[Attachment added to the letter [illegible] covering letter [illegible]]

7th February, 1958.

My dear Frank,

Miss Hunt has sent me a copy of your remarks on Lindley's review. I am glad you were moved to write them, for it was rather an offensive publication.

I wonder if any way can be found to induce him to try to substantiate his claim that apart from misprints anything in the book is mathematically erroneous, for he seems to argue that he has found an error, and, by a rather bold _non sequitur_, that this shows that all is erroneous.

I have looked at the example in Chapter V, Section 6, in which I illustrate the use of fiducially inferred probabilities as supplying Bayesian probabilities _a priori_, and it appears that the example is entirely correct, though I should probably have stressed, as in so many other places, that for fiducial inference exhaustive estimation is required if the argument is to be mathematically exact. Anyway, on reflection I do not propose to alter a word of the existing paragraph for the next edition, but to add what I have drafted on the enclosed sheet. I hope this may induce you to look at it again and see what you think.

I have been giving permission for reproduction of tables quite freely, and I suppose Oliver and Boyd must decide whether they should follow you to India.

Sincerely yours,

Enc.

p.128. In F. Yates file, April 1958 Re SMSI (located [illegible] but possibly out of order [illegible])

6.1

A second example, extending Bayes' method a little more widely is as follows.

~~6. Observations of two kinds~~

~~It has been shown that observations of different kinds may justify conclusions involving uncertainty at different levels. It will be of some interest to consider the logical situation when observations of two such kinds are both available.~~

~~For example~~, Let us suppose it to be possible from one source of data to determine how many of a random sample exceed a fixed standard value ($\xi$), the proportion in the population sampled ~~which exceed~~ less than this value being unknown, as in Bayes' problem without knowledge *a priori*. If *a* values out of (a+b) fall short of the standard, we then have the likelihood statement for this unknown (p), namely

$$e^L \propto p^a q^b, \tag{93}$$

or

$$L = a \log p + b \log q + \text{a constant}, \tag{94}$$

in which expressions in terms of any parameter $\alpha$, in terms of which *p* is known, can be substituted for *p* and *q*, so as to give the likelihood of all values of $\alpha$; but there is in the data no basis for making probability statements determining the probability that $\alpha$, or *p*, should lie between assigned limits.

It might be, for example, that

$$p = \tfrac{1}{2}\{1 + \tanh(\xi - \alpha)\} \tag{95}$$

or

$$\xi - \alpha = \tfrac{1}{2}(\log p - \log q), \tag{96}$$

so that $\alpha$ is the unknown mean of a distribution of <u>x</u> with frequency element

$$\tfrac{1}{2}\,\mathrm{sech}^2(x-\alpha)\,dx, \qquad (97)$$

and our first means of observation merely sifted out and counted the members of a sample below a certain fixed size.

Let us suppose now that the size to which the seive is set had been determined by a sample of a single unit accurately measured and found to be $x_1$. Such a measurement would supply fiducial probability statements about $\alpha$, of no high precision, indeed, for only one observation has been made, but definitely of the form

$$\Pr(\alpha < \alpha_1) = \tfrac{1}{2}\{1-\tanh(x_1-\alpha_1)\} \qquad (98)$$

being valid for any value $\alpha_1$ whatever, and therefore constituting a complete frequency distribution for $\alpha$ in the light of the single observation.

The precision of such a determination may however be greatly increased by combination with data obtained by the first method. The method of combination differs from that of Bayes, for a statement of fiducial probability <u>a posteriori</u> requires that Exhaustive ~~Sufficient~~ estimation should be possible, which is not always the case. ~~Moreover, the data postulated by Bayes comprise not merely a probability distribution a priori, but a prior act of random sampling independent of the sampling which provides the data.~~

In this case the combination offers no difficulty, for the probability of observing the whole sequence of observations is

$$\frac{n!}{a!\ b!} p^a q^b \cdot \frac{du}{2 \cosh^2 u}, \tag{99}$$

where

$$u = x_1 - \alpha \tag{100}$$

or

$$\frac{n!}{a!b!} \cdot \frac{e^{(a-b)u} du}{2^{n+1} \cosh^{n+2} u} . \tag{101}$$

So that

$$\Pr \{\alpha > (x_1 - v)\} \tag{102}$$

may be equated to

$$\int_{-\infty}^{v} \frac{e^{(a-b)u} du}{2^{n+1} \cosh^{n+2} u} \cdot \frac{n!}{a!b!} \tag{103}$$

or to

$$P = \frac{2 \cdot n!}{a!b!} \int_0^T \frac{t^{2a+1} dt}{(1+t^2)^{n+2}} \tag{104}$$

where

$$T = e^v, \tag{105}$$

and the probability statement is valid for all values of $v$.