November 4, 1938

Dear Bliss,

Your letter of October 12th gives me a good deal to think about, and there are still some points that I should like to feel clearer about before writing about them. However, there are a few that I can mention at once: in your first example I encounter the difficulty that, for the one degree of freedom for regression on specific gravity, the value given under XY - 1.60675 is not the geometric mean of the value under X^2 .61164 and under Y^2 5.54080 (04580), so one of these three figures must be wrong. Actually I have replaced the value under X^2 by ~~x~~.51164, and conpensated this by increasing the contribution of departures from regression to .58637. On these figures I still find that I nearly check your variance ratios. If I choose a compound X+LY to maximise the ratios of the sum of squares for regression on that for error, I get a variance ratio of about 100, which, of course, is now for 2 degrees of freedom. We compare this with what you get by maximising the departure from regression relative to error and here get a ratio ~~of~~ nearly 180 for the one degree of freedom for regression and nearly 12 for the three degrees of freedom which departure from regression now contains. Although my ratios thus agree quite well with yours, I am curious to know how you maximised the ratio of departures from regression to error, for this leads to a quadratic equation involving respectively maximum and minumum ratios.

The fact, however, that maximising the ratios does give a significant variance ratio when allowance is made for the three degrees of freedom, proves that the differences between treatments, though largely, are not wholly accounted for by differences in specific gravity. There is, I think, no paradox in the fact that, if L is computed so as to maximise the regression, the variance ratio in respect of regression is smaller than if L is computed to maximise departures from regression, for there is nothing to prevent the value of L found by maximising the regression from being the same as either that which maximises or that which minimises departures from regression, and in such a case we should have the same sums of squares ascribable to different numbers of degrees of freedom. I am puzzled that, in maximising either the three degrees of freedom for treatments or the two for departures, you seem to use only X^2 and Y^2 terms. Let me know what you have been doing, i.e., whether you have been following some method given by me or breaking out into something which is unfamiliar to me, I believe the same point affects the second example.

In this second example also I am puzzled to guess your reason for forming a discriminant, since the relative value of the preparation would seem to be assignable only when the commercial value of "perfect", "stung" and "wormy" apples respectively are known. These would, I suppose, give the correct compound to apply to X and Y. I quite understand your procedure in eliminating W. In view of the fact that, owing to regression, the different comparisons between treatments which one might choose to make in reviewing the results have different precisions, I do not think that an exact treatment of significance, applicable in general, could be carried out except by choosing these particular components and finding whether they were significantly distinguishable by using the discriminant function appropriate for them, e.g., ~~choosing~~ comparing M and SA with a view to seeing whether any feature expressible in X and Y will show a significant difference between them, but such specific tests seem to be pointless in this particular experiment.

Yours sincerely,