Copy of memo submitted to Directorate of Atomic Energy, DSIR, in Sept., 1943.

Should be 1944

THE ACCELERATION OF PARTICLES TO VERY HIGH ENERGIES.

Introduction.

The properties of elementary particles with energies up to 10 or 20 million electron-volts (10-20 MeV.) have been investigated extensively during recent years. In the war period this work has been intensified in some directions, although it has lapsed in others. A great deal of work remains to be done, but it is probable that the main outlines of nuclear physics in this region are now clear. In any event there is ample equipment in existence in USA and elsewhere to fill in most of the gaps in our knowledge, while much will be accomplished in the government laboratories which will be set up in various countries to exploit the possibilities of nuclear fission.

The greatest hope for an increase in fundamental understanding lies in experiments at energies above 1000 MeV. Cosmic radiation offers a source of particles with energies in this region, or higher, but, due to the low intensity and the uncertainties about the nature of any individual particle, there are difficulties in the proper interpretation of experimental results. Investigators in this field of physics have shown remarkable ingenuity and patience and very striking results have been obtained. However, the rate of progress could be greatly accelerated and obscure points could be much more easily settled, if there were available a method for accelerating particles of known kind to known energies in this region. It is certain that new and important phenomena would be discovered because of the greater intensity and the freedom from obscurity as to the kind and energy of the bombarding particles, while knowledge of the fundamental properties of these primary particles would reflect on the whole of nuclear physics.

The induction accelerator, or betatron, undoubtedly affords the simplest system yet devised for the acceleration of electrons to energies as high as 100 MeV. However, both mass and cost of equipment to reach 1000 MeV. are prohibitive. An examination of the possibility of producing pulses of particles by 'coreless' betatrons, using very large currents from charged condensers or from special short-circuit machines of the type used by Kapitza, shows that this approach involves very large scale equipment and presents formidable problems of engineering. Similarly, the extension of the cyclotron principle to extreme energies appears to be prohibitive in cost and to be difficult of solution when the particles have acquired velocities close to that of light.

In what follows we describe a method employing some of the principles of each of these systems, which is reasonable in cost and which can be applied in a modern laboratory.

The new method.

The essential feature of the proposals is that the particles should be constrained to move in a circle of constant radius, thus enabling the use of an annular ring of magnetic field of the correct form but over a total volume which is small enough to require only moderate power for its excitation. The magnetic field would be varied in such a way that the radius of curvature remains constant as the particles gain energy through successive accelerations by an alternating electric field applied between coaxial hollow electrodes, as in the cyclotron. The varying magnetic field performs the function of the guiding field in the betatron, but the acceleration is provided by an applied potential rather than by a changing flux. In this way it is possible to apply much higher accelerations per revolution. The changing magnetic field can be produced by an application of modern pulse technique as developed for radar purposes, while the accelerating potential can be provided by the same general method. Essentially, very large powers are available during the acceleration of a single burst of particles, a relatively long quiescent period between pulses reducing the average power consumed to a reasonable value.

At energies of 1000 MeV., or more, electrons and protons do not differ markedly in effective mass or velocity. Hence the greatest differences in technique required for the two particles will exist at the lower range of energies through which the particles are accelerated. These differences render it necessary to proceed in somewhat different ways in the case of electrons and protons.

Electrons.

The velocity of electrons approaches that of light for quite moderate energies. Further acceleration causes only a second order change in velocity and the increased momentum is present almost entirely as increased mass of the particles. Hence the frequency of revolution in an orbit of constant radius is very nearly constant and it becomes possible to employ an alternating potential of constant frequency on the accelerating electrodes. The small change in velocity can be compensated by a corresponding adjustment of the magnetic field and a consequent small change in radius of path. Thus, if we inject at an energy of 1.25 MV, where the velocity is about 0.95c, we can preserve constant frequency thereafter if we are prepared to allow the radius of the path to increase by 5 percent, i.e. by 20 cms. in the 400 cms we have specified. If we can inject at still higher energies the radius adjustment becomes much smaller. It is a comparatively simple matter to inject by means of an impulse generator to give any voltage up to about 2 million volts. In what follows we assume that the initial energy of the electrons is about 1.6 MeV., i.e. that the velocity is 0.97c.

The magnetic field.

The analysis of Kerst and Serber, Phys. Rev. 60, p.53,1941, shows that for stability of orbits against radial and axial oscillations the field must decrease radially at a rate $H \sim 1/r^n$, where n is less than 1 for radial stability, and $n > 0$ for axial stability. For the first betatron, on account of a restricted axial direction, n was chosen to be 2/3. This seems a good compromise for our case, as the large radius and relatively small gap, make it essential to avoid large axial oscillations.

Radius. We assume a maximum field of 10,000 gauss. This is reasonably removed from the saturation flux of iron and allows of later increase to deal with the acceleration of protons. We have the following expressions for the energy and momentum of a particle,

$$E = \frac{m_o C^2}{\ell} \left(\frac{1}{\sqrt{1-\beta 2}} - 1 \right),$$

$$Hr = \frac{mv}{\ell} = \frac{m_o C}{\ell} \left(\frac{\beta}{\sqrt{1-\beta^2}} \right).$$

Eliminating $\frac{1}{\sqrt{1-\beta^2}}$ we obtain

$$E = C\,\frac{Hr}{\beta} - \frac{m_o C^2}{\ell}\,.$$

Now for electrons $\frac{m_o C^2}{\ell} = 0.51$ MeV. and since at the energies we are considering $v \simeq C$, $\beta \simeq 1$,

$$E = (3 \times 10^{-4}\ Hr - 0.51)\ \text{MeV.}$$

For $E = 10^9$ eV the second term is negligible, so that

$$E = 3 \times 10^{-4}\ Hr.$$

Hence for $H = 10{,}000$ gauss

$$r = 330 \text{ cms.}$$

We choose $r = 400$ cms. to ensure that the magnet is suitable also for protons.

Larmor Frequency. At electron velocities approaching C the frequency ν_o of rotation in an orbit of radius r is $C/2\pi r$

$$\nu_o = 1.2 \times 10^7 \text{ when } r = 400 \text{ cms.}$$

$$= \frac{H}{2\pi} \cdot \frac{\ell}{m_o} \sqrt{1-\beta^2} \quad \text{for lower velocities.}$$

Variation of field with time. If the accelerating electrode system is supplied with high frequency a.c. of constant frequency and amplitude, and if the particles maintain a constant phase relationship with respect to the electric field, the same energy will be added each revolution and hence the energy will increase linearly with time, as in the cyclotron. Since at high energies Hr is proportional to the energy, the magnetic field also will increase linearly with time. More generally we have:

$$\frac{\delta E}{\delta H} = \frac{E_2 - E_1}{H_2 - H_1} = \underline{\frac{m_o C^2}{\ell} \left(\frac{1}{\sqrt{1-\beta_2^2}} - \frac{1}{\sqrt{1-\beta_2^2}} \right)}$$

$$= \frac{2\pi \nu_o \frac{m_o}{\ell} \left(\frac{1}{\sqrt{1-\beta_2^2}} - \frac{1}{\sqrt{1-\beta_2^2}} \right)}{\frac{C^2}{2\pi\nu_o}}$$

$$= 0.12 \text{ MeV/gauss if } \nu_o = 1.2 \times 10^7.$$

Over the whole range of energy the field will increase at a constant rate given by this expression, i.e. if the energy added per revolution is E' MeV., the increase of field per revolution is given by

$$H' = 8.3E'.$$

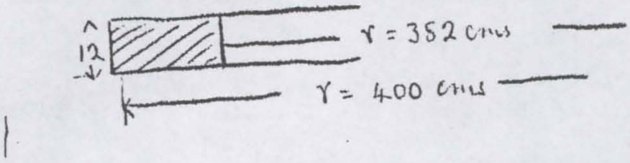
Hence the rate of increase of magnetic field is

$$\begin{aligned} \frac{\delta H}{\delta t} = \nu_o H' &= 8.3 \times 1.2 \times 10^7 E', \\ &= 10^8 E' \text{ gauss per second,} \\ &= 100E' \text{ gauss per microsecond.} \end{aligned}$$

We shall see later that it should be a simple matter to add an energy of 0.25MeV. per revolution. Then, the magnetic field must increase at a rate of 25 gauss per microsecond. The time for acceleration to 1000 MeV. will then be 400 microseconds. For various reasons, which we shall discuss, it may not be possible to utilize as high a rate of rise of field, in which case the energy added per revolution becomes still more conservative.

It would be difficult to produce a strictly linear rise of field with time, but E' can be modulated by the magnet current itself in such a way as to keep the particles in step with the a.c. potential.

Methods of producing the changing magnetic field. If the initial electrons are injected at an energy of 1.6 MeV., ($v/c = 0.97$), the radius of cruvature will be 12 cms. less than the final radius of 400 cms. The magnetic field must have the correct form over about 18 cms. in order to cater for eccentric orbits arising from divergence of the initial beam. The axial dimension of the beam must be about 12 cms. for the same reason. Hence the problem is that of producing a field of the correct form over an area



The decrease of field between the inner and outer radii will be about 3 percent.

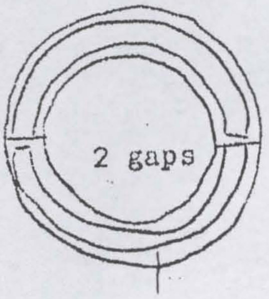
Iron cored magnet. It is obvious that the design of an iron-cored magnet to produce a field of the correct form is a simple matter. However, the rate of change of field is such that the magnet must be built of laminated iron. Rough design shows that the mass of laminated steel required would be about 360 tons, a formidable amount of high grade material. At the same time the leakage flux cannot be so much reduced over that of an air-cored coil, that the power required for excitation is significantly smaller. For these reasons we have not proceeded further with consideration of magnets employing iron, though we may wish to return to these later.

Air-cored magnet. The precise design of a system of current carrying conductors which will produce a field of the desired magnitude and shape has not yet been worked out. However, sufficient has been done to determine the order of magnitude of the power required. This is large, but not by any means impossible.

The electrical properties of the coils required will not differ much from those of a pair of concentric coils of outer diameter 405 cm and inner diameter 375 cm., consisting each of 11 turns over a length of 45 cms., the coils being connected in series and in opposition. The inductance of such a system is about 2000 microhenries, the current required to produce 10,000 gauss at the centre is 45,000 amperes. The stored energy is thus about 2×10^6 watt-seconds, the voltage at 500 cycles to pass 45,000 amperes is about 250,000 volts. In practice we would use a higher current, by paralleling turns, and a lower voltage. It is apparent, however, that the momentary power needed is of the order of 10^6 kilowatts, which we must obtain from charged condensers or from a short circuited alternator. The condensers or machine required to produce the magnetic field are by far the biggest items of equipment with which we are faced.

The voltage required to force the current to rise through the coil at the required rate can be reduced if we are prepared to allow a longer time for increase of the magnetic field to its maximum value. Doubling the time decreases the voltage to half and halves the power, though the total energy supplied remains the same, if we neglect ohmic losses. Thus, if the energy is stored on condensers consisting of units in series-parallel, it is a simple matter to adjust the time of rise by a rearrangement of the condensers alone. For experimental work, therefore, we would prefer to have the flexibility of a condenser bank rather than the less elastic machine.

Accelerating Electrodes. The particles may be accelerated by passage across one or more gaps between electrodes, across which an a.c. voltage is applied at the proper frequency.



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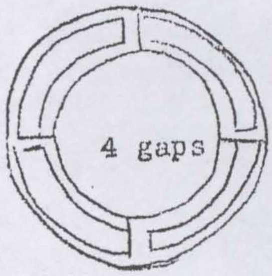

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The A.C. frequency is, for the Larmor frequency, $\nu_0 = 1.2 \times 10^7$.

1) $\nu = \nu_0 = 1.2 \times 10^7$ cycles/second, $\lambda = 25$ metres.
2) $\nu = 2\nu_0 = 2.4 \times 10^7$ ' ' $\lambda = 12.5$ metres.
3) $\nu = 4\nu_0 = 4.8 \times 10^7$ ' ' $\lambda = 6.25$ metres.
etc.

The gap voltages required to add an energy of 0.5 MeV per revolution are

1) 250,000 volts,
2) 125,000 '
3) 62,500 '

These frequencies and voltages can be obtained by coupling to an oscillator operating continuously, and their production is especially easy with pulsed operation.

It will be necessary to take precautions to avoid eddy currents generated in the mass of the electrodes by the varying magnetic field. This can be prevented by laminated construction and by use of materials with sufficiently high bulk resistance - e.g. graphite.

Method of injection and of removal of particles. Electrons may be injected into the system from outside by a process the reverse of that used in the cyclotron for the extraction of particles, i.e. by deflecting them electrically into a path of correct curvature and position. However, the potential required to extract them in this way is so great as to be quite impracticable. It is proposed that the axis of revolution of the electrons be shifted suddenly by passing a pulse of current through conductors placed eccentrically to the normal orbits. The particles can, in this way, be made to issue at a selected spot. The pulse required is very short - it need last only for a single revolution, i.e. for $\sim 2 \times 10^{-7}$ secs, and hence the energy required is not large.

Protons. The acceleration of protons to extremely high energies presents the additional problem over acceleration of electrons of having to deal with a very wide range of velocities.

Protons can be accelerated to an energy of about 45 M.e.V. in the 60 inch cyclotron. They could be injected into the new accelerating equipment with this energy, i.e. with a velocity of 0.3C. At an energy of 1000 M.e.V. the velocity has risen to 0.88c, or at 1200 M.e.V., which we hope to reach, $v = 0.9C$.

It is seen that we must cater for a velocity change by a factor 3: i.e. the period of revolution in an orbit of constant radius will decrease by a factor 3. This means that during the acceleration the frequency of the A.C. applied to the accelerating electrodes must increase by a factor 3.

For an orbit of radius 400 cms. the frequency of revolution will change from $\nu_0 = 3.6 \times 10^6$ to $\nu_0 = 1.1 \times 10^7$ revolutions per second. Accordingly, we must apply to the simplest electrode system high frequency A.C. which varies in wavelength in the right way during the acceleration between $\lambda = 84$ metres and $\lambda = 27$ metres.

It is unlikely that the range of variation in frequency can be obtained by purely electronic methods. It can be achieved by mechanical tuning by a combination of rotating condenser and inductance, but the time required for the sweep cannot be much less than $\frac{1}{100}$ second.

Accordingly we visualize for the present a system in which the oscillating electrode circuit is itself the tank-circuit of a grounded grid oscillator, the frequency being swept over the range required by mechanical variation of loading capacity and inductance.

In all other respects the acceleration of protons is very similar to the acceleration of electrons. The magnetic field must now rise in about $\frac{1}{100}$ second to approximately 15,000 gauss.