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Towards an estimation framework for some problems in computer vision

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ABSTRACT

This thesis is concerned with fundamental algorithms for estimating parameters of geometric models that are particularly relevant to computer vision. A general framework is considered which accommodates several important problems involving estimation in a maximum likelihood setting. By considering a special form of a commonly used cost function, a new, iterative, estimation method is evolved. This method is subsequently expanded to enable incorporation of a so-called ancillary constraint. An important feature of these methods is that they can serve as a basis for conducting theoretical comparison of various estimation approaches.

Two specific applications are considered: conic fitting, and estimation of the fundamental matrix (a matrix arising in stereo vision). In the case of conic fitting, unconstrained methods are first treated. The problem of producing ellipse-specific estimates is subsequently tackled. For the problem of estimating the fundamental matrix, the new constrained method is applied to generate an estimate which satisfies the necessary rank-two constraint. Other constrained and unconstrained methods are compared within this context. For both of these example problems, the unconstrained and constrained methods are shown to perform with high accuracy and efficiency.

The value of incorporating covariance information characterising the uncertainty of measured image point locations within the estimation process is also explored. Covariance matrices associated with data points are modelled, then an empirical study is made of the conditions under which covariance information enables generation of improved parameter estimates. Under the assumption that covariance information is, in itself, subject to estimation error, tests are undertaken to determine the effect of imprecise information upon the quality of parameter estimates. Finally, these results are carried over to experiments to assess the value of covariance information in estimating the fundamental matrix from real images. The use of such information is shown to be of potential benefit when the measurement process of image features is considered.

DECLARATION

This thesis contains no material that has been accepted for the award of any other degree or diploma in any University or other tertiary institution. To the best of my knowledge and belief, it contains no material previously published or written by any other person, except where due reference is made in the text.

I give consent for this thesis, when deposited in the University library, to be available for loan and photocopying.

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PUBLICATIONS

In undertaking research which contributes to this thesis, a number of papers were published.

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[4] W. Chojnacki, M. J. Brooks, A. van den Hengel, and D. Gawley. A fast MLE-based method for estimating the fundamental matrix. In B. Mercer, editor, Proceedings of the *IEEE International Conference on Image Processing (ICIP-01)*, Thessaloniki, Greece, 7-10 October, 2001, vol 2, pp. 189-192. IEEE Computer Society, 2001.

[5] W. Chojnacki, M. J. Brooks, A. van den Hengel, and D. Gawley. From FNS to HEIV: A link between two vision parameter estimation methods. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 26(2):264-268, 2004.

[6] W. Chojnacki, M. J. Brooks, A. van den Hengel, and D. Gawley. A new constrained parameter estimator for computer vision applications. *Image and Vision Computing*, 22(2):85-91, 2004.

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NOTATION

$\boldsymbol{a} \in \mathbb{R}^p$	:	column vector of length p
$\boldsymbol{A} \in \mathbb{R}^{n \times m}$	:	$n \times m$ dimensional matrix
$\boldsymbol{I}_p, \boldsymbol{I}$	:	$p \times p$ identity matrix (generally p is inferred from context)
$\boldsymbol{A}^\top$	:	transpose of $\boldsymbol{A}$
$\boldsymbol{A}^{-1}$	:	inverse of $\boldsymbol{A}$
$\boldsymbol{A}^{-}$	:	Moore-Penrose inverse of $\boldsymbol{A}$
$\lvert\boldsymbol{A}\rvert$	:	determinant of $\boldsymbol{A}$
$\lVert\boldsymbol{A}\rVert_{\mathrm{F}}$	:	Frobenius norm of $\boldsymbol{A}$
$\mathrm{rank}(\boldsymbol{A})$	:	rank of $\boldsymbol{A}$
$\mathrm{diag}(\sigma_1, \ldots, \sigma_n)$	:	$n \times n$ matrix with $\sigma_1, \ldots, \sigma_n$ along the diagonal and zeros elsewhere
$\mathrm{tr}(\boldsymbol{A})$	:	trace of matrix $\boldsymbol{A}$
$\mathrm{vec}(\boldsymbol{A})$	:	vectorisation of $\boldsymbol{A}$
$\partial_{\boldsymbol{x}} J$	:	gradient **row** vector $[\frac{\partial J}{\partial x_1}, \ldots, \frac{\partial J}{\partial x_n}]$
$\partial^2_{\boldsymbol{x}\boldsymbol{x}} J$	:	Hessian matrix of J, $\left[\frac{\partial^2 J}{\partial x_i \partial x_j}\right]_{1 \leq i,j \leq n}$.
$\boldsymbol{H}_J$	:	alternative notation for the Hessian matrix of $J(\cdot)$
$p(X)$	:	probability of event X
$\mathsf{E}[\mathbf{x}]$	:	expected value of the random variable $\mathbf{x}$
$\boldsymbol{A} \otimes \boldsymbol{B}$	:	Kronecker product of $\boldsymbol{A}$ and $\boldsymbol{B}$