The Stability of Multiple Wing-tip Vortices

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Contents

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Abstract

Over the last forty or so years interest in the study of wing-tip vortices has increased, primarily due to the introduction of larger passenger aircraft and their subsequent interaction with smaller aircraft. The vortices generated by these larger aircraft present a problem in two main areas; the wake hazard problem, where other aircraft can be subjected to the large tangential velocities of the vortex, and the interaction with ground based features of vortices created during landing and take-off. The first of these is particularly dangerous close to the ground when aircraft are in a high lift configuration at take-off and landing. As the vortices effectively scale with aircraft wing span, significant encounters between large vortices and smaller aircraft have been documented over the years. An example of one such documented wake vortex interaction incident can be found in Ogawa [45].

In this study, the system of vortices are described as classical Batchelor vortices (or linear superpositions thereof) which are then subjected to small perturbations. By discretising the domain and solving for the eigenvalues of the system it is possible to ascertain the stability characteristics of the flow as a function of the system parameters which include the axial wave-number, the spacing of the vortices, their cross-flow decay rate and their axial strength.

We first consider the inviscid instability of multiple tip vortices, an approximation which is valid in the limit of large Reynolds numbers. In this limit the stability of the flow is dominated by the axial component of the basic vortex flow. The governing equations of continuity and momentum are reduced to a second order partial differential equation (PDE). This equation is solved numerically to determine which vortex configurations produce the greatest instability growth rate. These results are extended to consider the effect of compressibility on the inviscid instability. Finally

we consider the effects of viscosity on the stability of the full Batchelor similarity solution which results in a second order PDE in four dependent variables.

The stability equations are solved both globally (for the entire eigenspectra) and locally (for a single eigenvalue in a pre-determined region) using codes that run in both serial and parallel form. The numerical methods are based on pseudo-spectral discretisation (Chebyshev polynomials for Cartesian and radial directions and Fourier for azimuthal) in the global scheme, the eigenvalues being recovered either with a QZ algorithm or a shift-and-invert Arnoldi algorithm. For the local scheme, fourth order centred finite-diffences are used in conjunction with an iterative eigenvalue recovery method.