Xiaodan Zhu, Yuanqing Xia and Senchun Chai, Peng Shi

A finite-frequency domain approach to fault detection filter design for vehicle active suspension systems

Proceedings of the 11th World Congress on Intelligent Control and Automation, 2014 / pp.485-491

DOI: In Press

A Finite-Frequency Domain Approach to Fault Detection Filter Design for Vehicle Active Suspension Systems*

Xiaodan Zhu, Yuanqing Xia and Senchun Chai
School of Automation
Beijing Institute of Technology
Beijing 100081, China
zhuxiaodan1027@gmail.com
xia_yuanqing@bit.edu.cn
chaisc97@bit.edu.cn

Peng Shi
School of Electrical and Electronic Engineering
University of Adelaide
Adelaide SA 5005, Australia
College of Engineering and Science
Victoria University
Melbourne, Vic. 3000, Australia
peng.shi@adelaide.edu.au

***Abstract*— This paper is concerned with the problem of fault detection filter for vehicle active suspension systems. The aim of this paper is to design a fault detection filter in finite-frequency domain. A sufficient condition for residual system with the prescribed H_∞ performance index is derived based on the generalized Kalman-Yakubovich-Popov (KYP) lemma. In view of the obtained condition, the fault detection filter is designed in middle-frequency domain. Simulation results show the effectiveness and potential of the proposed results.**

***Index Terms*— Finite-frequency domain. Fault detection. Filter design. Vehicle active suspension systems. H_∞ performance index.**

I. INTRODUCTION

For many years, a hot research topic on vehicle suspensions has appeared because vehicle suspensions play an important role in ride comfort, vehicle safety, road damage minimization and the overall vehicle performance. Vehicle suspension system is mainly composed of wishbone, spring, shock absorber to transmit and filter all forces between body and road. To satisfy these requirements, many types of suspension systems have currently considered, such as passive [2], semi-active [3], [4], active suspensions [5], [6]. It has been well known that active suspensions have a great potential in meeting the tight performance requirements demanded by users.

Most of the approaches are considered in the full frequency domain. Active suspension systems, however, may belong to certain frequency band and ride comfort is known to be frequency sensitive. In the vertical direction, the human body is much sensitive to vibrations of $4-8$Hz from the ISO2631. Hence, it has more practical values to enhance the vehicle suspension performance in finite frequency. For vehicle active suspension systems, the problem of multi-objective control [7] has been proposed via designing load-dependent controllers. Finite frequency H_∞ control of vehicle active suspension systems [8] has been dealt with. In [9], active fault-tolerant control for vehicle active suspension systems has been solved. H_∞ control problem of time-delay active vehicle suspensions [10] is investigated.

The approach for finite frequency domain is to generalize the fundamental machinery, the KYP lemma. The KYP lemma shows the equivalence between a frequency domain inequality for a transfer function and a linear matrix inequality associated with its state-space realization. Recently, Iwasaki and Hara have made a very significant development about the generalized KYP lemma [11]. The equivalence is established between a frequency domain property and a LMI in a finite frequency range. The generalized KYP lemma is used to solve the analysis and synthesis problems in practical applications. In [12], fuzzy filter design for nonlinear systems in finite-frequency domain has been considered. The problem of feedback control synthesis for multiple frequency domains [13] has been solved. For continuous-time T-S fuzzy systems [14], H_∞ state feedback controller has been designed in finite frequency domain. H_∞ filtering for uncertain 2-D systems and LPV systems with sensor faults has been given in [15] and [16], respectively.

In a wide range of industrial processes, there have been intensive research results on fault detection and isolation algorithms and their applications. Many scholars keep a watchful eye on the fault detection. The source of false alarm is mainly unknown inputs, uncertainties, faults, disturbances and so on in any industrial systems. The most common method is that a residual signal is constructed and compared with a predefined threshold by designing a state observer or filter. The approach is named as the model-based approach. When the residual evaluation function is larger than the threshold, an alarm is produced. The filter has been designed to solve the fault detection problem of networked control systems modeled [17], Markovian jump singular systems with intermittent measurements [18], discrete-time switched singular time-

*This work of Yuanqing Xia was supported by the National Basic Research Program of China (973 Program) (2012CB720000), the National Natural Science Foundation of China (61225015, 60974011), the Ph. D. Programs Foundation of Ministry of Education of China (20091101110023, 20111101110012), and Beijing Municipal Natural Science Foundation (4102053, 4101001).

delay systems [19], and fuzzy jump systems [20]. However, by using the generalized KYP Lemma, many results of fault detection have been shown. The problem of fault detection for sampled-data systems [21] has been solved. For T-S fuzzy discrete systems [22], fault detection filter has been designed. In [23], the fault detection filter problem of networked control systems with missing measurements has been considered. The fault detection observer design problem for linear delta operator system, linear continuous-time systems have been proposed in [24], [25], respectively. However, to the best of the authors' knowledge, the problem of fault detection for vehicle active suspension systems in finite-frequency domain has not yet been fully investigated, which motivates the research in this paper.

In this paper, the problem of fault detection filter for vehicle active suspension systems in finite-frequency domain is studied. The objective of this paper is to design a fault detection filter. A sufficient condition of residual system with the prescribed H_∞ performance is derived in term of the generalized Kalman-Yakubovich-Popov (KYP) lemma. By means of the obtained condition, the form of the fault detection filter is given in middle-frequency domain. Simulation results illustrate the effectiveness and potential of the proposed results.

II. Problem Formulation and Preliminaries

This paper is concerned with the quarter car model [7], [8], [9] shown in Fig. 1.

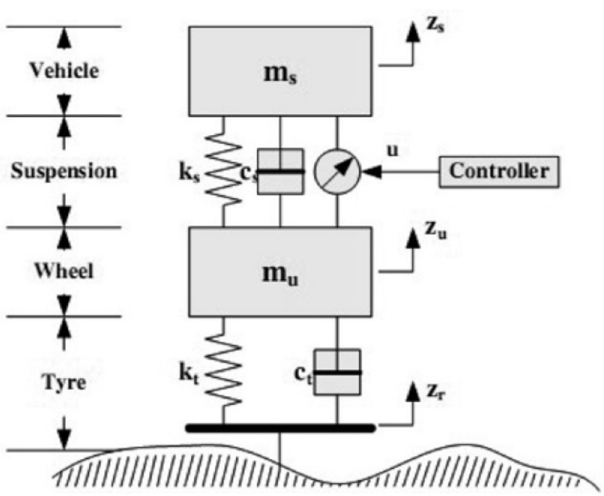

Fig. 1. The quarter car model

We can obtained the ideal dynamic equations of the sprung and unsprung masses as follows:

$$m_s\ddot{z}_s(t) + c_s[\dot{z}_s(t) - \dot{z}_\mu(t)] + k_s[z_s(t) - z_\mu(t)] = \mu(t)$$
$$m_\mu\ddot{z}_\mu(t) + c_s[\dot{z}_\mu(t) - \dot{z}_s(t)] + k_s[z_\mu(t) - z_s(t)]$$
$$+k_t[z_\mu(t) - z_r(t)] + c_t[\dot{z}_\mu(t) - \dot{z}_r(t)] = -\mu(t)$$

where m_s notates the sprung mass; $m\mu$ notates the unsprung mass; c_s and k_s are damping and stiffness, respectively. k_t and c_t mean compressibility and damping of the pneumatic tyre, respectively; z_s and z_u notate the displacements of the sprung and unsprung masses, respectively; z_r is the road displacement input; μ is the active input of the suspension system.

Set

$$x_1(t) = z_s(t) - z_\mu(t), x_3(t) = \dot{z}_s(t)$$
$$x_2(t) = z_\mu(t) - z_r(t), x_4(t) = \dot{z}_u(t)$$

where $x_1(t)$ is the suspension deflection, $x_2(t)$ denotes the tire deflection, $x_3(t)$ denotes the sprung mass speed and $x_4(t)$ is the unsprung mass speed.

Define the disturbance inputs is $d(t) = \dot{z}_r(t)$ and the control output

$$z(t) = \ddot{z}_s(t), y(t) = \begin{bmatrix} \frac{z_s(t)-z_\mu(t)}{z_{max}} & \frac{k_t(z_\mu(t)-z_r(t))}{(m_s+m_\mu)g} \end{bmatrix}^T$$

where $z_s(t) - z_\mu(t) \le z_{max}$ means the structural features of the vehicle also constrain the amount of suspension deflection, with z_{max} is the maximum suspension deflection.

Based on the dynamic characteristic of the active suspension system, the vehicle suspension control model is described as follow:

$$\begin{aligned} \dot{x}(t) &= A_1x(t) + B_1\mu(t) + D_1d(t) \\ y(t) &= A_2x(t) \\ z(t) &= A_3x(t) + B_3\mu(t) \end{aligned} \tag{1}$$

where

$$x(t) = \begin{bmatrix} x_1(t) & x_2(t) & x_3(t) & x_4(t) \end{bmatrix}^T$$
$$A_1 = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -\frac{k_s}{m_s} & 0 & -\frac{c_s}{m_s} & \frac{c_s}{m_s} \\ \frac{k_s}{m_u} & -\frac{k_\mu}{m_u} & \frac{c_s}{m_\mu} & -\frac{c_s+c_t}{m_\mu} \end{bmatrix}$$
$$B_1 = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_s} \\ -\frac{1}{m_\mu} \end{bmatrix}, D_1 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ \frac{c_t}{m_\mu} \end{bmatrix}$$
$$A_2 = \begin{bmatrix} -\frac{k_s}{m_s} & 0 & -\frac{c_s}{m_s} & \frac{c_s}{m_s} \end{bmatrix}$$
$$A_3 = \begin{bmatrix} \frac{1}{z_{max}} & 0 & 0 & 0 \\ 0 & \frac{k_t}{(m_s+m_\mu)g} & 0 & 0 \end{bmatrix}$$
$$B_3 = \begin{bmatrix} \frac{1}{m_s} \end{bmatrix}$$

If the fault occurs, (1) becomes the following form:

$$\begin{aligned} \dot{x}(t) &= A_1x(t) + B_1\mu(t) + D_1d(t) + F_1f(t) \\ y(t) &= A_2x(t) \\ z(t) &= A_3x(t) + B_3\mu(t) + F_3f(t) \end{aligned} \tag{2}$$

For system (2), construct the fault detection filter as follows:

$$\begin{array}{l}\dot{\hat{x}}(t) = A_f\hat{x}(t) + B_f y(t)\\ r(t) = C_f\hat{x}(t) + D_f y(t)\end{array} \tag{3}$$

For system (2), our aim is to construct a fault detection filter in order to get the residual generation. For a given stable weighting matrix $W_f(s)$, the weighted fault $\hat{f}(s) = W_f(s)f(s)$ is introduced to enhance the performance. Supposed that $W_f(s)$'s minimal realization is as follows

$$\begin{array}{l}\dot{x}_f(t) = A_w x_f(t) + B_w f(t)\\ r_f(t) = C_w x_f(t) + D_w f(t)\end{array} \tag{4}$$

Define $x_e(t) = \begin{bmatrix} x(t)^T & \hat{x}(t)^T & x_f(t)^T \end{bmatrix}^T$ and $e(t) = r(t) - r_f(t) - z(t)$, and then the residual system is obtained that

$$\begin{array}{l}\dot{x}_e(t) = A_e x_e(t) + B_e\nu(t)\\ e(t) = C_e x_f(t) + D_e\nu(t)\end{array} \tag{5}$$

where

$$\begin{array}{l}
A_e = \begin{bmatrix} A_1 & 0 & 0\\ B_f A_2 & A_f & 0\\ 0 & 0 & A_w \end{bmatrix}\\
B_e = \begin{bmatrix} B_1 & D_1 & F_1\\ 0 & 0 & 0\\ 0 & 0 & B_w \end{bmatrix}\\
C_e = \begin{bmatrix} D_f A_3 - A_3 & C_f & -C_w \end{bmatrix}\\
D_e = \begin{bmatrix} -B_3 & 0 & -D_w - F_3 \end{bmatrix}\\
\nu(t) = \begin{bmatrix} \mu(t)^T & d(t)^T & f(t)^T \end{bmatrix}
\end{array}$$

Lemma 2.1: (Projection Lemma) Given a symmetric matrix ψ and two matrix Γ and Λ, the problem

$$\psi + \Gamma\chi\Lambda^T + \Lambda\chi^T\Gamma < 0$$

is solvable with respect to the decision matrix χ if and only if

$$\Gamma^{\perp}\psi\Gamma^{\perp T} < 0 \quad and \quad \Lambda^{\perp}\psi\Lambda^{\perp T} < 0$$

Lemma 2.2: (Finsler's Lemma) Letting $\eta \in \mathbb{R}^n$, $P = P^T \in \mathbb{R}^{n\times n}$, $H \in \mathbb{R}^{m\times n}$ such that rank(H)=r¡n the following statements are equivalent:

1) $\eta^T P\eta < 0$ for all $\eta \neq 0$, $H\eta = 0$;

2) $\exists X \in \mathbb{R}^{n\times m}$ such that $P + XH + H^T X^T < 0$.

Lemma 2.3: (Generalized KYP Lemma) Consider the following system:

$$\begin{array}{l}\dot{x}(t) = Ax(t) + B\mu(t)\\ y(t) = Cx(t) + D\mu(t)\end{array} \tag{6}$$

with transfer function matrix $G(s) = C(sI - A)^{-1}B + D$. Let a symmetric matrix Π is given, the following statements are equivalent:

- The finite frequency inequality

$$\begin{bmatrix} G(j\omega)\\ I \end{bmatrix}^T \Pi \begin{bmatrix} G(j\omega)\\ I \end{bmatrix} < 0, \varpi_1 \le \omega \le \varpi_2 \tag{7}$$

- Hermitian matrix P and Q, $Q > 0$, then

$$\begin{bmatrix} A & B\\ I & 0 \end{bmatrix}^T \Theta \begin{bmatrix} A & B\\ I & 0 \end{bmatrix} + \begin{bmatrix} C & D\\ 0 & I \end{bmatrix}^T \Pi \begin{bmatrix} C & D\\ 0 & I \end{bmatrix} < 0 \tag{8}$$

where

$$\Theta = \begin{bmatrix} -Q & P + j\varpi_c Q\\ P - j\varpi_c Q & -\varpi_1\varpi_2 Q \end{bmatrix}$$

with $\varpi_c = (\varpi_1 + \varpi_2)/2$.

Definition 2.1: Consider system (2), given a scalar $\gamma > 0$, the fault detection filter (3) is designed such that the following conditions hold:

(i)Residual system (5) with $\nu(t) = 0$ is asymptotically stable.

(ii)Under zero initial condition, residual system (5) satisfies

$$H_\infty \triangleq \sup_{\omega} \sigma_{max}(G_{e\nu}) < \gamma$$

III. Main results

A. Fault Detection Filter Design

Theorem 3.1: Consider system (2). For the given scalar $\gamma > 0$ and the middle-frequency rang $\varpi_1 \le |\omega| \le \varpi_2$, system (5) is asymptotically stable and satisfies H_∞ performance index if there exist symmetric matrices P and $Q > 0$, matrices X, scalars q, p satisfying $qp + pq < 0$ such that the following inequalities holds:

$$\begin{bmatrix} -Xq - qX^T & P + Xp + qX^T A_e\\ * & -A_e^T Xp - pX^T A_e \end{bmatrix} < 0 \tag{9}$$

$$\begin{bmatrix} -Q & P + j\varpi_c Q + M & 0 & 0\\ * & -\varpi_1\varpi_2 Q - A_e^T M - M^T A_e & -M^T B_e & C_e^T\\ * & * & -\gamma^2 I & D_e^T\\ * & * & * & -I \end{bmatrix} < 0 \tag{10}$$

Proof: First of all, it is shown that system (5) is asymptotically stable.

Consider the following Lyapunov functional:

$$V(t) = X_e^T(t)PX_e(t) \tag{11}$$

Taking the derivatives of $V(t)$ along the trajectory system (11), we can have that

$$\dot{V}(t) = A_e^T P + PA_e \tag{12}$$

There exists

$$A_e^T P + PA_e < 0 \tag{13}$$

such that $\dot{V}(t) < 0$. Form (13), it is easy to obtain that

$$\begin{bmatrix} A_e^T & I \end{bmatrix} \begin{bmatrix} 0 & P \\ P & 0 \end{bmatrix} \begin{bmatrix} A_e \\ I \end{bmatrix} < 0 \tag{14}$$

Since that $\begin{bmatrix} qI \\ -pI \end{bmatrix}^{\perp} = \begin{bmatrix} pI & qI \end{bmatrix}$ and $qp + pq < 0$, we can have that

$$\begin{bmatrix} pI & qI \end{bmatrix} \begin{bmatrix} 0 & P \\ P & 0 \end{bmatrix} \begin{bmatrix} pI \\ qI \end{bmatrix} = (qp + pq)P < 0 \tag{15}$$

Notice that $\begin{bmatrix} A_e^T & I \end{bmatrix}^{\perp} = \begin{bmatrix} -I \\ A_e^T \end{bmatrix}$. According to Projection Lemma, we can have

$$\begin{bmatrix} 0 & P \\ P & 0 \end{bmatrix} + \begin{bmatrix} -I \\ A_e^T \end{bmatrix} X \begin{bmatrix} qI \\ -pI \end{bmatrix}^T + \begin{bmatrix} qI \\ -pI \end{bmatrix} X^T \begin{bmatrix} -I \\ A_e^T \end{bmatrix}^T < 0 \tag{16}$$

after some matrix manipulation, it is proven that (9) holds.

Next, we can show that system (5) satisfies the H_∞ index bound γ.

Let $\Pi = \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix}$, (7) of Lemma 2.3 is rewritten

$$G^T(j\omega)G(j\omega) < \gamma^2 I \tag{17}$$

that is, sup $\sigma_{max}(G(j\omega)) < \gamma$.

Hence, (17) holds if and only if there exist P, Q such that

$$\begin{bmatrix} A_e & B_e \\ I & 0 \end{bmatrix}^T \begin{bmatrix} -Q & P + j\varpi_c Q \\ P - j\varpi_c Q & -\varpi_1\varpi_2 Q \end{bmatrix} \begin{bmatrix} A_e & B_e \\ I & 0 \end{bmatrix} + \begin{bmatrix} C_e & D_e \\ 0 & I \end{bmatrix}^T \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \begin{bmatrix} C_e & D_e \\ 0 & I \end{bmatrix} < 0 \tag{18}$$

then (18) have the following form

$$\Upsilon_1^T(\Upsilon_2^T \Theta \Upsilon_2 + \Upsilon_4^T \Pi \Upsilon_4)\Upsilon_1 < 0 \tag{19}$$

where

$$\Upsilon_1 = \begin{bmatrix} A_e & B_e \\ I & 0 \\ 0 & I \end{bmatrix}, \Upsilon_2 = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \end{bmatrix}$$

$$\Upsilon_4 = \begin{bmatrix} 0 & C_e & D_e \\ 0 & 0 & I \end{bmatrix}$$

Let $\Gamma^{\perp} = \Upsilon_1^T = \begin{bmatrix} A_e^T & I & 0 \\ B_e^T & 0 & I \end{bmatrix}$, then

$$\Gamma = \begin{bmatrix} I & -A_e & -B_e \end{bmatrix}^T \tag{20}$$

By using Projection Lemma, (19) holds if the following inequality holds:

$$\Upsilon_2^T \Theta \Upsilon_2 + \Upsilon_4^T \Pi \Upsilon_4 < \Gamma M \varphi^T + \varphi M^T \Gamma^T \tag{21}$$

Define

$$\varphi^T = \begin{bmatrix} 0 & -I & 0 \end{bmatrix} \tag{22}$$

and substituting (20) and (22) into (21), then by some matrix manipulations and Schur complement, it is obtained that (10) holds. The proof is finished. ■

In the following, Theorem 3.2 is shown to obtain the parameters of filter (3).

Theorem 3.2: Consider system (2). For the given scalar $\gamma > 0$, system (5) is asymptotically stable and satisfies H_∞ performance index in the middle-frequency rang $\varpi_1 \leq |\omega| \leq \varpi_2$ if there exist scalars q, p satisfying $qp + pq < 0$, matrices X_1, X_2, X_3, M_1, M_2, M_3, M_4, A_{f0}, B_{f0}, C_{f0}, D_{f0}, symmetric matrices

$$P = \begin{bmatrix} P_1 & P_2 & P_3 \\ * & P_4 & P_5 \\ * & * & P_6 \end{bmatrix}, Q = \begin{bmatrix} Q_1 & Q_2 & Q_3 \\ * & Q_4 & Q_5 \\ * & * & Q_6 \end{bmatrix} > 0$$

such that (24) and (24) hold.

$$\begin{bmatrix} -q(X_1 + X_1^T) & -q(M_2^T + X_2) & 0 & \Psi_{14} & \Psi_{15} & P_3 \\ * & -q(M_2 + M_2^T) & 0 & \Psi_{24} & \Psi_{25} & P_5 \\ * & * & -q(X_3 + X_3^T) & P_3^T & P_5^T & P_6 + pX_3 + qX_3^T A_w \\ * & * & * & \Psi_{44} & \Psi_{45} & 0 \\ * & * & * & * & -p(A_{f0}^T + A_{f0}) & 0 \\ * & * & * & * & * & -p(X_3^T A_w + A_w^T X_3) \end{bmatrix} < 0 \tag{23}$$

$$\begin{bmatrix} -Q_1 & -Q_2 & -Q_3 & \Xi_{14} & \Xi_{15} & \Xi_{16} & 0 & 0 & 0 & 0 \\ * & -Q_4 & -Q_5 & \Xi_{24} & \Xi_{25} & \Xi_{26} & 0 & 0 & 0 & 0 \\ * & * & -Q_6 & \Xi_{34} & \Xi_{35} & \Xi_{36} & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44} & \Xi_{45} & -\varpi_1\varpi_2 Q_3 & -M_1^T B_1 & -M_1^T D_1 & -M_1^T F_1 & \Xi_{410} \\ * & * & * & * & \Xi_{55} & -\varpi_1\varpi_2 Q_5 & -M_3^T B_1 & -M_3^T D_1 & -M_3^T F_1 & C_{f0}^T \\ * & * & * & * & * & \Xi_{66} & 0 & 0 & -M_4^T B_w & -C_w^T \\ * & * & * & * & * & * & -\gamma^2 I & 0 & 0 & -B_3^T \\ * & * & * & * & * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & * & * & * & * & -\gamma^2 I & \Xi_{410} \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \tag{24}$$

where

$$\Psi_{14} = P_1 + pX_1 + qX_1^T A_1 + qB_{f0}A_2$$
$$\Psi_{15} = P_2 + pX_2 + qA_{f0}$$

$$\Psi_{24} = P_2^T + pM_2 + qX_2^T A_1 + qB_{f0}A_2$$
$$\Psi_{25} = P_4 + pM_2 + qA_{f0}$$
$$\Psi_{44} = -p(X_1^T A_1 + B_{f0}A_2) - p(X_1^T A_1 + B_{f0}A_2)^T$$
$$\Psi_{45} = -pA_{f0} - p(X_2^T A_1 + B_{f0}A_2)^T$$
$$\Xi_{14} = P_1 + M_1 + j\varpi_c Q_1, \Xi_{15} = P_2 + M_3 + j\varpi_c Q_2$$
$$\Xi_{24} = P_2^T + M_2 + j\varpi_c Q_2^T, \Xi_{25} = P_4 + M_2 + j\varpi_c Q_4$$
$$\Xi_{16} = P_3 + j\varpi_c Q_3, \Xi_{26} = P_5 + j\varpi_c Q_5$$
$$\Xi_{34} = P_3^T + j\varpi_c Q_3^T, \Xi_{35} = P_5^T + j\varpi_c Q_5^T$$
$$\Xi_{36} = P_6 + M_2 + j\varpi_c Q_6$$
$$\Xi_{44} = -\varpi_1\varpi_2 Q_1 - (A_1^T M_1 + A_2^T B_{f0}^T) - (A_1^T M_1 + A_2^T B_{f0}^T)^T$$
$$\Xi_{45} = -\varpi_1\varpi_2 Q_2 - (A_1^T M_3 + A_2^T B_{f0}^T + A_{f0})$$
$$\Xi_{55} = -\varpi_1\varpi_2 Q_4 - (A_{f0} + A_{f0}^T)$$
$$\Xi_{66} = -\varpi_1\varpi_2 Q_6 - (A_w^T M_4 + M_4^T A_w)$$
$$\Xi_{410} = (D_{f0}A_2 - A_3)^T, \Xi_{910} = -F_3^T - D_w^T$$

Moreover, if (24) and (24) are feasible, then a suitable filter satisfies

$$\begin{array}{l} A_f = M_2^{-T} A_{f0}, C_f = C_{f0} \\ B_f = M_2^{-T} B_{f0}, D_f = D_{f0} \end{array} \quad (25)$$

Proof: Define

$$X = \begin{bmatrix} X_1 & X_2 & 0 \\ M_2 & M_2 & 0 \\ 0 & 0 & X_3 \end{bmatrix}, M = \begin{bmatrix} M_1 & M_3 & 0 \\ M_2 & M_2 & 0 \\ 0 & 0 & M_4 \end{bmatrix}$$

substituting P, Q, X, A_e, B_e, C_e, D_e into (9), let

$$\begin{array}{l} A_f^T = A_{f0}^T M_2^{-1}, C_f = C_{f0} \\ B_f^T = B_{f0}^T M_2^{-1}, D_f = D_{f0} \end{array} \quad (26)$$

we can obtain (24) holds.

Similar, substituting P, Q, M, A_e, B_e, C_e, D_e into (10) and considering (26), we can obtain (24) holds. The proof is finished. ■

B. Threshold Design

In order to detect the faults, the widely adopted approach is to choose an appropriate threshold J_{th} and determine the evaluation function $J(r)$. That is, a threshold J_{th} and a residual evaluation function $J(t)$ can be chosen:

$$\begin{array}{c} J(r) = \|r(t)\|_{2,\tau} = \sqrt{\int_0^\tau r^T(t) r(t) \mathrm{dt}} \\ J_{th} = \sup_{\mu(t), d(t) \in L_2, f(t)=0} J(r) \end{array} \quad (27)$$

where τ is the evaluation time window.

Based on this, the occurrence of the sensor faults can be detected via the following logic rule:

$$J_r(t) < J_{th} \Rightarrow No\ Faults$$
$$J_r(t) \geq J_{th} \Rightarrow Faults \Rightarrow Alarm$$

which means that the faults are detected by comparing the residual evolution function and the predefined threshold.

Remark 3.1: In this paper, we show a bridge between fault detection and vehicle active suspension systems. By designing a filter , the problem of fault detection for vehicle active suspension systems can be solved. Some comparison results are given in TABLE I.

TABLE I
COMPARISONS OF THE EXISTING RESULTS

Reference	Contribution
[8]	Finite Frequency H_∞ control
[9]	Active fault-tolerant controller design
This paper	Fault detection filter design

IV. ILLUSTRATIVE EXAMPLES

In this section, we give an example to illustrate that the proposed method is effective. The quarter-car model parameters are considered as follows: $m_s = 320kg$, $m_\mu = 40kg$, $k_s = 18kN/m$, $k_\mu = 200kN/m$, $k_t = 200kN/m$, $c_s = 1kN/m$, $c_t = 10Ns/m$, $z_{max} = 100mm$. Based on the parameters, the system matrices is easily obtained. It is widely accepted that ride comfort is closely related to the body acceleration in frequency band $4-8$Hz. The weighted matrix of the fault is supposed to be $W_f(s) = \frac{5}{s+5}$ with the minimal realizations $A_w = -5$, $B_w = 5$, $C_w = 1$, $D_w = 0$. Consider $F_1 = \begin{bmatrix} 0 & 0 & -2 & -2 \end{bmatrix}^T$, $F_3 = -2$ when $\gamma = 0.5$, we can have

$$A_f = \begin{bmatrix} -2.5347 & -0.2571 & 0.9986 & -0.9994 \\ 0.8009 & -1.1232 & -0.5686 & 0.7492 \\ -0.6409 & 0.2030 & -1.2378 & -0.7255 \\ 0.6115 & 0.4901 & -1.0688 & -0.9370 \end{bmatrix}$$
$$B_f = \begin{bmatrix} -253.3453 & -4.4550 \\ 51.6753 & -63.2421 \\ -69.1644 & -1.2997 \\ 102.1435 & 102.4626 \end{bmatrix}$$
$$C_f = \begin{bmatrix} 0.8413 & 0.1199 & 0.1009 & -0.1344 \end{bmatrix}$$
$$D_f = \begin{bmatrix} 103.3864 & 14.2191 \end{bmatrix}$$

To show the effectiveness of the design, the control input is assumed to be $\mu(t) = 0.5sin(2\pi t)$, the fault signal is given as follows:

$$f(t) = \begin{cases} 1, 5s < t < 20s \\ 0, \quad else \end{cases}$$

and the disturbance $d(t)$ is shown in Fig. 2. Fig. 3 shows the residual evaluation function $J_r(t)$. The simulation results show that $J_r(9.7) = 9.4988 < 9.345$ when $t = 9.7$s, which means that the fault $f(t)$ can be detected $4.7s$ after its occurrence.

V. CONCLUSION

This paper dealt the problem of fault detection filter for vehicle active suspension systems in finite-frequency domain. Based on the generalized KYP lemma, this paper's aim is that the fault detection filter is designed in the middle-frequency domain such that the residual systems are asymptotically stable and meet the prescribed H_∞ performance index. The fault detection filter is shown in the form of linear matrix inequalities. Simulation results are show the effectiveness and potential of the proposed results.

REFERENCES

[1] M. M. M. Salem and A. A. Aly, "Fuzzy control of a quarter-car suspension system," *World Academy of Science, Engineering and Technology*, vol. 53, pp.258-263, 2009.

[2] A. F. Naude and J. A. Snyman, "Optimisation of road vehicle passive suspension systems-part 1: optimization algorithm and vehicle model," *Applied Mathematical Modelling*, vol. 27, no. 4, pp.249-261, 2003.

[3] S. B. Choi and W. K. Kim, "Vibration control of a semi-active suspension featuring electrorheological fluid dampers," *Journal of Sound and Vibration*, vol. 234, no. 3, pp.537-546, 2000.

[4] T. J. Gordon and R.S. Sharp, "On improving the performance of automotive semi-active suspension systems through road preview," *Journal of Sound and Vibration*, vol. 217, no. 1, pp.163-182, 1998.

[5] H. Chen and K. Guo, "Constrained H_∞ control of active suspensions: An LMI approach," *IEEE Transaction on Control Systems Technology*, vol. 13, no. 3, pp.412-421, 2005.

[6] H. Kim, H. Yang, and Y. Park, "Improving the vehicle performance with active suspension using road-sensing algorithm," *Computers & Structures*, vol. 80, pp.1569-1577, 2002.

[7] H. Gao, J. Lam, and C. Wang, "Multi-objective control of vehicle active suspension systems via load-dependent controllers," *Journal of Sound and Vibration*, vol. 290, iss. 3-5, pp.645-675, 2006.

[8] W. Sun, H. Gao, and O. Kaynak, "Finite Frequency H_∞ control for vehicle active suspension systems," *IEEE Transactions on Control Systems, Technology*, vol. 19, no. 2, pp.416-422, 2011.

[9] J. Qiu, M. Ren, Y. Zhao, and Y. Guo, "Active fault-tolerant control for vehicle active suspension systems in finite-frequency domain," *IET Control Theory and Applications*, vol. 5, iss. 13, pp.1544-1550, 2011.

[10] H. Du and N. Zhang, "H_∞ control of active vehicle suspensions with actuator time delay", *Journal of Sound and Vibration*, vol. 301, iss. 1-2, pp.236-252, 2007.

[11] T. Iwasaki and S. Hara, "Generalized KYP Lemma: Unified frequency domain inequalities with design applications," *IEEE Transactions on Automatic Control*, vol. 50, no. 1, pp.41-59, 2005.

[12] D. Ding and G. Yang, "Fuzzy filter design for nonlinear systems in finite-frequency domain," *IEEE Transactions on Fuzzy Systems*, vol. 18, no. 5, pp.935-945, 2010.

[13] T. Iwasaki and S. Hara, "Feedback control synthesis of multiple frequency domain," *International Journal of Robust and Nonlinear Control*, vol. 17, pp.415-434, 2007.

[14] H. Wang, L. Peng, H. Ju, and Y. Wang, "H_∞ state feedback controller design for continuous-time T-S fuzzy systems in finite frequency domain," *Information Sciences*, vol. 223, pp.221-235, 2013.

[15] X. Li and H. Gao, "Robust finite frequency H_∞ filtering for uncertain 2-D systems: The FM model case," *Automatica*, vol. 49, pp.2446-2452, 2013.

[16] H. Wang and H. Ju, "Reliable H_∞ filtering for LPV systems with sensor faults in finite frequency domain," *International Journal of Systems Science*, vol. 44, no. 12, pp.2310-2320, 2013.

[17] Z. Mao, B. Jiang, P. Shi, "H_∞ fault detection filter design for networked control systems modelled by discrete Markovian jump systems," *IET Control Theory and Applications*, vol. 1, no. 5, pp.1336-1343, 2007.

[18] X. Yao, L. Wu, W. Zheng, "Fault detection filter design for Markovian jump singular systems with intermittent measurements," *IEEE Transactions on Signal Processing*, vol. 59, no. 7, pp.3099-3109, 2011.

[19] J. Lin, S. Fei, Z. Gao, J. Ding, "Fault detection for discrete-time switched singular time-delay systems: an average dwell time approach," *International Journal of Adaptive Control and Signal Processing*, accepted, 2012.

[20] S. He and Fei Liu, "Filtering-based robust fault detection of fuzzy jump systems," *Fuzzy Sets and Systems*, vol. 185, pp.95-110, 2011.

[21] P. Zhang, S. X. Ding, G. Z. Wang, and D. H. Zhou, "A frequency domain approach to fault detection in sampled-data systems," *Automatica*, vol. 39, pp.1303-1307, 2003.

[22] H. Yang, Y. Xia, and B. Liu, "Fault detection for T-S fuzzy discrete systems in finite frequency domain," *IEEE Transactions on Systems, Man and Cybernetics-Part B: Cybernetics*, vol. 41, no. 4, pp.911-920, 2011.

[23] Y. Long and G. Yang, "Fault detection in finite frequency domain for networked control systems with missing measurements," *Journal of the Franklin Institute*, vol. 350, iss. 9, pp.2605-2626, 2013.

[24] H. Yang, Y. Xia, and J. Zhang, "Generalised finite-frequency KYP lemma in delta domain and applications to fault detection," *International Journal of Control*, vol. 84, no. 3, pp.511-525, 2011.

[25] H. Wang and G. Yang, "A finite frequency domain approach to fault detection observer design for linear continuous-time systems," *Asian Journal of Control*, vol. 10, no. 5, pp.1-10, 2008.

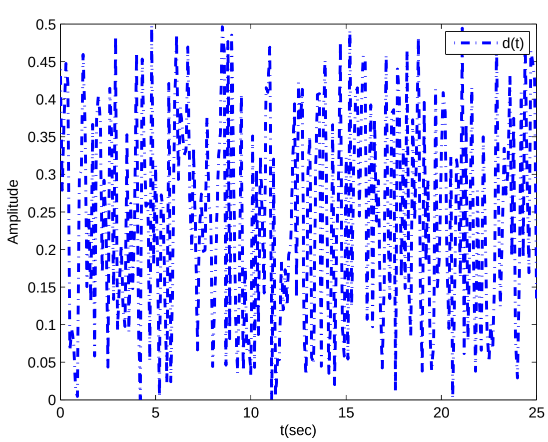

Fig. 2. The disturbance $d(t)$

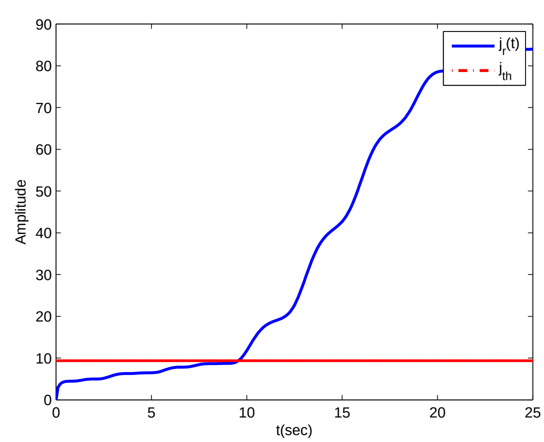

Fig. 3. The residual evaluation function $J_r(t)$