Perturbing Singular Systems and the Correlating of Uncorrelated Random Sequences

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Abstract. A stochastic process may be used to combine sequences with zero autocorrelation to give an autocorrelated sequence. We study this simple paradigm of irreversible mixing.

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We consider two independent binary random number generators with zero autocorrelation, with a logic "1" occurring with probability q_1 and q_2, respectively. These are mediated by a two-state discrete-time hidden Markov model. When this is in state i ($i = 1, 2$), source i is observed. When the chain is in state i it makes a change of state with probability α_i. The mediation results in an observed binary sequence $Z(t)$ which does not have zero autocorrelation.

An interesting feature of this system is that the autocorrelation of the observations need not be continuous in the parameters of the hidden Markov model in the limit as the system decouples. Indeed, the direction of approach to the uncoupled limit is also relevant. Further, the autocorrelation between consecutive observations can be maximized over the parameter space in the approach to the uncoupled limit.

The system is related to Parrondo games, where switching (either on a random or a deterministic basis) between losing games can give rise to a winning game. Parrondo games in turn are useful toy models for considering various phenomena in physics and nanotechnology.

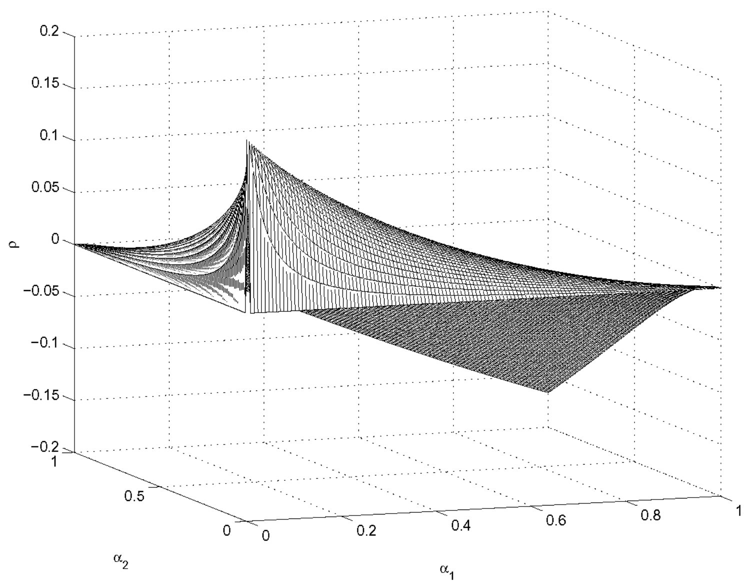

FIGURE 1. A mesh plot of $\rho\,[Z(t+1), Z(t)]$ as a function of α_1 and α_2 for $q_1 = 0.2$ and $q_2 = 0.6$. Note the steep gradients near $[\alpha_1, \alpha_2] = [0, 0]$. We call this point the "pinnacle." The value of ρ at $[0,0]'$ is zero. Nonzero limits are obtained with any linear approach to the pinnacle other than along an axis.

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