May 4, 1937

Dear Immer,

I am sorry to have left your letter so long unanswered. In your analysis the entry for varieties within sets, 72 degrees of freedom, seems to contain some extraneous material:

(a) the difference between checks (c - c') appears four times. Of these four degrees of freedom one is, therefore, allocated to the difference~~s~~ between these checks, and three to error. V(b)

(b) There are also four other contrasts, such as:

17(c + c') - 2S(17 varieties).

The sum of these gives the one degree of freedom ~~for~~ the contrast between the pair of checks and the whole group of 68 test varieties, while the three discrepancies among them belong to the contrast between different sets of 17. These must be combined with the three degrees of freedom already ascribed to sets to give three for sets, after the new information is included, and three for error (V_a)

In combining the yields of the varieties in different sets I think the following are independent parts of $x_a - x_b$

$$(x_a - \bar{s}_1) + (\bar{s}_1 - \bar{s}_2) - (x_b - \bar{s}_2)$$

Of these the first and last have variances

$$\frac{16}{17} V_b$$

while the second has variance

$$\frac{2}{17} V_a ,$$

When the two variances have been correctly estimated; in all

$$2 \left[16V_b + \frac{V_a}{17} \right] \quad 2\,\frac{16 V_b + V_a}{17}$$

I hope this will make sense of your discussion with Goulden.

What Yates has called the Pseudo-Latin Square, or a quasi-factorial Latin square lattice, published in the Annals of Eugenics, seems to me quite the best ~~estimate~~ design yet put forward for variety trials. It would take 81* varieties in 5 replications without checks, which seems to be about what you want.

I am glad to hear about Anderson's lectures.

Yours sincerely,

* or 64 varieties in 9 replications. The arrangements are easily assigned using the completely orthogonalized 8 X 8 & 9 X 9 square given in my 2nd Edition

Varieties within sets (72) contains some pure errors

The difference between clusters $c - c'$ appears to this, giving 3 pure errors, this can be removed.

It also contains the contrasts

$$17(c+c') - 2S_1 \;(17 \text{ varieties})$$

$$17(c+c') - 2S_2 \;(\qquad)$$

$$17(c+c') - 2S_3 \;(\qquad)$$

$$17(c+c') - 2S_4 \;(\qquad)$$

The sum of these gives the 1 degree of freedom for the contrast between clusters and their varieties, but the 3 discrepancies among these belong to sets, that is within sets of clusters, varieties to sets. This gives 3 degrees of freedom available if sets when both sources of variation are used, and 3 for errors.

For comparing the yields of varieties in different sets $x_a - x_b$; of which the following are independent parts

$$(x_a - \bar{x}_1) + (\bar{x}_1 - \bar{x}_2) + -(x_b - \bar{x}_2)$$

of these the first and last have variances

$$\frac{16}{17} V_b$$

while the second has variance

$$\frac{2}{17} V_a$$

when the two varieties have been separately [illegible], in all

$$2\left\{\frac{16V_b + V_a}{17}\right\}$$