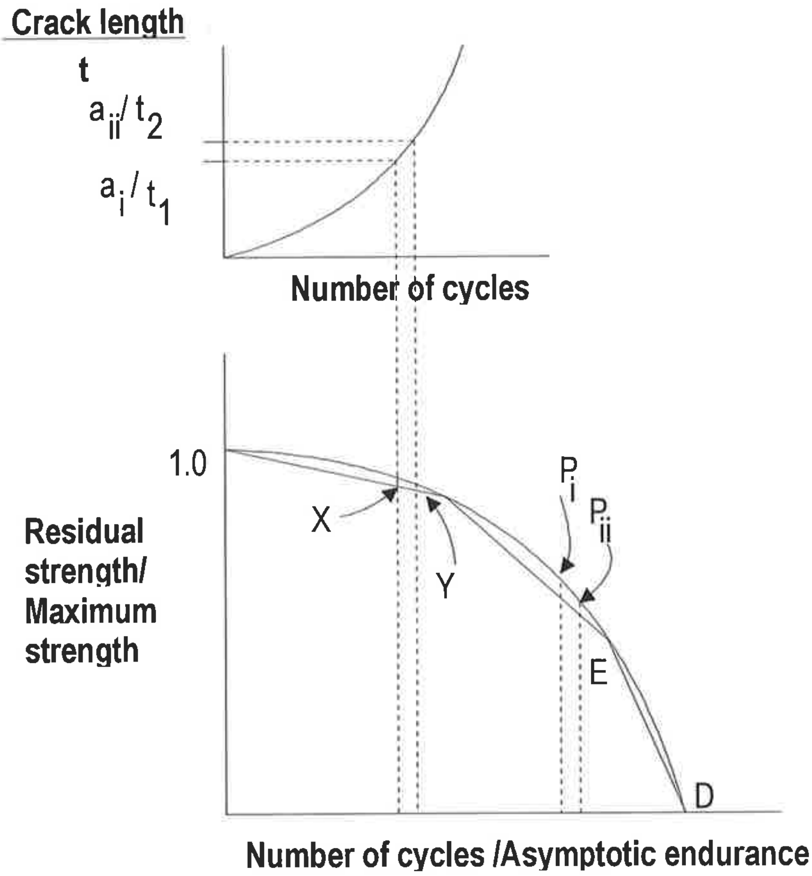



Fig.8.8 Iterative procedure of carrying out design

### 8.2.4.3.3 *Iteration procedure to find design plate thickness*

Assume that the design curve of a component of given shape is known. In order to start designing for thickness, $t_1$ in Fig.8.8 is chosen arbitrarily. The corresponding initial crack length $a_1$ is determined from S/N curves. For this given $a_i/t_1$ ratio in Fig.8.8, the corresponding point X in the normalised curve in Fig.8.8 is determined. The curve XD gives the shape of the residual strength variation for a component of initial crack length/thickness ratio $a_i/t_1$. This residual strength curve is used to find the unstable crack propagation strength of a component at the end of the fatigue life, which is shown as '$P_i$' in Fig.8.8, using Eqn.7.6. The unstable crack propagation strength of the component being known the plastic deformation strength can be found using Eqn.7.10. The lower of the two values gives the residual strength of the component, $P_a$. If the maximum overload the component need to resist at the end of the fatigue life is known in terms of shear flow $Q_o$, a new estimate of the plate thickness $t_2$ can be determined by dividing this shear force $Q_o$ by the residual strength '$P_a$'. The initial crack length $a_{ii}$ corresponding to the component of thickness $t_2$ is obtained from S/N curves. The whole procedure described previously is repeated in Fig.8.8: for the given $a_{ii}/t_2$ ratio the corresponding point 'Y' in the normalised residual strength curve in Fig.8.8 is determined; the curve YD now gives the shape of the unstable crack propagation residual strength variation of a component of initial crack length/thickness ratio $a_{ii}/t_2$; the reduction at the end of the fatigue life in the unstable crack propagation residual strength due to the application of fatigue loads '$P_{ii}$' is determined; the unstable crack propagation strength of the component is then compared with the plastic deformation strength given by Eqn.7.10 to determine the residual strength $P_b$; another new estimate of the thickness $t_3$ can be determined by dividing this shear force $Q_o$ by the stress $P_b$. Iterations are carried out in this manner till successive iterations yield similar thickness ie. $T_n$-$T_{n-1}$=0 where n is the number of iteration.

## 8.6 Conclusions

Present methods of design of components include a fatigue design for strength and a static design for endurance. These designs are carried out separately, and any variation of strength with fatigue loading is neglected. Therefore design is carried out according to the curve shown in Fig. 2.38. These curves are an approximation of the actual variation of the residual strength with the number of cycles. The residual strength variation of a component due to constant amplitude loading has been obtained from fracture mechanics as shown in Chapter 5. It was discussed in Chapter 5 that such curves can be linearised to obtain the change of the residual strength of a component with variable amplitude loading. In this chapter, it has been shown that a normalised constant amplitude residual strength curve drawn for a component of initial crack length/thickness ratio 'k' can adequately represent the residual strength variation of all components having a crack length/thickness ratio greater than k and for all stress ranges. Such a normalised residual strength curve has been linearised to allow for variable stress ranges and used for design. Design is carried out using an iterative process where the thickness of the component which can withstand the maximum overload applied to the component at the end of the fatigue life is determined. The design procedure developed is accurate compared to present methods since design is based on the actual variation of strength and the design procedure can be used by practising engineers with ease.

# Chapter 9
# Application of residual strength approach for fatigue analysis

## 9.1 Introduction

In Chapters 3 to 8 it has been shown how the residual strength variation of a component can be determined and used for assessment and design. In Chapter 7, the procedure of assessing components is discussed. Thus the initial crack length is determined and the unstable-crack-propagation residual-strength-variation for constant amplitude loading is determined. The residual strength variation is then linearised and the linear portions are considered to be the residual strength variations of idealised welds. The magnification factors and initial strengths of these idealised welds are determined in order to provide assessment equations from which the condition of a component at any intermediate period of its fatigue life can be determined.

In order to design a component a normalised residual strength curve is used as described in Chapter 8. This curve can then be used to determine the residual strength variation of a curve of any plate thickness for any sequence and combination of stress range. Design is carried out using an iterative method where first the thickness of a component is estimated, then the corresponding initial crack length is determined, and as the initial crack length/thickness ratio is now known, the residual strength variation can be determined from the normalised curve. Using the residual strength curve, it is determined whether the initial estimate of thickness chosen can adequately withstand the maximum overload at the end of the fatigue life. Iterations are carried until an adequate thickness is determined.

The procedures of assessment and design developed in Chapters 3 to 8 are based on S/N curves, plastic deformation theory and fracture mechanics. The methods developed are illustrated in this chapter using practical components in order to obtain realistic results. It will be shown that the theory provides much more information than existing techniques and can be used by practising engineers with ease.

In this chapter, a stiffener weld is assessed to determine its residual strength, remaining life and crack length under fatigue loading. The assessment procedure has been specifically adapted for assessing stiffener welds in bridges. Inspection periods are then determined for the component and finally it is shown how the stiffener weld can be designed.

## 9.2 Assessment example

The stiffener weld shown in Fig.2.9 has been assessed here. It is assumed that the component has a thickness of 22mm along the section the crack develops in Fig.2.9 and that the component is composed of A-36 steel that has a fracture toughness of 1401 $N/mm^2$. A-36 steel belongs to the ferrite-pearlite category which has a value of $m = 3$ and a value of $C = 2.3x10^{-13}$. Assessment is first carried out for a given load pattern and using presently available techniques. Assessment is then carried out using the residual strength technique discussed in Section 7.5 and a comparison is made between the methods. Finally, it is shown how the technique developed in Section 7.5 can be used to determine the condition of stiffener welds in bridges.

### 9.2.1 Present Methods of Assessment (see Appendix A.1.1.a for calculations)

Consider the stiffener weld described in Section 9.2 to be subjected to the forces given in Table 9.1. The component is assessed by standard codes using the Palmgren-Miner's solution. According to Palmgren Miner, the damage caused to a component due to the application of a number of stress ranges is given by the accumulated damage law

$$\frac{N_1}{E_1} + \frac{N_2}{E_2} + \ldots + \frac{N_n}{E_n} = A \qquad (9.1)$$

where $N_1$, $N_2$, .. $N_n$ are the number of cycles of stress range $\Delta\sigma_1$, $\Delta\sigma_2$ and $\Delta\sigma_n$ applied, while $E_1$, $E_2$, .. $E_n$ is the endurance of the component corresponding to the stress ranges $\Delta\sigma_1$, $\Delta\sigma_2$ and $\Delta\sigma_n$ and A is the damage caused.

Table 9.1 Stresses applied with corresponding frequency and endurance

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| Stress range in $N/mm^2$ | 30 | 20 | 15 | 10 | 7.5 |
| Number of application in million | 28.2 | 37.5 | 28.2 | 37.5 | 18.6 |
| Endurance in millions | 56.3 | 189.9 | 450 | 1518 | 3600 |

The endurance corresponding to the stress ranges 30, 20, 15, 10 and 7.5 $N/mm^2$ is obtained from the codes (Eqn.2.28) as 56.3 million, 189.9 million, 450 million, 1518 million and 3600 million respectively as shown in Table 9.1. These endurances are substituted as $E_1$, $E_2$,..$E_n$ in Eqn.9.1. The number of applications in the second row of Table 9.1 can be substituted in Eqn.9.1 to get the value of A as 0.79.

According to Miner's law the component would fail when the value of A is 1. Thus it can be said that according to present design techniques, the stiffener weld is near failure.

### 9.2.2 Assessment using Residual Strength Technique (see Appendix A.1.1.b for calculations)

In this section, a component has been assessed using the residual strength technique discussed in Section 7.5. The first step in the assessment procedure involves determining the initial crack. A technique of determining the initial crack length of components from S/N curves has earlier been discussed in Section 6.3 of Chapter 6. In Section 6.4.3, it is shown how the crack length of different components varies with their thickness. Figure 6.6 shows the variation of the initial crack length with thickness for a stiffener weld. For a component of thickness 22mm, the initial crack length is given as 0.28 mm.

The initial crack length of the component now being known, the next step is the determination of a non-linear unstable crack propagation residual strength curve for a unit stress range by taking increments in crack length and numerically integrating. The residual strength curve is shown in Fig.9.1 where the initial strength of the component due to unstable crack propagation is 890 $N/mm^2$ and the asymptotic endurance of the component is 15.2E+12.

The residual strength variation is now linearised. The number of regions into which the residual strength curve is linearised depends on the shape of the curve showing the variation of the magnification factor against crack length/thickness. Figure 2.16a gives the variation of the magnification factor of the component against crack length/thickness for a stiffener weld. The value of M decreases in the first region, varies slowly in the second region and increases in the third region. These three regions are separated by crack-length/thickness values of 0.17 and 0.56.

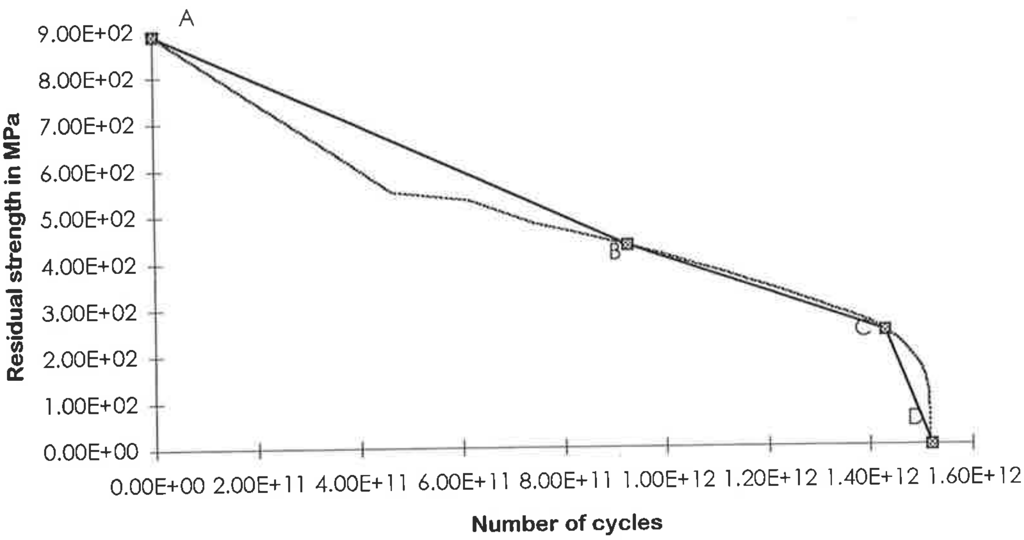



Fig.9.1 Residual strength variation of stiffener weld

For a 22mm thick plate, the corresponding cracks sizes that separate the three regions are 3.64 mm and 12.2 mm. The corresponding unstable-crack-propagation residual-strength is determined from Eqn.2.7 as 434 $N/mm^2$ and 247 $N/mm^2$. Thus the residual strength curve can be linearised between the portions of the initial strength of 890 $N/mm^2$, the intermediate strengths of 434 $N/mm^2$ and 247 $N/mm^2$ and the final strength of zero. The three linear portions are shown in Fig.9.1.

Each linear portion of the linearised residual strength curve is considered to be a part of the residual strength variations of an idealised weld. Hence the residual strength curve of the real component in Fig.9.1 can be represented by the linear residual strengths of idealised components as shown in Fig.9.2. The magnification factor of each of the idealised welds can be determined from their individual slopes G

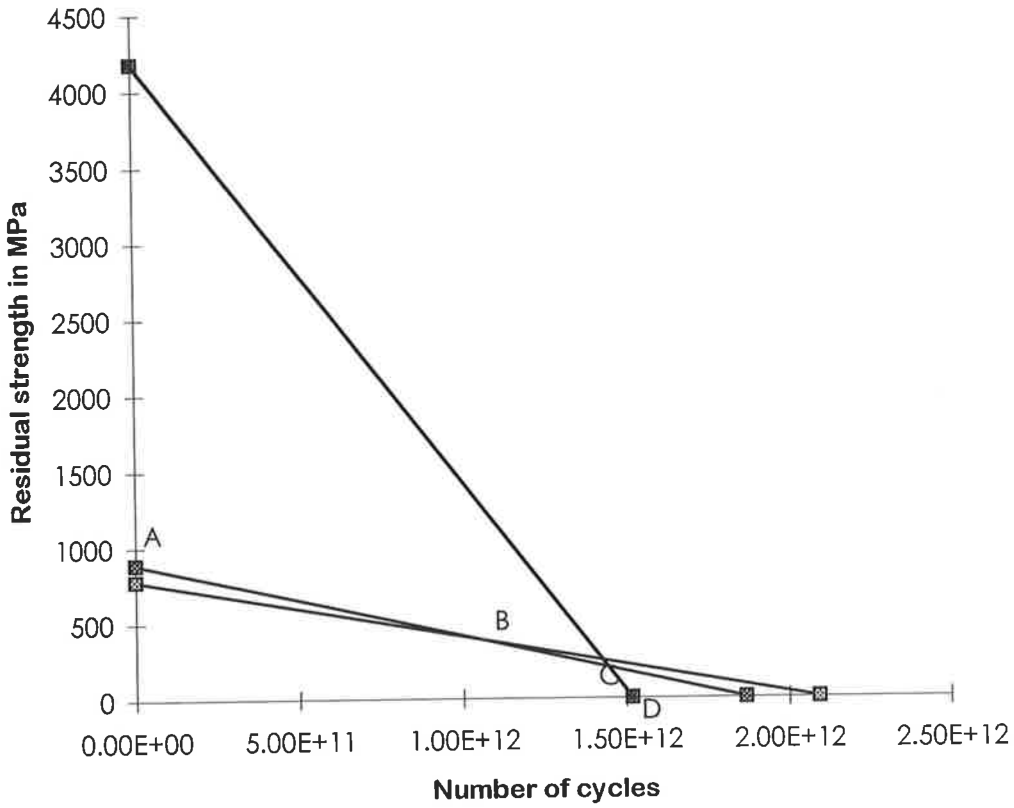



Fig.9.2. Residual strength variation of idealised components

in Fig.9.1 using $G = 2/\Pi(\Delta\sigma)^3 CM^2 K_{IC}$ (Eqn.3.21a) which for a unit stress range becomes $G = 2/\Pi CM^2 K_{IC}$. Thus the magnification factor $M_1$ of the first idealised weld is calculated as 0.99, the value of $M_2$ of the second idealised weld as 0.86, while the value of $M_3$ for the third idealised weld is 2.33. Apart from determining the magnification factor of each idealised weld, it is also necessary for analysis purposes to know the initial strength of each of the three idealised welds. These initial strengths are derived from Fig.9.1 as 890 N/mm$^2$ for the first weld, 777 N/mm$^2$ for the second weld and 4176 N/mm$^2$ for the third idealised weld.

The initial strengths and magnification factors having now been determined, the component can now be assessed for any load pattern. The component has been assessed here for the forces given in Table 9.1. The total number of cycles of loading in Table 9.1 is 150 million and the effective stress range has been calculated using Eqn.4.22 as 20 N/mm$^2$.

The assessment is started by substituting the properties of the first idealised weld into Eqn.7.6. Thus substituting $P_s$ as 890 N/mm$^2$ and M as 0.99, we can calculate $P_m$ as 299 N/mm$^2$. As the value of 299 N/mm$^2$ is less than the value of 434 N/mm$^2$, (corresponding to the point B in Fig.9.2 which separates the linear portion AB and BC in Fig.9.2), the reduction in strength is now calculated using the properties of the second idealised weld. Thus substituting $P_s$ as 777 N/mm$^2$, M as 0.86, the value of $P_m$ is calculated as 332 N/mm$^2$. As this value is greater than 247 N/mm$^2$, it gives the correct value of residual strength.

Having now determined the unstable crack propagation strength, the crack length of the component is determined from Eqn.7.7. However, it is to be noted that $M_m$ in Eqn.7.7 depends on the crack length $a_m$ and both are unknowns at this stage. Therefore, Eqn.7.7 can only be used to give the parameter $M_m\sqrt{a_m}$ and not the individual value of the crack lengths. In order to find the crack length, the values of $M_m$ and $a_m/t$ obtained from Fig.2.16a are reorganised to determine the curve of

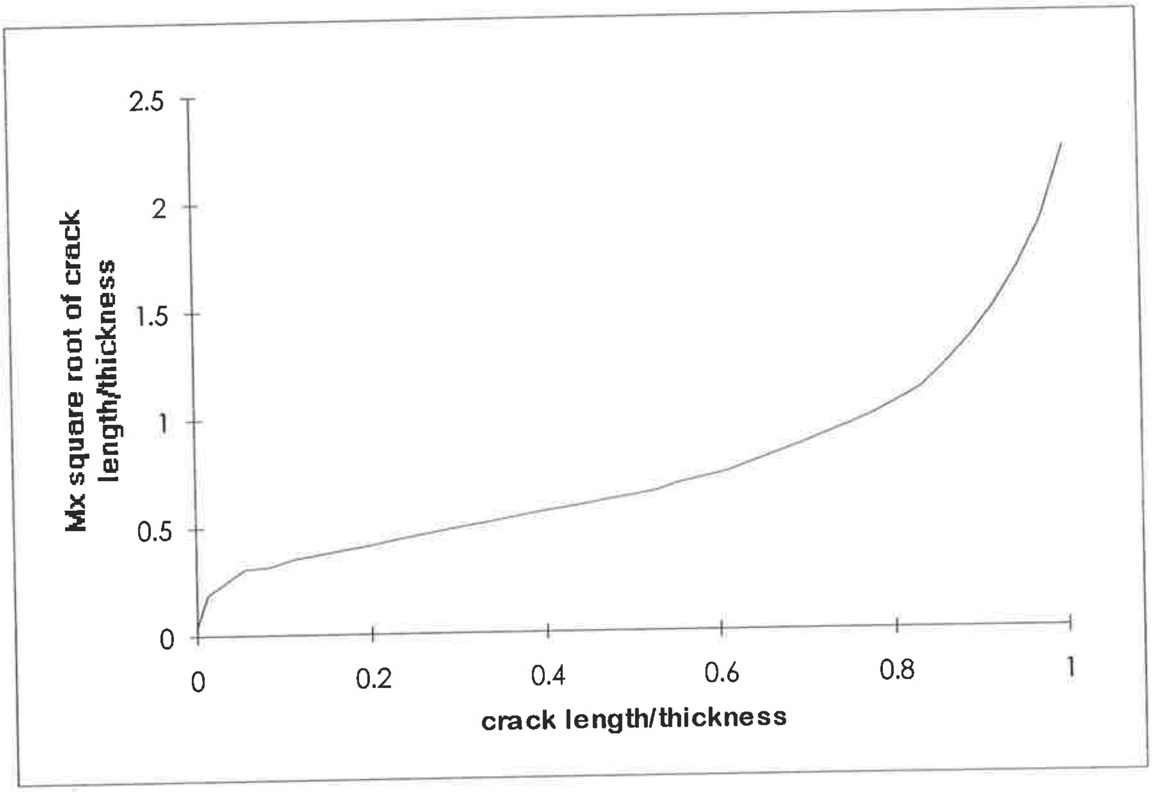



Fig.9.3 Figure showing $M\sqrt{\frac{a}{t}}$ verses a/t

$M_m\sqrt{\frac{a_m}{t}}$ verses $a_m/t$ shown in Fig.9.3. For this analysis, Eqn.7.7 gives the value of $M_m\sqrt{a_m}$ and for this value of $M_m\sqrt{a_m}$, Fig. 9.3 gives a/t and hence for a thickness of 22 mm the crack size is calculated as 7.33 mm. The crack length of the component now being known, the Von-mises plastic deformation strength is calculated using Eqn.7.8 as 317 N/mm$^2$ and the Tresca plastic deformation strength is calculated using Eqn.7.9 as 275 N/mm$^2$.

Having analysed the component according to present assessment techniques based on Palmgren-Miner's accumulated damage law and according to the residual strength assessment approach, a comparison can be made of both the methods. Palmgren-Miner gives a vague idea of the condition of the structure in the form of a ratio which considers the component to be very close to failure. The residual strength approach to fatigue design gives us the unstable crack propagation strength, the crack length and the plastic deformation strength of the component.

### 9.2.3 Assessment Example for Bridges (see Appendix.A.1.2 for calculations)

The residual strength technique of assessing components has been specifically adapted for bridges as discussed in Section 7.6 of Chapter 7. The stiffener weld assessed in Section 9.2.2 is analysed here. Table 9.2 gives the stress ranges $\Delta\sigma$ and frequencies N to which a component is subjected when a single standard vehicle passes along the bridge. The constant F in Eqn.2.39 is calculated from these stress ranges and frequencies as described in Section 2.7.3. Table 9.3 gives the weight ratio W and probability of occurrence B of vehicles of different weights, as described in Section 2.7.3 from which the constant L in Eqn.2.40 can be derived. In this section, the condition of a component in a bridge that has been subjected to 100 million cycles, is repaired or subjected to load restrictions then the condition of the component can still be determined for the component.

The assessment procedure of the component having been subjected to 100 million applications is started by using the properties of the first idealised weld developed in Section 9.2.2. After being subjected to 100 million vehicle traversals, the strength is calculated from Eqn.7.20 as 384 N/mm$^2$. As this value is less than 434 N/mm$^2$ that corresponds to the point B in Fig.9.2, the residual strength is calculated as 395 N/mm$^2$ using the properties of the second idealised weld. The value of $M_m\sqrt{a_m}$ is derived from Eqn.7.21 from which the value of the crack length is calculated from Fig.9.3 as 5.33 mm. The corresponding plastic deformation strength according to Von-mises is calculated using Eqn.7.22 as 361 N/mm$^2$ and the strength according to Tresca is calculated using Eqn.7.23 as 312 N/mm$^2$.

Table 9.2 Stress spectrum

| Level (1) | Stress range in MPa (2) | Number of applications (3) | $N(\Delta\sigma)^3$ (4) |
|---|---|---|---|
| 1 | 20 | 1 | $0.8 \times 10^4$ |
| 2 | 15 | 4 | $1.35 \times 10^4$ |
| 3 | 10 | 16 | $1.6 \times 10^4$ |
| | | | $3.75 \times 10^4$ |

The technique developed here for assessing components in bridges is capable of extreme flexibility. This is illustrated here by showing how the technique can cater for different conditions. The component considered has already been subjected to 100 million traversals, where the loads and force spectrums was given by Table 9.2 and

Table 9.3 Load spectrum

| Level (1) | Weight ratio (W) (2) | Probability of occurrence (B) (3) | $BW^3$ (4) |
|---|---|---|---|
| 1 | 6.5 | 0.00002 | 0.0055 |
| 2 | 5.0 | 0.00010 | 0.0125 |
| 3 | 2.0 | 0.01000 | 0.0800 |
| 4 | 1.0 | 0.13988 | 0.1399 |
| 5 | 0.5 | 0.25000 | 0.0313 |
| 6 | 0.2 | 0.60000 | 0.0048 |
| | | $\sum = 1$ | $\sum 0.274$ |

9.3. Let us assume that some repair work is carried out at this stage so that the stresses acting on the stiffener weld are reduced by 10%. The value of F in Table 9.2 then changes from $3.75 \times 10^4$ to $3.75 \times (0.9)^3 \times 10^4$ or $2.73 \times 10^4$. It is assumed that the loads and stresses acting on the component do not change for the next 20 million vehicle traversals. Next in order to increase the life of the bridge the vehicles passing through the bridge for the next 10 million vehicle traversals it will be assumed that a weight restriction prevents the heaviest vehicle given by level 1 in Table 9.2 from entering the bridge and hence causes the value of L to change to 0.27.

The residual strength variation of the component in the bridge, after it has been subjected to these different variations of loading, is derived from Eqn.7.22 as 313 $N/mm^2$. The residual strength being known the corresponding crack length is calculated using Eqn.2.24 and Fig.9.3 as 8.54 mm while the corresponding plastic deformation strength due to Von-mises is calculated from Eqn.2.26 as 291 $N/mm^2$ and the strength due to Tresca from Eqn.2.27 is calculated as 252 $N/mm^2$.

The remaining life of the component is determined by following the unstable-crack-propagation strength curve. The unstable crack propagation strength, after it has been subjected to these different variations of loading, is controlled by the second idealised weld and is calculated as 313 $N/mm^2$ as discussed in the previous paragraph. The second idealised weld (Fig.9.2) controls the unstable crack propagation strength until it reduces to 247 $N/mm^2$. Thereafter, the component follows the residual strength variation of the third idealised weld till the strength reduces to the maximum load applied at which the component fails. Let 195 $N/mm^2$ be the maximum stress range to which the component is subjected. The remaining life has been calculated here that this stress is due to unstable crack propagation failure. The number of vehicle traversals required for the strength to reduce from a value of 313 $N/mm^2$ to a value of 247 $N/mm^2$ is calculated substituting the properties of the second idealised weld in Eqn.7.20 as 23.9 million traversals. The number of vehicle traversals required for the strength to reduce from 247 N/mm$^2$ to 195 N/mm$^2$ is derived from the third idealised weld and is calculated as 25.6 million traversals. Therefore the total remaining life calculated using the variation of the unstable crack propagation strength is 49.5 million traversals.

## 9.3 Inspection periods (see Appendix A.1.3 for calculations)

A procedure for determining inspection periods for welded components in bridges based on the unstable crack propagation strength has earlier been discussed in Section 7.7. In this section inspection periods will be determined for a stiffener weld in the bridge being assessed. The component in the bridge is considered to be subjected to the loading patterns given in Table 9.2 and 9.3.

The inspection periods have been determined here for a constant reduction in the unstable-crack-propagation strength of 5% of the initial strength. As the initial strength of the component is 890 N/mm$^2$, inspection is carried out each time there is a reduction of strength of 5% of 890 N/mm$^2$ ie. 44.5 N/mm$^2$. Substituting the value of 890 N/mm$^2$ for $P_s$ in Eqn.7.20 and the value of (890-44.5)=845.5 N/mm$^2$ as $P_m$ (Eqn.7.29) the first inspection interval is calculated as 8.80 million traversals as shown in Fig.9.4. This rate of inspection will remain unchanged for the entire period during which the component follows the residual strength variation of the first idealised weld ie. till the strength reduces to 434 N/mm$^2$ (B in Fig.9.4). Substituting the value of $P_s$ as 890 N/mm$^2$ and $P_m$ as 434 N/mm$^2$ (Eqn.7.30) one can determine this period, for which the rate of inspection is constant, as 90.2 million traversals as shown in Fig.9.4.

After this, the inspection period of the component will be determined following the second idealised weld. The rate of inspection of a component as it follows the second idealised weld is given by substituting $P_s$=777 N/mm$^2$ and $P_m$=777-44.5=732.5 N/mm$^2$ (Eqn.7.31) as 11.5 million traversals. This value of 11.5 million traversals gives the rate of inspection for the entire period during which the component follows the residual strength variation of the second idealised weld, ie. till the strength reduces to 247 N/mm$^2$. Substituting the value of $P_s$=777 N/mm$^2$ (Fig.9.2) and $P_m$=247 N/mm$^2$ (Eqn.7.32) one can determine the period since the component was first installed during which the component follows the second idealised weld as 139 million traversals (Fig.9.4).

After this the inspection periods of the components is determined by following the residual strength variation of the third idealised weld. The rate of inspection of the component as it follows the third idealised weld is given by substituting $P_s$=4176 N/mm$^2$ (Fig.9.2) and $P_m$=247-44.5 N/mm$^2$ (Eqn.7.33) in Eqn.7.20 as 1.5 million traversals. The component needs to be inspected at this rate till it fails.

Thus the assessment procedure developed can adequately predict the condition of a component in a bridge after a given number of vehicle traversals. It can predict the crack length, the plastic deformation strength, the unstable crack propagation strength, and the remaining number of vehicle traversals the component can sustain before it fails. It not only predicts the condition of the component after being subjected to a given constant load pattern, but also can predict the condition when there is a change in the load pattern in the form of stresses applied to the bridge or change in the pattern of vehicles traversing the bridge. It can also be used to predict the intervals in terms of vehicle traversals after which inspection needs to be carried out.

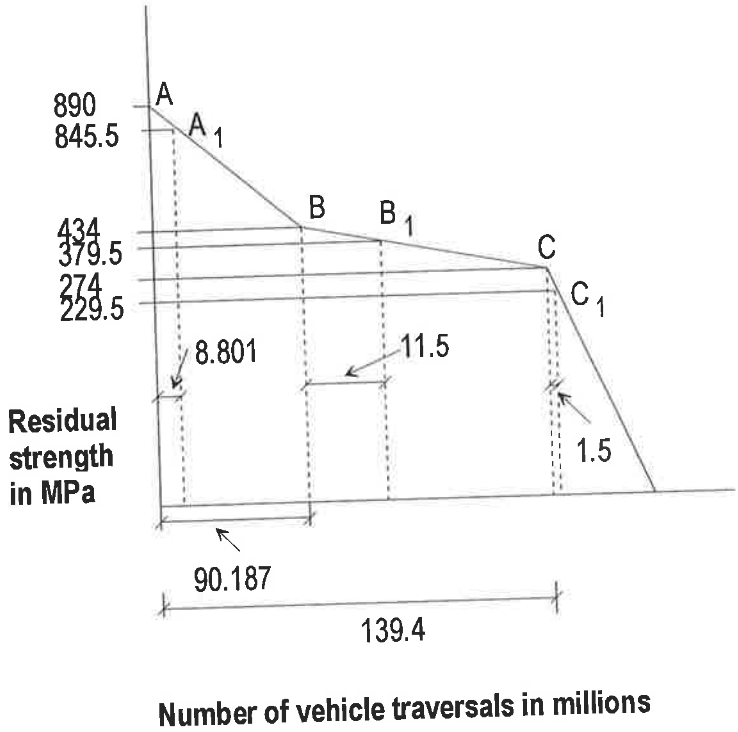



Fig.9.4 Determining inspection periods

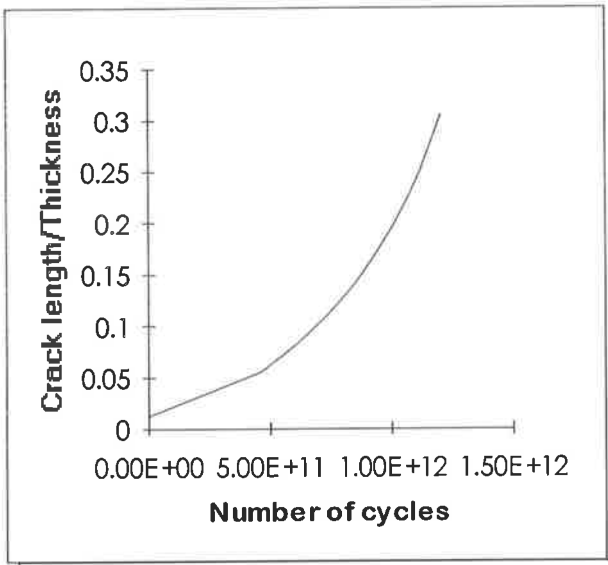
## 9.4 Design example (see Appendix A.1.4 for calculations)

The same shape of stiffener weld that was earlier assessed has been designed here for an effective stress range of 20 N/mm$^2$ and 160 million vehicle traversals. The component is considered to have the same material properties as the one assessed and is assumed that the component must be able to sustain a maximum shear flow of 5000 N/mm at the end of the fatigue life. Design has been carried out here according to the procedure described in Section 8.5. The procedure involves determining the design curve which is then used to find the design thickness using an iterative method.

A design curve of crack-length/thickness against number of cycles of a 22mm thick component with an initial crack length of 0.28 mm is shown in Fig.9.5. Design is carried out using an iterative procedure as earlier discussed in Section 8.5. The iteration is started by considering a 22mm thick plate having an initial crack length of

0.28 mm as earlier discussed. For this component the value of $P_s$ is determined and from Fig.9.5 the corresponding values of $P_1$ and $P_2$ are found out. Using the properties of the first idealised weld in Eqn.7.6, the unstable crack propagation strength of the component has been deduced as 260 N/mm$^2$ after application of a equivalent stress range of 20 N/mm$^2$ for 160 million cycles. Since this is less than $P_1$, the strength of the component is found using the second idealised weld as 302 N/mm$^2$, which being greater than $P_2$ gives the unstable crack propagation strength of the component. The corresponding crack length is found using Eqn.7.7 and Fig 9.3 as 8.36 mm. The Von-mises plastic deformation strength is calculated using Eqn.7.8 as 295 N/mm$^2$ while Tresca's plastic deformation strength is calculated using Eqn.7.9 as 255 N/mm$^2$ which is considered to be the strength of the component.

A second value of the thickness is calculated by dividing the maximum shear flow overload by the strength of the component. Dividing the shear flow of 5000 N/mm by Tresca's plastic deformation strength of 255 N/mm$^2$gives a thickness of 19.6 mm. For this thickness the corresponding initial crack is determined from Fig.6.6 as 0.31 mm. For this component the value of $P_s$ is determined and from Fig.9.5 the corresponding values of $P_1$ and $P_2$ are found out. The unstable crack propagation strength is found to lie between $P_1$ and $P_2$ and is determined using the properties of the second idealised weld as 285 N/mm$^2$. The corresponding crack length is found using Eqn7.7 and Fig.7.18 as 7.64 mm. The plastic deformation strength due to Von-mises is found using Eqn.7.8 as 290 N/mm$^2$ and that due to Tresca is found using Eqn.7.9 as 252 N/mm$^2$.



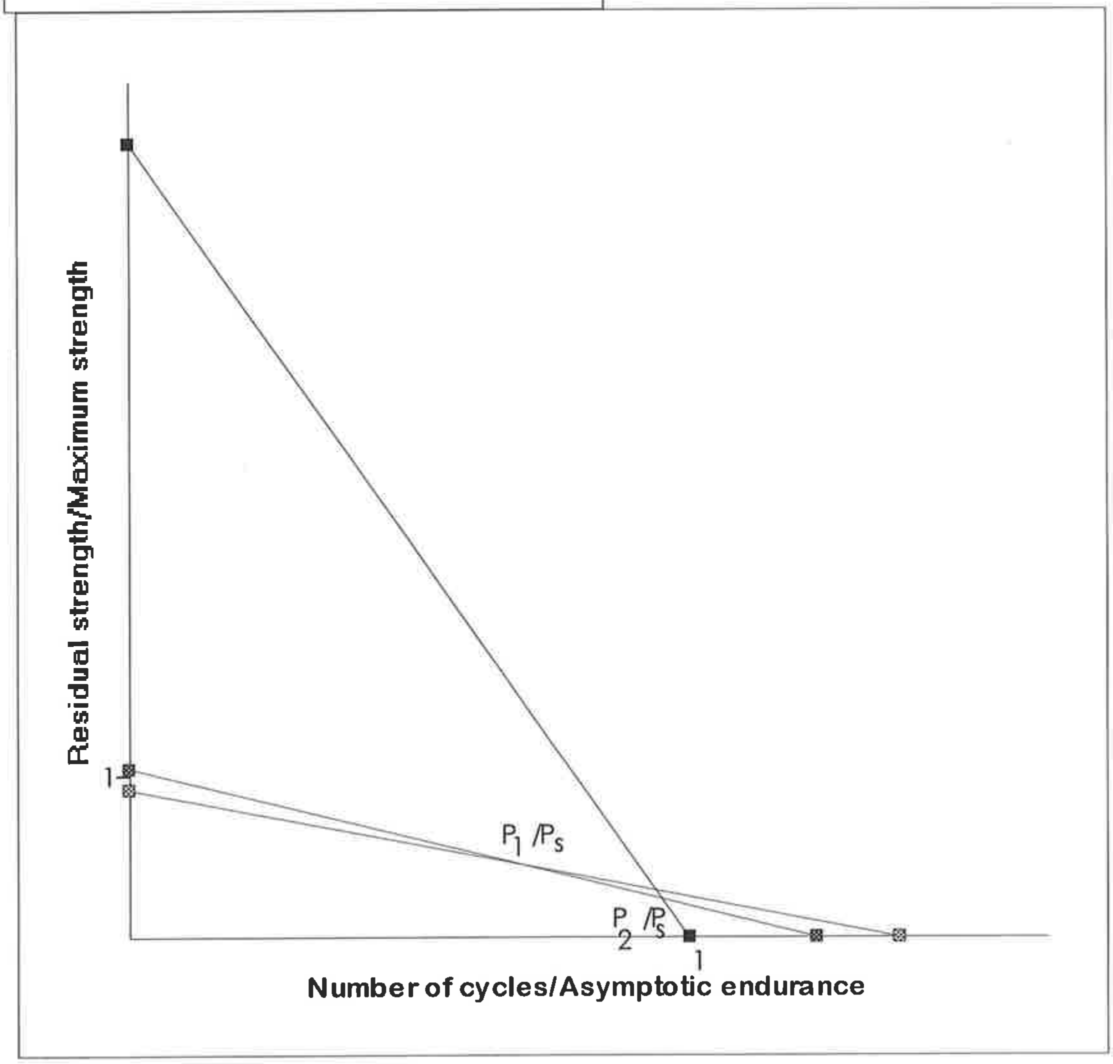




Fig.9.5a Curve showing variation of crack length with number of cycles

Fig.9.5b Normalised residual strength curve

A third value of the thickness is calculated by dividing the shear flow of 5000 N/mm by a stress of 252 N/mm$^2$ to give a thickness of 19.8 mm. For this thickness the initial crack length is determined from Fig.6.6 as 0.3 mm. For this value of initial crack length, the strength of the component is found as 289 N/mm$^2$ using the properties of the second idealised weld in Eqn.7.6. The corresponding crack length is found using Eqn.7.7 and Fig.9.3 as 7.72 mm, while the Von-mises plastic deformation strength is 290 N/mm$^2$ and the Tresca plastic deformation strength is 252 N/mm$^2$. The thickness of the component is calculated by dividing the shear flow of 5000 N/mm by the residual strength 252 N/mm$^2$ as 19.8 mm. This is the same value of thickness obtained from the last iteration and therefore gives the design thickness.

## 9.5 Conclusions

The assessment and design techniques developed in this thesis for welded structural components has been illustrated in this chapter. The assessment technique developed provides much more information on the condition of the component as compared to present techniques. The design procedure developed can be considered to be better than present methods as it is based on the actual variation of residual strength of a component. The assessment and design methods uses material properties that are commonly available and involves simple calculation that can be carried out with ease by practising engineers. The method is based on established fracture mechanics principles and established S/N curves data.

# Chapter 10
# Conclusions and Recommendations

## 10.1 Conclusion

In Civil Engineering practice, components subjected to fatigue loads are designed and assessed using S/N curves and Palmgren-Miner's summation. S/N curves give only the fatigue life of the component and Palmgren-Miner's solution give only the ratio of the fatigue life expended. In this thesis, a procedure for designing and assessing components based on residual strength has been developed. The technique developed are better than these existing techniques giving us more information about the condition of the component and can be used easily by practising engineers.

The basic principles on which the assessment and design technique is founded include determining the residual strength variation of a component for constant amplitude loading from fracture mechanics. Also, it is shown that when the constant amplitude residual strength variation is linear, then the reduction in strength due to a number of stress ranges can easily be determined and does not depend on the sequence of loads. However, when the constant amplitude residual strength curve is non-linear, then the reduction in strength due to a number of stress ranges depends on the sequence of loads and cannot be easily determined.

These basic principles once established have been used to develop a assessment and design technique for welded components based on residual strength. The first step is to develop the residual strength curve for welded components. Welded components can fail by unstable crack propagation failure or plastic deformation failure and it is shown how the residual strength curves can be developed for these two types of failure. However, in order to develop these residual strength curves the initial crack length of components need to be known which has been found from S/N curves. Having determined the initial crack length of components the residual strength variation of components can be obtained.

The aim has been to develop a simple technique of assessment which can be easily used by practising engineers. The technique is therefore based on linear curves which allows easy determination of the reduction in strength when a component is subjected to different stress ranges. Since the residual strength curve of welded components are generally non-linear, the unstable crack propagation curve which can be easily linearised based on fundamental fracture mechanics principles has been broken into linear segments. An assessment procedure has been developed based on the linearised unstable crack propagation curve. However instead of failing by unstable crack propagation a component can fail due to plastic deformation, in which case the strength of the component needs to be calculated from plastic deformation failure criteria. This has been allowed for in the technique determined. A simple hand technique has therefore been developed, which can be used to give the residual strength, the crack length, the remaining life of the component at any intermediate stage of the fatigue life. The unstable crack propagation curve has also been used to determine inspection periods. As in the case of the assessment procedure a design procedure is developed based on the unstable crack propagation curve and allowing for plastic deformation failure. This procedure designs welded components more efficiently compared to present design techniques since the actual residual strength variation is followed during design compared to present techniques which assumes the residual strength variation to be unaltered during the fatigue life.

The following work is considered original in this research:-

1) Procedure of determining the residual strength curve from fracture mechanics.

2) The residual strength curve is linear for a component failing by unstable crack propagation and having a constant value of the magnification factor M.

3) When the constant amplitude residual strength curve is non-linear, the reduction in strength depends on the sequence in which loads are applied while when the residual strength curve is linear the reduction in strength does not depend on the sequence of loads applied. As in practise it is not possible to determine the reduction in strength cycle by cycle, it is deduced that linear curves must be used to determine the reduction in strength.

4) As a welded component fails by unstable crack propagation failure or plastic deformation failure, the residual strength due to these two modes needs to be found out, the lower value giving the residual strength of the component.

5) S/N curves are used to calculate the initial strength of the component.

6) The unstable crack propagation curve can be linearised using fracture mechanics principles.

7) The straight portions of a linearised unstable crack propagation curve can be considered to be a part of the residual strength variation of an idealised weld.

8) Developing a procedure for assessing structural components that is based on the unstable crack propagation curve.

9) For a component of a given initial-crack-length/thickness ratio, the non-dimensionalised residual strength variation does not change whatever the thickness or stress range applied to the component.

10) The non-dimensionalised residual strength of a component of a given shape can be used to determine the design thickness of a component that is subjected to fatigue loads and a maximum overload.

## 10.2 Recommendation for future work

It is recommended that future work involving this research should aim to generalise the method as much as possible so that the method can be standardised in future.

The assessment procedure developed here involves determining the residual strength variation of a component whose dimensions are already known. The design procedure is developed for a component whose shape is known or in other words whose dimensions are known as a ratio of each other. In order to generalise the procedure for a given type of weld, for example a stiffener weld, it is worth determining the variation in residual strength as the shape or ratio of dimensions are changed. If the normalised residual strength variation does not change significantly with a change in dimensions, then such shapes of residual strength curve can represent all shapes of a particular type of component. Hence, a particular component can be assessed and designed using this residual strength variation.

On the other hand, if the normalised residual strength variation does not change with a variation in shape then a relation need to be established between changes in shape and changes in normalised residual strength curve. Thus a family of normalised residual strength curves can represent all shapes of a given component. Hence a particular component can be assessed or designed using these residual strength variations.

The relation between residual strength and shape should be investigated for different components and it should be determined if there is any similarity in the relation between residual strength and shear for different components. If any similarity is established, then an attempt can be made to establish a general relationship between the residual strength and shape of these different components thus generalising the technique further.

# Bibliography

AASHTO: Guide specifications for fatigue design of steel bridges, *American Association of State Highway and Transportation Officials*,1989.

Adams, N.J.I.: Crack growth in the vicinity of welds, J.Struct. Engrg., *Welding research supplement*, 508s-512s, Nov. 1973.

Ahlskog, J.J.: Bridge Management-The answer to the challenge, *Bridge evaluation, repair and rehabilitation*. Edited by A.S. Nowak. Kluwer Academic Publishers, 1990.

Ayyub, B.M. and Halder, A.: Practical structural reliability techniques, *J.Strut.Engrg., ASCE,* 110 (8), 1984.

Albrecht, P. and Yamada, K.: Rapid calculation of stress intensity factors, *J.Strut.Engrg., ASCE* ,(2), 377-389,1977.

Argon, A.S: In corrosion fatigue, ed, Devereux O.F., McEvily A.J., Staehle R.W., *NACE* 1972, p.176.

Australian standard AS 4100-1990: Steel structures, *Standards Association of Australia*, Standard house, North Sydney, NSW, Australia, 1990.

Barsom, J.M.: Fatigue crack propagation in steels of various yield strengths, *Transaction of the ASME, Journal of Engineering for Industry*, Series B, 93, No.4, Nov.1971.

Barsom, J.M.: Development of the AASHTO fracture toughnessfor bridge steels, *Journal of Engineering Fracture mechanics*, 1975, p.605.

Barsom, J.M., Imhof.Jr, E.J. and Rolfe, S.T.: Fatigue crack propagation in high yield strength steel, *Engineering fracture mechanics*, 2, No.4, June 1971.

Barsom, J.M. and Rolfe, S.T.:- $K_{Ic}$ transition temperature behavior of A517-F steel, *Journal of Engineering Fracture Mechanics*, 2, 1971.

Boresi, A.P. and Sidebottom, O.M.: *Advanced mechanics of materials*, Fourth Edition, 1985.

Bowman, M.D. and Munse, W.H.: Fatigue behavior of welded steel butt joints containing artificial discontinuities, *Welding research supplement*, Feb. 1983, pp 36s-44s.

Broek, D.: *Elementary engineering fracture mechanics*, Martinus Nijhoff Publishers, 1986, Dordrecht, The Netherlands.

Brown, W.F. and Srawley, J.E.: Plain strain crack toughness testing of high strength metallic materials, *ASTM STP 410*, pp.11-28, 1967.

BS 5400: Steel, concrete and composite bridges. Part 10: Code of practice for fatigue, 1980. *British Standards Institution*, London, England.

BS PD 6493 :Guidance on methods for assessing the acceptability of flawsin fusion welded structures. *British Standards Institution*, London, U.K, 1991.

Bucci, R.J., Greene, B.N. and Paris, P.C.: Fatigue crack propagation and fracture toughness of 5Ni and 9Ni steel at cryogenic temperatures, *ASTM STP 536, American society of testing of materials*, Philadelphia, 1973.

Buch, A.: *Fatigue strength calculation*, Aedermansdorf, Trans Tech publications, 1988.

Burns, D.J., Lambert, S.B., Mohaupt, U.H.: Crack growth behavior and fracture mechanics approach, *Steel in marine structures*, edited by C.Woodhock and J.de Back, Elsevier Science Publishers, B.V., Amsterdam, 1987.

Castiglioni, C.A.: Numerical simulation of fatigue crack growth, *IABSE Workshop*, Lausanne, 1990.

Campbell, J.E., Gerberich W.W., Underwood, J.H.: Application of fracture mechanics for selection of metallic structural materials, *American Society of Metals*, Ohio, 1982.

Clark, W.G. Jr.: Fatigue crack growth charecteristics of heavy section ASTM A533 grade B Class 1 Steel Weldments, ASME paper No. 70-PVP-24, *American society of mechanical engineers*, New York, 1970.

Cox, E.P.: A methodology for characterising weldment fracture behavior, *Welding Journal*, 1987, pp 39s-49s.

Dijkstra, O.D., Snider, H.H., Van rongen H.J.M.: Assessment of remaining fatigue life of defective welded joints, *IABSE Workshop*, Lausanne, 1990.

Eisenstadt, M.V.: *Introduction to mechanical properties of material*, The Macmillan Company , New York, 1979.

Elber, W.: The significance of fatigue crack closure, Damage tolerance in Aircraft structures, *ASTM STP 486*, pp 230-247, 1971, Philadelphia.

Erdogan, F.: Crack propagation theories, *NASA-CR-901*, 1967.

Eurocode No.3: Common unified rules for steel structures: *Commission of the European Communities*, 1992, Luxembourg.

Ewalds, H.L. and Wanhill, R.J.H.: *Fracture mechanics*, Delft, Netherlands, Delftse Uitgererers Maatschappij, 1984.

McClintock, F.A.: Plasticity aspects of fracture, *Fracture, An advanced treatise, Volume 3*, Engineering fundamentals and environmental effects, Edited by H.Liebowitz, 1968.

Feddersen, C.E.: Discussion in plain strain crack toughness testing of high strength metallic materials, *ASTM STP 410*, pp.77-9, 1967.

Feddersen, C.E.: Evaluation and prediction of the residual strength of centre cracked tension panels, Damage Tolerence in Aircraft Structures, *ASTM STP 486, American Society of Testing and Materials*, 1971, pp.50-78.

Fisher, J.W., Albrecht, P., Yen, B.T., Klingerman, D.J., McNamee, B.M.: Fatigue strength of steel beams with welded stiffeners and attachments, *NCHRP Report 147*, Highway Research Board, National Research Council, Washington, D.C., 1972.

Fisher, J.W., Frank, K.H., Hirt, M.A., McNamee, B.M.: Effects of weldments on fatigue strength of steel beams, *NCHRP Report 102*, Highway Research Board, National Research Council, Washington , D.C., 1970.

Fisher, J.W., Menzemer, C.C. and Yen, B.T.: Fatigue crack detection and repair of steel bridge structures, *IABSE Workshop*, Lausanne, 1990.

Forman, R.G., Kearney, V.E., Engle, R.M.: Numerical analysis of crack propagation in a cyclically loaded structure, *ASME Trans.J.Basic Eng.* 89D, 1967, p 459.

Fuchs, H.O., and. Stephens, R.I.: *Metal fatigue in Engineering*, A Wiley-Interscience Publication, John Wiley and Sons, 1980.

Gere, J.M. and Timoshenko, S.P.: *Mechanics of materials*, Boston, Massachusetts, 1984.

Ghosh, A., Oehlers, D.J., Wahab, M.A.: Linear fracture envelopes for fatigue assessment of welds in bridges, *Structural Engineering and Mechanics Journal*, July, 1996.

Ghosh, A., Oehlers, D.J. and Wahab, M.A.: A New Fatigue Design and Analysis Approach Based on Residual Strength, *Research report no. R108*, October, Department of Civil and Environmental Engineering, University of Adelaide, 1993.

Ghosh, A., Oehlers, D.J., Wahab, M.A.: Assessing the remaining strength and life of bridge structures that are subjected to fatigue loads, *Australasian Structural Engineering Conference,* Sydney, Sept., 1994.

Ghosh, A., Oehlers, D.J., Wahab, M.A.: Fatigue design of steel and composite bridges for greated reliability and reduced inspection periods, *5th East Asia-Pacific Structural Conference*, Wollongong, 1995.

Ghosh, A., Oehlers, D.J., Wahab, M.A.: Linear failure envelopes for the assessment of welds in bridges, *International Conference on Structural Stability and Design*, Sydney, 1995.

Ghosh, A., Oehlers, D.J., Wahab, M.A.: Adapting S/N data for fatigue design and assessment, *14 th Australasian Conference on the mechanics of Structures and Materials,* Hobart, 1995.

Green, A.E. and Sneddon, I.N.: The stress distribution in the neighbourhood of a flat elliptical crack in an elastic solid, *Proc.Cambridge Phil.Soc.,*46 (1950) pp.159-164.

Griffith, A. A.: The phenomena of rupture and flow in solids, *Phil.Trans.Roy.Soc.of London,* A 221, 1921, pp 163-197.

Gurney, T.R.: *Fatigue of welded structures,* Cambridge University Press, Second Edition, 1979.

Gurney, T. R.: Some fatigue tests on automatic longitudinal butt welds made from one side, *Welding Research,* Vol 2, No.4, 1972, pp 29-36.

Gurney, T.R.: Cumulative damage calculations taking account of low stresses in the spectrum, *Welding Research*, Vol.6, No.2, 1976, pp-51-76.

Harrison, J.D.: An analysis of the fatigue behavior of cruciform joints, *Metal construction and British welding Journal,* pp 333-335, 1969.

Harrison, J.D.: Basis for a proposed acceptance standard for weld defects, Part 2:, Slag inclusions, *Metal construction and British welding Journal*, pp 262-267, 1972.

Hertzberg, R.A.: *Deformation and fracture mechanics of engineering materials*, New York, Wiley, 1983.

Hirt, M.A., and Fisher, J.W.: Fatigue crack growth in welded beams, *Journal of Engineering fracture mechanics*, Vol.5, No.2, June, 1973.

Irwin, G.R., Liebowitz, H., and Paris, P.P.: A mystery of fracture mechanics, *Engineering fracture mechanics*, Vol.1, 1968.

Irwin, G.R.: The crack extension force for a part through crack in a plate, *Trans.ASME, J.Appl.Mech.*, 1962, pp.651-654.

Isida, N.: On determination of stress intensity factors for some common structural problems, *Engrg.Fract.Mech.*, 2, pp.61-79, 1970.

Irwin, G.R.: Analysis of stresses and strains near the end of a crack traversing a plate, *Transactions, American society of Mechanical Engineers*, Series E, Vol.24, No.3, Sept., 1957.

Jack, A.R. and Price, A.T.: The use of crack initiation and growth data in the calculation of fatigue life of specimens containing defects., *Metal construction and British welding Journal*, Nov, 1971, pp 416-419.

James, L.A.: Fatigue crack Propagation behavior of Several Pressure Vessel Steels and Weldments, *Welding research supplement*, December, 1977.

Jergeus, H.A.: A simple formula for the stress intensity factors for cracks at side notches, *Int.J.Fracture*, 14, R113-R116, 1978.

Johnson, W. and Mellor, P.B.: *Plasticity for mechanical engineers*, D. Van Nostrund Company Ltd, 1962.

Johnson, R. P.:-*Composite structures of steel and concrete*, Boston, Blackwell Scientific, 1994.

Karlsson, A. and Backlund, J.: Summary of SIF graphs for cracks emanating from circular holes, *Int.J.Fracture*, 14, 585-586, 1978.

Klesnil, M. and Lukas, P.: *Fatigue of metallic materials*. Elsevier Scientific Publishing Company, 1980.

Knesl, Z., Marek, P., Polcar, P., Vlk, M.: Remaining fatigue life of a spherical joint, *IABSE Workshop*, Lausanne 1990.

Knott, J.F: *Fundamentals of fracture mechanics*, London, Butterworth, 1973.

Larsen, J.M. and Annis, C.J.Jr.: Observation of crack retardation resulting from load sequencing chareteristic of military gas turbine operation, in Effect of Load Spectrum Variables on Fatigue Crack Initiation and Propagation, *ASTM STP 714*, pp.91-107, 1980.

Larsen, J.M and Nicholas, T.: Cumulative damage modelling of fatigue crack growth in turbine engine materials, *Engng. Fract. Mech.*, 22, pp.713-30, 1985.

Lawrence, F.V.: Estimation of fatigue crack propagation life in butt welds, *Welding Journal*, Vol.52, No.5, 1973, pp.212-220s.

Lawrence, F.V. and Munse, W.H.: Fatigue crack propagation in butt welds containing joint penetration defects, *Welding Journal*, Vol.52, No.5, 1973, pp.221-225s.

Linda, R.L: Fatigue crack growth of weldments, fatigue and fracture testing of weldments, ASTM STP 1058, *American Society for Testing of Materials*, 1990, pp.16-33.

Maddox, S.J.: Calculating the fatigue strength of a welded joint using fracture mechanics, *Metal construction and British welding Journal*, August, 1970.

Maddox, S.J.: Assessing the significance of flaws in welds subjected to fatigue, *Welding research supplement*, Vol.53, No.9, 1974, p-401s.

Maddox, S.J.: *Fatigue strength of welded structures*. Second edition, Abington publishing, 1991.

Mattos, R.J. and Lawrence, F.V.: Estimation of fatigue crack initiation life in welds using low cycle fatigue concepts, *SP-424, Society of Automotive Engineers*, 1979.

*Manual for maintenance inspection of bridges*, AASHTO, Washington, D.C., 1983.

McClintock, F.A.: Plasticity aspects of fracture, *Fracture-An Advanced treatise*, edited by H.Liebowitz, Academic press, New York and London, 1971.

McClintock, F.A.: On notch sensitivity. *Welding research supplement*, pp-202s, May, 1961.

McClintock, F.A., and Irwin, G.R.: Plasticity aspects of fracture mechanics, *ASTM STP 381*, 1965, pp 84-113.

Meguid, S.A.: *Engineering fracture mechanics*, New York, Elsevier, Applied Science, 1989.

Miner, M.A.: Cumulative damage in fatigue, *J.appl.Mech*, Vol.12, No.3, Sept. 1945, pp.159-164.

Moses, F.: Bridge load models for fatigue, *IABSE Workshop*, Lausanne 1990.

Oehlers, D.J.: Deterioration in strength of stud shear connectors in composite bridge beams, *J.Strut.Engrg, ASCE*, 1990, 3417-3431.

Oehlers, D.J.: Residual strength of structural components subjected to cyclic loads, *J.Strut.Engrg., ASCE*, 1992, 2645-2658.

Oehlers, D.J.: Design and assessment of shear connectors in composite bridge beams, *J.Strut.Engrg, ASCE*, Feb. 1995.

Oehlers, D.J., Ghosh, A., Wahab, M.A.: Fatigue design and analysis of structural components based on residual strength, *Research report no. R114*, March, Department of Civil and Environmental Engineering, University of Adelaide, 1994.

Oehlers, D.J., Ghosh, A., Wahab, M.A.: A residual strength approach to fatigue design and analysis, *J.Strut.Engrg. American Society of Civil Engineers*, Sept, 1995.

Oehlers, D.J. and Bradford, M.A.: *Composite steel and concrete structural members-Fundamental behaviour*, Permagon Press, U.K, 1995.

Oehlers, D.J., Ghosh, A., Wahab, M.A.: Assessment of the performance of welds in existing structures subjected to fatigue loads, *Third International Conference on Bridge Management*, E & F.N. Spon, U.K., 1996.

Oehlers, D.J.,Wahab, M., Ghosh A.: Fatigue design and assessment based on the residual strength of a component, *13th Australian Conference on the mechanics of Structures and Materials*, Wollongong, July, 1993.

Otegui, J.L., Kerr, H.W., Burns, D.J., Mohaupt, U.H.: Fatigue crack initiation from defects at weld toes in steel, *Int.J.ves.and Piping*, 1989, pp. 385-417.

Paris, P.C. and Erdogan, F.: A critical analysis of crack propagation laws, *J.Bas.Engng, Transactions, ASME,* 85, No.4528, 1963.

Paris, P.C., Gomez, M.P. and Anderson, W.E.: A rational analytic theory of fatigue, *The trend in engineering*, 13, 1961, pp.9-14.

Paris, P.C.and Sih, G.C.: Stress analysis of cracks, *ASTM STP 391*, 1965, pp.30-81.

Pellini,.W.S.: Design options for selection of fracture control procedures in the modernization of codes, rules and standards, in Proceedings: *Joint United States-Japan Symposium on Application of Pressure Component Codes*, Tokyo, Japan, March 13-15, 1973.

Pollard, B. and Cover, R.J.: *Fatigue of steel weldments*, Nov, 1972, pp-544s-554s.

Roberts, R., Irwin G.R., Krishna, G.V. and Yen, B.T.: Fracture toughness of bridge steels-phase II final report, *Fritz Engineering Laboratory Report No. 379.2*, Lehigh University, Bethlehem, June 1974.

Rolfe, S.T. and Barsom, J.M.: *Fracture and fatigue control in structures*, Prentice-Hall, 1977, New Jersey.

Rooke, D.P. and Cartwright, D.J.: *Compendium of stress intensity factors,* London, HMSO, 1974.

Rice, J.R.: The line spring model for surface flaws. The surface crack, physical problems and computational solutions, *Presented at the winter annual meeting of the American Society of Mechanical Engineers*, New York, 1972.

Schijve, J.: The stress ratio effect on fatigue crack growth in 2024-T3 Alclad and a relation to crack closure, Delft University of Technology, *Department of Aerospace Engineering, Memorandum M-336,* August 1979.

Schijve, J.: The stress intensity factor for small cracks at notches, *Delft University of Technology, Rep. LR-330*, 1981.

Sedlacek, G.: Determination of the residual safety and service life of old steel bridges and conclusions for the design of modern bridges, *The design life of structures*, Blackie and Son Ltd, London, 1992.

Skallerud, B., Eide, O.I., Berge, S.: Fatigue crack growth in complex tubular joints, *IABSE Workshop,* Lausanne 1990.

Skorupa, M: Fatigue crack initiation life prediction for welded joints by low cycle fatigue approach, *Fracture Engineering Material Structures*, Vol.13, No.6, pp-597-613, 1990.

Skorupa, M.: Fatigue life prediction of cruciform joints failing at the weld toe, *Welding research supplement,* August, 1992, pp-269s

Skorupa, M., Braam, H. and Prij, J: Applicability of approximate $K_I$ solutions towards cracks at weld toes, *Engineering fracture mechanics*, Vol 26, No. 5, pp.669-681, 1987.

Smith, R.A. and Miller, K.J.: Prediction of fatigue regimes in notched components, *Int.J.Mech.Sci*, 19,11-22, 1977.

Smith, I.F.C. and Smith, R.A.: Fatigue crack growth in a fillet welded Joint, *Engrg Fract Mechanics,* Vol.18, N0.4, pp. 861-893, 1983.

Sneddon, I.N.: The distribution of stress in the neighbourhood of a crack in an elastic solid, *Proc.Roy.Soc.London A 187*, 1946, pp.229-260.

Suresh, S: *Fatigue of materials*, Cambridge, Cambridge University Press, 1992.

Tada, H., Paris, P., and Irwin, G.R.: *The stress analysis of cracks handbook*, Del Research Corporation, Hellertown, Pa., 1973.

Testin, R.A., Yung, J.Y., Lawrence, F.V., Rice, R.C.: Predicting the fatigue resistence of steel weldments, *Welding research supplement*, April, 1987.

Tsai, C.L. and Tsai, M.J.: Significance of weld undercut in design of fillet welded T-joints, *Welding research supplement*, February, 1984.

Timoshenko, S.P. and Goodier, J.N.: *Theory of elasticity*, Third edition, McGraw-Hill, 1970.

Wahab, M., Oehlers, D.J. and Ghosh, A.: Residual strength approach for fatigue design of structural components, *5th Australian Aeronautical Conference*, Melbourne, Sept., 1993.

Wheeler O.E.: Spectrum loading and crack growth, *J.Basic Emgrg, Trans. ASME*, 94, pp.181-6, 1972.

Yazdani N. and Albrecht, P.: Risk analysis of fatigue failure of highway steel bridges, *Journal of structural Engineering, ASCE*, Vol.113, No.3, March, 1987.

Yamada, K: Measurement of service stresses and fatigue life evaluation of bridges, *IABSE Workshop*, Lausanne, 1990.

Zhao, Z., Halder, A., and Breen, F.L.: Fatigue evaluation of steel bridges, *J. Struct.Engrg., ASCE*, 120 (5), 1608-1623, 1993.

Zhao, Z., Halder, A., and Breen, F.L.: Fatigue reliability updating through inspection of steel bridges, *J. Struct.Engrg., ASCE*, 120 (5), 1624-1642, 1993.

# Appendix A.1

## A.1.0 Introduction

The calculations corresponding to the examples in Chapter 9 are given in this appendix. In Section A.1.1 a stiffener weld of A-36 steel is analysed. The fracture toughness of the component under fast loading is considered to be 45 $MN/m^{3/2}$ or 1401 $N/mm^{3/2}$. A-36 is considered to be of ferrite-pearlite category which has a value of m=3 and a value of $C=2.3x10^{-13}$. In Section A.1.1 a given loading is chosen and assessment is carried out for a 22 mm thick component by presently available techniques and by the residual strength technique discussed in Section 7.5 of this thesis. In Section A.1.2 of this appendix it is shown how the same stiffener weld described above can be assessed after being in service in a bridge according to a procedure discussed in Section 7.6 of the thesis. In Section A.1.3 the inspection periods for this component when in service in a bridge is determined according to a producer discussed in Section 7.7. Lastly in Section A.1.4 a stiffener of A-36 steel is designed to withstand some specified loads.

## A.1.1 Assessment example for components

In this section of the appendix a component subjected to the loads given in Table 9.1 is assessed according to present methods (A.1.1.a) and according to the assessment technique developed (A.1.1.b).

### A.1.1.a Assessment of a component according to present available techniques.

Endurance of a component of type D (BS 5400, 1980) in Table 2.1 subjected to a stress range of 30 $N/mm^2$ is given by Eqn.2.30 as

$$E_1 = \frac{K_o \times \Delta_d \times \left(\frac{22}{t}\right)^{0.25}}{(\Delta\sigma)^3} = \frac{3.99 \times 10^{12} \times (0.617)^2}{(30)^3} = 56.3 \times 10^6 \text{ cycles.}$$

Similarly endurance of the same component when subjected to a stress range of 20 N/mm$^2$ is given by Eqn.2.30 as 189 x 10$^6$ cycles, endurance when subjected to a stress range of 15 N/mm$^2$ is given as 450 x 10$^6$ cycles, endurance when subjected to a stress range of 10 N/mm$^2$ is given as 1518 x 10$^6$ cycles and endurance when subjected to a stress range of 7.5 N/mm$^2$ is given as 3600 x 10$^6$ cycles.

According to Miner's law (Eqn.9.1)

$$\frac{N_1}{E_1}+\frac{N_2}{E_2}+\frac{N_3}{E_3}+\ldots= A$$

Therefore for the given example the damage is

$$\frac{28.2}{56.3}+\frac{37.5}{189.9}+\frac{28.2}{450}+\frac{37.5}{1518}+\frac{18.6}{3600}=0.79$$

Thus according to Miner's law almost four/fifth of the total life of the component has been expended.

## A.1.1.b Assessment using new residual strength technique

The initial crack length of a stiffener of thickness 22 mm is obtained from Fig.6.6 as 0.28 mm. The magnification factor M curve of a stiffener weld is shown in Fig.2.16. The crack length/thickness ratios 0.17 and 0.55 separate the M-curve into three regions. For a component of 22 mm thickness these crack length/thickness ratios corresponds to crack lengths of 3.64 mm and 12.2 mm. Since the residual strength curves are based on the M-curve, the residual strengths at crack lengths 3.64 mm and 12.2 mm separate the residual strength curve into three different portions.

The residual strength at crack length 3.64 mm is given by Eqn.2.30 as

$$\sigma = \frac{K_{IC}}{M\sqrt{\Pi a}} = \frac{1401}{0.955\sqrt{\Pi \times 3.64}} = 434 \text{ N/mm}^2.$$

Similarly the residual strength at crack length 12.2 mm is given by Eqn.2.30 as 247 N/mm$^2$.

The variation of residual strength against number of cycles of the stiffener weld is found out as described earlier in Section 5.2. Thus the residual strength of a component of initial crack 0.28 mm is found out using Eqn.2.30. This residual strength corresponds to zero cycles. Knowing the residual strength and number of cycles corresponding to a crack length of 0.28 mm a point can be plotted on the residual strength against number of cycles curve. Then an increment of 0.25 mm is considered and the number of cycles required by the initial crack of 0.28 mm to propagate to a crack of 0.28+0.25=0.53 mm is determined using Eqn.2.19. The strength of the component of crack length 0.53 mm is found out using Eqn.2.30. Knowing the number of cycles and residual strength corresponding to a crack length of 0.53 mm a point is plotted on the residual strength against number of cycles curve. Further increments of 0.25 mm are taken and points corresponding to these crack lengths are plotted. This process is continued till the length of the crack reaches the thickness of the component. All the points plotted are joined to give the residual strength against number of cycles curve. This curve has been shown in Fig.9.1.

It has earlier been discussed that the crack lengths 3.64 mm and 12.2 mm corresponding to the residual strengths 434 $N/mm^2$ and 247 $N/mm^2$ separate the residual strength curve into three different regions. As discussed in Section 7.4 these three regions have been linearised as shown in Fig.9.1 and the linearised curve is used for easy assessment.

The three linear portions forms a part of the residual strength variations of idealised welds. The magnification factor of these idealised welds is next determined.

Slope G of the residual strength variation of the first idealised weld is given by Fig.9.1 as $9.26 \times 10^{11}/(890-434)$.

G is given by Eqn.3.21a as $G = 2/\Pi C M^2 K_{IC} = \dfrac{2 \times 10^{13}}{M^2 \times 2.3 \times \Pi \times 1401}$

Equating these two values of G the magnification factor of the first idealised weld is determined as M=0.986.

Slope G of the residual strength variation of the second idealised weld is given by Fig.9.1 as $5.04 \times 10^{11}/187$.

G is given by Eqn.3.21a as $G = 2/\Pi C M^2 K_{IC} = \dfrac{2 \times 10^{13}}{M^2 \times 2.3 \times \Pi \times 1401}$

Equating these two values of G the magnification factor of the second idealised weld is determined as M=0.856.

Slope G of the residual strength variation of the third idealised weld is given by Fig.9.1 as $0.9 \times 10^{11}/247$.

G is given by Eqn.3.21a as $G = 2/\Pi C M^2 K_{IC} = \dfrac{2 \times 10^{13}}{M^2 \times 2.3 \times \Pi \times 1401}$

Equating these two values of G the magnification factor of the third idealised weld is determined as M=2.328.

By extending the linear portions of the linearised curve in Fig.9.1 so that they touch the axes, the residual strength variation of the three idealised welds is determined. This has been shown in Fig.9.2. The initial strengths of the three idealised welds have been determined from similar triangles as 890 N/mm$^2$, 777 N/mm$^2$ and 4176 N/mm$^2$.

The residual strength variation shown in Fig.9.2 is next used to assess the condition of the weld after it has been subjected to the loads given in Table 9.1.

It is to be noted that the total number of loads application according to Table 9.1 is 150 million. The effective stress range is calculated using Eqn.4.22 as 20 N/mm$^2$.

The assessment is started by substituting the properties of the first idealised weld into Eqn.7.6.

Eqn.7.6 is given as

$$1-\frac{P_m}{P_s}=\frac{N_e(\Delta\sigma_e)^3 CM^2\Pi K_{IC}}{2P_s}$$

Substituting the values $P_s$=890 N/mm$^2$, $N_e$=150 million, $\Delta\sigma_e$=20 N/mm$^2$, M=0.986

Eqn.7.6 can be written as

$$1-\frac{P_m}{890}=\frac{150\times10^6\times(20)^3\times2..3\times10^{-13}\times(0.986)^2\times\Pi\times1401}{2\times890}$$

or $P_m$=299.5 N/mm$^2$.

Since the value of 299.5 N/mm$^2$ is less than 434 N/mm$^2$ the first idealised weld does not give the strength of the component and the properties of the second idealised weld are used to give the required residual strength

Substituting the values $P_s$=777 N/mm$^2$, $N_e$=150 million, $\Delta\sigma_e$=20 N/mm$^2$, M=0.856
Eqn.7.6 can be written as

$$1-\frac{P_m}{777}=\frac{150\times10^6\times(20)^3\times2..3\times10^{-13}\times(0.856)^2\times\Pi\times1401}{2\times777}$$

or $P_m$=332 N/mm$^2$.

The value of 332 N/mm$^2$ is greater than 247 N/mm$^2$ and gives the correct value of residual strength.

In order to calculate the crack length we use the stress intensity equation (Eqn.2.24) which is given as $K_{IC} = M\sigma\sqrt{\Pi a}$

Substituting $K_{IC}$=1401 N/mm$^{3/2}$ and $\sigma$=332 N/mm$^2$ we get

$$1401 = 332 \times \sqrt{\Pi} \times M\sqrt{a}$$

which gives $M\sqrt{\Pi a} = 2.38$

Since the thickness t is 22mm, $M\sqrt{\frac{a}{t}}$=0.51.

The corresponding value of a/t is determined from Fig.9.3 as 0.33 and the crack length for a component 22 mm thick is 7.33 mm. The von-Mises plastic deformation strength for the component of tensile strength 412 N/mm$^2$ is calculated using Eqn.7.8 which is given as

$$P_{mp} = \frac{2}{\sqrt{3}} \times f_u \times \left(1 - \frac{a}{t}\right)$$

$$= \frac{2}{\sqrt{3}} \times 412 \times \left(1 - \frac{7.33}{22}\right)$$

=317 N/mm$^2$.

The Tresca plastic deformation strength is calculated using Eqn.7.9 which is given as

$$P_{mp} = f_u \times \left(1 - \frac{a}{t}\right)$$

$$= 412 \times \left(1 - \frac{7.33}{22}\right)$$

=274 N/mm$^2$.

The Tresca plastic deformation strength of 274 N/mm$^2$ being the lowest gives the real strength of the component.

The residual strength approach of assessment as shown in this example gives the crack length and residual strength of a component in contrast to Miner's method which only gives an approximate value of the fatigue life expended.

## A.1.2 Assessment examples for bridges

In this Section of the appendix the same stiffener, 22 mm thick of A-36 steel is assessed when used in a bridge. Initially the component is assessed after 100 million vehicle traversals. The frequency of stress ranges on the weld if a standard fatigue vehicle passes through the bridge is given by Table 9.2 and the load pattern is given by Table 9.3.

After application of 100 million cycles the residual strength of the component is calculated using the properties of the first idealised weld in Eqn.7.20 which is given as

$$TFL = \left(1 - \frac{P_m}{P_s}\right)\frac{2P_s}{CM^2 \Pi K_{IC}}$$

Substituting the values F=3.75 x $10^4$, L=0.274, T=100 x $10^6$, C=2.3 x $10^{-13}$, M=0.986, $K_{IC}$=1401 N/mm$^{3/2}$, $P_s$=890 N/mm$^2$ we get

$$3.75 \times 10^4 \times 0.274 \times 10^8 = \left(1 - \frac{P_m}{890}\right)\frac{2 \times 890}{2 \times (0.986)^2 \times \Pi \times 1401}$$

or $P_m$=384 N/mm$^2$

Since 384 N/mm$^2$ is less than 434 N/mm$^2$, the next step is to calculate the residual strength of the component using the properties of the second idealised weld in Eqn.7.20

Substituting the values F=3.75 x $10^4$, L=0.274, T=100 x $10^6$, C=2.3 x $10^{-13}$, M=0.856, $K_{IC}$=1401 N/mm$^{3/2}$, $P_s$=777 N/mm$^2$ Eqn.7.20 can be written as

$$3.75 \times 10^4 \times 0.274 \times 10^8 = \left(1 - \frac{P_m}{777}\right)\frac{2 \times 777}{2 \times (0.856)^2 \times \Pi \times 1401}$$

or $P_m$=395 N/mm$^2$.

The value of $M\sqrt{a}$ is derived from Eqn.2.24 as being equal to 2.

For a component of 22 mm thickness $M\sqrt{\frac{a}{t}} = 0.426$.

The corresponding value of a/t is determined from Fig.9.3 as 0.24. Hence the crack length for a 22mm thick component = 0.24 x 22 =5.3 mm. The von-Mises plastic deformation strength for the component of tensile strength 412 N/mm$^2$ is calculated using Eqn.7.8 which is given as

$$P_{mp} = \frac{2}{\sqrt{3}} \times f_u \times \left(1 - \frac{a}{t}\right)$$

$$= \frac{2}{\sqrt{3}} \times 412 \times \left(1 - \frac{5.3}{22}\right)$$

$=361$ N/mm$^2$.

The Tresca plastic deformation strength is calculated using Eqn.7.9 which is given as

$$P_{mp} = f_u \times \left(1 - \frac{a}{t}\right)$$

$$= 412 \times \left(1 - \frac{5.3}{22}\right)$$

$=312$ N/mm$^2$

The Tresca plastic deformation strength of 312 N/mm$^2$ being the lowest gives the strength of the component. This strength of the component has reduced to this value after being subjected to 100 million cycles of the load pattern given by Table 9.2 and 9.3.

Next the condition of the welded component is assessed after the bridge has been repaired which causes the stress on the bridge to be reduced by 10% and a load restriction has been placed on the bridge so that the loads in level 1 of Table 9.3 is prevented from entering the bridge. The bridge is in the repaired condition for the next 20 million traversals following which the weight restriction is placed on the bridge for the next 10 million traversals.

The strength of the component after being subjected to these load patterns is given by Eqn.7.28 which is given as

$$P_s - P_{mx} = \sum_{i=1}^{i=x} (TFL)_i \frac{CM^2 \Pi K_{IC}}{2}$$

Since after application of 100 million cycles the strength of the component was given by the second idealised weld, the start to the problem is made with the properties of the second idealised weld to find the strength. The value of TFL has changed three times in this problem. For the first 100 million cycles TFL is given as

$$(TFL)_1 = 3.75 \times 10^4 \times 0.274 \times 10^8$$

For the next 20 million traversals the stresses on the component is reduced by 10 %. Thus the value of F given by Table 9.2 changes from 3.75 x $10^4$ to a value of $(0.9)^3$x3.75x $10^4$=2.74x$10^4$ and TFL is given as

$$(TFL)_2 = 2.74 \times 10^4 \times 0.274 \times 20 \times 10^6$$

For the next 10 million traversals a weight restriction is placed on the bridge. Thus the value of L changes from 0.274 to 0.27 and TFL is given as

$$(TFL)_3 = 2.74 \times 10^4 \times 0.27 \times 10^6$$

Substituting the values of TFL, $P_s$=777 N/mm$^2$, C=2.3 x 10$^{-13}$, M=0.856, $K_{IC}$=1401 N/mm$^{3/2}$, Eqn.7.28 can be written as

$$777 - P_{mx} = \frac{\left[1.0275 \times 10^{12} + 0.73675 \times 10^{11} + 1.498095 \times 10^{11}\right] \times 2.3x10^{-13} \times \Pi \times 1401 \times (0.856)^2}{2}$$

or $P_{mx}$=313 N/mm$^2$.

Since 313 N/mm$^2$ lies between 434 N/mm$^2$ and 247 N/mm$^2$ it gives the correct value of residual strength.

The value of $M\sqrt{a}$ is calculated using Eqn.2.24 as 2.53. Since the component is 22mm thick the value of $M\sqrt{\frac{a}{t}}$ is 0.55.

From Fig.9.3 we get the value of the crack length as 8.54 mm. The von-Mises plastic deformation strength for the component of tensile strength 412 N/mm$^2$ is calculated using Eqn.7.8 which is given as

$$P_{mp} = \frac{2}{\sqrt{3}} \times f_u \times \left(1 - \frac{a}{t}\right)$$

$$= \frac{2}{\sqrt{3}} \times 412 \times \left(1 - \frac{8.54}{22}\right)$$

=291 N/mm$^2$.

The Tresca plastic deformation strength is calculated using Eqn.7.9 which is given as

$$P_{mp} = f_u \times \left(1 - \frac{a}{t}\right)$$

$$=412 \times \left(1 - \frac{8.54}{22}\right)$$

=252 N/mm$^2$

The value of 252 N/mm$^2$ being the lowest gives the actual residual strength of the component.

The problem states that the maximum unstable crack propagation strength the component can withstand is 195 N/mm$^2$. Thus the remaining life can be found by calculating the number of vehicle traversals required for the strength to reduce from 313 N/mm$^2$ to 195 N/mm$^2$. The residual strength variation follows the second idealised weld as the strength reduces from 313 N/mm$^2$ to 247 N/mm$^2$ and the third idealised weld as the strength reduces from 247 N/mm$^2$ to 195 N/mm$^2$.

The number of traversals required for the strength to reduce from 313 N/mm$^2$ to 247 N/mm$^2$ is given using the properties of the second idealised weld in Eqn.7.20. Thus if $T_1$ is the number of traversals we get

$$1-\frac{247}{313}=\frac{T_1 \times 0.274\times 10^4 \times 0.27 \times 2.3\times 10^{-13} \times 1401 \times (0.856)^2 \times \Pi}{2\times 313}$$

from which we get

$T_1$=23.9 traversals.

The number of traversals required for the strength to reduce from 247 N/mm$^2$ to 195 N/mm$^2$ is given using the properties of the third idealised weld in Eqn.7.20. Thus if $T_2$ is the number of traversals we get

$$1-\frac{195}{247}=\frac{T_2 \times 0.274\times 10^4 \times 0.27 \times 2.3\times 10^{-13} \times 1401 \times (2.328)^2 \times \Pi}{2\times 247}$$

from which we get

$T_2$=25.6 traversals.

Therefore the remaining life is given as $T_1$+$T_2$=49.5 million traversals.

## A.1.3 Inspection Periods

Inspection is carried out each time there is a 5% reduction of the initial strength ie. each time there is a 5/100 x 890=44.5 N/mm$^2$ reduction in strength.

Substituting the values of $P_s$=890 N/mm$^2$ and $P_m$=845.5 N/mm$^2$, F=3.75x10$^4$, L=0.274, Eqn.7.20 can be written as

$$1-\frac{845.5}{890}=\frac{T\times\left(3.75\times10^{4}\right)\times0.274\times2.3\times10^{-13}\times\Pi\times1401\times\left(0.986\right)^{2}}{2\times890}$$

or T=8.8 million.

which is the inspection period of the first idealised weld.

This rate of inspection will remain unchanged for the entire period during which the component follows the residual strength variation of the first idealised weld ie. till the strength reduces to 434 N/mm$^2$. Substituting the values $P_s$=890 N/mm$^2$ and $P_m$=434 N/mm$^2$, F=3.75x10$^4$, L=0.274, Eqn.7.20 can be written as

$$1-\frac{434}{890}=\frac{T\times\left(3.75\times10^{4}\right)\times0.274\times2.3\times10^{-13}\times\Pi\times1401\times\left(0.986\right)^{2}}{2\times890}$$

or T=90.2 traversals.

Substituting the properties of the second idealised weld we get the inspection period of the second idealised weld. Thus substituting the values of $P_s$=777 N/mm$^2$ and $P_m$=732.5 N/mm$^2$, F=3.75x10$^4$, L=0.274, Eqn.7.20 can be written as

$$1-\frac{732.5}{777}=\frac{T\times\left(3.75\times10^{4}\right)\times0.274\times2.3\times10^{-13}\times\Pi\times1401\times\left(0.856\right)^{2}}{2\times777}$$

or T=11.5 million.

The rate of inspection will remain unchanged for the entire period during which the component follows the residual strength variation of the second idealised weld ie. till the strength reduces to 247 N/mm$^2$.

Substituting the values of $P_s$=777 N/mm$^2$ and $P_m$=247 N/mm$^2$, F=3.75x10$^4$, L=0.274, Eqn.7.20 can be written as

$$1-\frac{247}{777}=\frac{T\times\left(3.75\times10^{4}\right)\times0.274\times2.3x10^{-13}\times\Pi\times1401\times(0.856)^{2}}{2\times777}$$

or T=139 million.

The rate of inspection for the third idealised weld is determined by substituting the values of $P_s$=4176 N/mm$^2$ and $P_m$=4131.5 N/mm$^2$, F=3.75x10$^4$, L=0.274 in Eqn.7.20. Thus

$$1-\frac{4131.5}{4176}=\frac{T\times\left(3.75\times10^{4}\right)\times0.274\times2.3\times10^{-13}\times\Pi\times1401\times(2.328)^{2}}{2\times4176}$$

or T=1.57 million traversals.

## A.1.4 Design of welded components

The design process is started by assuming that the weld is 22 mm thick. The initial crack is found using Fig.6.6 as 0.28 mm. For a component of initial crack length 0.28 mm the magnification factor M is given by Fig.2.16a as 1.68.

The initial strength of this component is given by Eqn.2.24 as 890 N/mm$^2$. The corresponding values of $P_1$ and $P_2$ are found using Fig.9.5 as 434 N/mm$^2$ and 247 N/mm$^2$. The unstable crack propagation strength of the component after being subjected to 160 million cycles of the stress range of 20 N/mm$^2$ is found using the properties of the first idealised weld in Eqn.7.6. Thus

$$1-\frac{P_m}{777}=\frac{160\times10^{6}\times(20)^{3}\times2.3\times10^{-13}\times\Pi\times1401\times(0.986)^{2}}{2\times777}$$

or $P_m$=260 N/mm$^2$

Since the value of 260 N/mm$^2$ is less than 434 N/mm$^2$ the first idealised weld does not give the strength of the component and the properties of the second idealised weld are used to give the required residual strength. Thus

$$1-\frac{P_m}{777}=\frac{160\times10^{6}\times(20)^{3}\times2.3\times10^{-13}\times(0.856)^{2}\times\Pi\times1401}{2\times777}$$

or $P_m$=302 N/mm$^2$

The value of 302 N/mm$^2$ is greater than 247 N/mm$^2$ and gives the correct value of residual strength.

In order to calculate the crack length we use the stress intensity equation (Eqn.2.24) which is given as $K_{IC} = M\sigma\sqrt{\Pi a}$

Substituting $K_{IC}$=1401 N/mm$^{3/2}$ and $\sigma$=302 N/mm$^2$ we get

$$1401 = 302 \times \sqrt{\Pi} \times M\sqrt{a}$$

which gives $M\sqrt{\Pi a} = 2.6$

Since the thickness t is 22mm, $M\sqrt{\frac{a}{t}}$=0.56.

The corresponding value of a/t is determined from Fig.9.3 as 0.38. The crack length for a 22mm thick is 8.36 mm. The von-Mises plastic deformation strength for the component of tensile strength 412 N/mm$^2$ is calculated using Eqn.7.8. Thus

$$P_{mp} = \frac{2}{\sqrt{3}} \times f_u \times \left(1 - \frac{a}{t}\right)$$
$$= \frac{2}{\sqrt{3}} \times 412 \times \left(1 - \frac{8.36}{22}\right)$$
$$=295 \text{ N/mm}^2.$$

The Tresca plastic deformation strength is calculated using Eqn.7.9. Thus

$$P_{mp} = f_u \times \left(1 - \frac{a}{t}\right)$$
$$= 412 \times \left(1 - \frac{8.36}{22}\right)$$
$$=255 \text{ N/mm}^2.$$

Dividing the shear flow of 5000 N/mm by 255 N/mm$^2$ we get t=19.6 mm. The corresponding initial crack for a component of thickness 19.6 mm is found using Fig.6.6 as 0.31 mm.

Hence the value of $a_i/t$=0.016. The corresponding value of the magnification factor can be found from Fig.2.16a as 1.63.

The initial strength of the component is found from Eqn.2.24 as 871 N/mm$^2$. The corresponding values of $P_1$ and $P_2$ are found from Fig.9.5 as 424 and 241 N/mm$^2$. From Fig.9.5 if the initial strength of the first idealised weld is 871 N/mm$^2$, the initial strength of the second idealised weld is 760 N/mm$^2$. This strength reduces with the application of 160 million cycles of the stress range 20 N/mm$^2$. The reduction in strength is calculated using Eqn.7.20. Thus

$$1-\frac{P_m}{760}=\frac{160\times10^6\times(20)^3\times2..3\times10^{13}\times(0.856)^2\times\Pi\times1401}{2\times760}$$

or $P_m$=285 N/mm$^2$

The value of 285 N/mm$^2$ is greater than 241 N/mm$^2$ and gives the correct value of residual strength.

In order to calculate the crack length the stress intensity equation (Eqn.2.24) is used which is given as $K_{IC}=M\sigma\sqrt{\Pi a}$

Substituting $K_{IC}$=1401 N/mm$^{3/2}$ and $\sigma$=285 N/mm$^2$ we get

$$1401=285\times\sqrt{\Pi}\times M\sqrt{a}$$

which gives $M\sqrt{\Pi a}=2.77$

Since the thickness t is 22mm, $M\sqrt{\frac{a}{t}}$=0.622.

The corresponding value of a/t from Fig.9.3 is 0.39 and therefore for a thickness of 19.6 mm the crack length is 7.64 mm. The von-Mises plastic deformation strength for the component of tensile strength 412 N/mm$^2$ is calculated using Eqn.7.8 which is given as

$$P_{mp}=\frac{2}{\sqrt{3}}\times f_u\times\left(1-\frac{a}{t}\right)$$
$$=\frac{2}{\sqrt{3}}\times412\times\left(1-\frac{7.64}{19.6}\right)$$

=290 N/mm$^2$.

The Tresca plastic deformation strength is calculated using Eqn.7.9 which is given as

$$P_{mp}=f_u\times\left(1-\frac{a}{t}\right)$$
$$=412\times\left(1-\frac{7.64}{19.6}\right)$$
=252 N/mm$^2$.

Dividing the shear flow of 5000 N/mm by 252 N/mm$^2$ we get t=19.8 mm. The corresponding initial crack is found using Fig.6.6 as 0.30 mm.

Hence the value of $a_i/t$=0.015. The corresponding value of the magnification factor can be found from Fig.2.16a as 1.65.

The initial strength of the component is found from Eqn.2.24 as 875 N/mm$^2$. The corresponding values of $P_1$ and $P_2$ are found from Fig.9.5 as 426 and 243 N/mm$^2$. From Fig.9.5 if the initial strength of the first idealised weld is 875 N/mm$^2$, the initial strength of the second idealised weld is 764 N/mm$^2$. This strength reduces with the application of 160 million cycles of the stress range 20 N/mm$^2$. The reduction in strength is calculated using Eqn.7.20. Thus

$$1-\frac{P_m}{764}=\frac{160\times10^6\times(20)^3\times2.3\times10^{-13}\times(0.856)^2\times\Pi\times1401}{2\times764}$$

or $P_m$=289 N/mm$^2$

The value of 289 N/mm$^2$ is greater than 243 N/mm$^2$ and gives the correct value of residual strength.

In order to calculate the crack length the stress intensity equation (Eqn.2.24) is used which is given as $K_{IC} = M\sigma\sqrt{\Pi a}$

Substituting $K_{IC}$=1401 N/mm$^{3/2}$ and $\sigma$=289 N/mm$^2$ we get

$$1401 = 289\times\sqrt{\Pi}\times M\sqrt{a}$$

which gives $M\sqrt{\Pi a}$ = 2.74

Since the thickness t is 22mm, $M\sqrt{\frac{a}{t}}$ = 0.616.

The corresponding value of a/t from Fig.9.3 is 0.39 and therefore for a thickness of 19.8 mm the crack length is 7.72 mm. The von-Mises plastic deformation strength for the component of tensile strength 412 N/mm$^2$ is calculated using Eqn.7.8. Thus

$$P_{mp}=\frac{2}{\sqrt{3}}\times f_u\times\left(1-\frac{a}{t}\right)$$
$$=\frac{2}{\sqrt{3}}\times412\times\left(1-\frac{7.72}{19.8}\right)$$

=290 N/mm².

The Tresca plastic deformation strength is calculated using Eqn.7.9. Thus

$$P_{mp} = f_u \times \left(1 - \frac{a}{t}\right)$$
$$=412 \times \left(1 - \frac{7.72}{19.8}\right)$$
$$=252 \text{ N/mm}^2.$$

Dividing the shear flow of 5000 N/mm by 252 N/mm² we get t=19.8 mm which is the design thickness.

# Appendix A.2

This appendix gives the variation of crack length against number of cycles for examples in chapter 5. These include a curve of crack length against number of cycles for a cover plate 10 mm thick which is continuously being subjected to a stress of 0 to 20 N/mm$^2$ and a curve of crack length against number of cycles for a stiffener weld 10 mm thick which is being continuously being subjected to a stress amplitude of 0 to 20 N/mm$^2$.

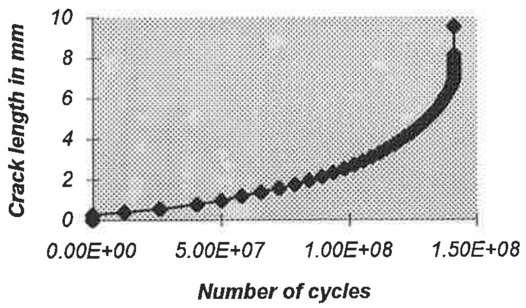



Fig.A.1 Figure showing variation of crack length with number of cycles of a cover plate of A-36 steel. The variation of residual strength with number of cycles of a cover plate of A-36 steel is given in Fig.5.4.

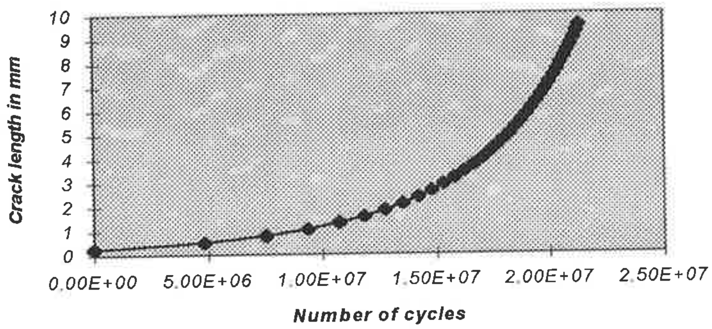



Fig.A.2 Figure showing variation of crack length against number of cycles of a stiffener weld of 4340 steel. The variation of residual strength against number of cycles of a stiffener weld of 4340 steel is shown in Fig.5.5.