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A hybrid model predictive control scheme for energy and cost savings in commercial buildings: simulation and experiment

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A hybrid model predictive control scheme for energy and cost savings in commercial buildings: simulation and experiment

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***Abstract*— This paper presents a hybrid model predictive control (MPC) scheme for energy-saving control in commercial buildings. The proposed method combines a linear MPC with a neural network feedback linearisation (NNFL) method. The control model for the linear MPC is developed using a simplified physical model, while nonlinearities associated with the building system are handled by an affine recurrent neural network (ARNN) model through system feedback. The proposed MPC integrates several advanced air-conditioning control strategies, such as an economizer control, an optimal start-stop control, and a pre-cooling control. The developed MPC has been tested in the check-in hall of T-1 building, Adelaide Airport, through both simulation and field experiment. The result shows that the proposed control scheme can achieve a considerable amount of savings without violating occupants' thermal comfort.**

Fig. 1: Check-in hall at level-2 of Adelaide airport

I. INTRODUCTION

Buildings are responsible for 40% of the energy consumption and 33% of carbon dioxide emissions in the world [1]. Within the buildings, almost half of the energy use is related to heating, ventilation, and air conditioning (HVAC) systems. Reducing the building energy costs has become an urgent task due to the increasing environmental concerns and energy prices.

The traditional HVAC control strategies, such as proportional-integral-derivative (PID) control and on/off control, use the current indoor temperature as an input to control local actuators such as chilled water valves. Due to the thermal inertia of the buildings, HVAC may respond to indoor temperature change with a significant time delay. This causes over-heating (cooling), high on-peak electricity demand and poor thermal comfort in the buildings. In recent years, researchers have demonstrated that these issues can be solved by implementing model predictive control (MPC) [2], [3], [4], [5], [6]. MPC utilises information of weather forecast, occupancy prediction, and time-varying electricity price, to minimise the energy costs and improve the thermal comfort in buildings.

To build control-oriented models for MPC, resistance-capacitance (RC) networks, based on the first principle of thermal dynamics, are employed for the modelling of the thermal dynamics of the building [7], [8], [6], [9], [10]. RC networks use lumped capacitance and resistance in an analogy electric circuit to represent the thermal elements of a building. The resultant models can be transformed into state-space forms, so that a standard MPC can be utilised. However, when implemented for real-world buildings, such approaches face application problems, mainly because the thermal dynamics of a real building is nonlinear in characteristic and contains several uncertainties. An alternative modelling approach is artificial neural network (ANN) model. It has been shown that ANN models outperform both physical and statistic models in modelling building temperature [11], [12]. It can also be used to design nonlinear MPC for real buildings [13]. However, when ANNs are used in an MPC, two major drawbacks arise: 1. A non-convex optimisation problem must be solved to calculate the control sequence, which is computationally demanding. 2. Model performance cannot be guaranteed when the system is running outside the operational range.

This paper aims to propose an MPC scheme based on neural network feedback linearisation to achieve energy and cost savings in commercial buildings. The approach exploits the universal non-linear approximation ability of ANN and reliability of the classical MPC techniques. The control model is built using a simplified physical model, which allows a linear programming optimisation to be applied. Nonlinearity of the system is handled separately using an affine neural network model. Although this control scheme has been studied before [14], [15], it has not been used for the building energy control yet. We use such an MPC framework to evaluate the energy saving potential of two advanced air-conditioning control approaches: An optimal start-stop MPC and a pre-cooling MPC, by both simulation and experiment.

II. MODELLING

A. Thermal dynamics modelling

The test building is the T1 building of Adelaide Airport, South Australia. The check-in hall located at level 2 of the building is selected as the experimental area. Fig. 1 shows the external appearance of the investigated zones. The

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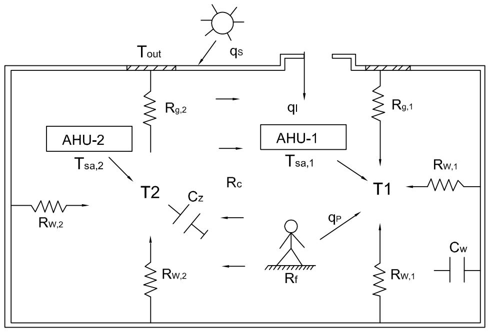


Fig. 2: Thermal network diagram for a double-zone case

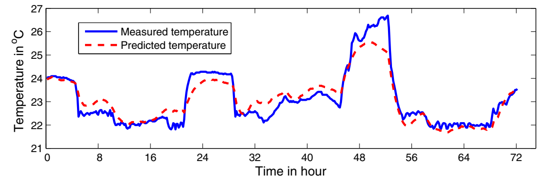


Fig. 3: Simulation results of using RC model

selected zones are isolated from the outdoor environment by a large glass facade to the north. The investigated zones are lightweight in structure, because it has a significant thermal coupling with the outdoor environment and the adjacent space. The uncertainties such as solar radiation, internal gain, leakage and thermal interaction make the modelling work very difficult. In this work, thermal building models are represented by a second order RC model. The thermal network used to represent the building system is depicted in Fig. 2. Before building the RC model, the following assumptions were made:

1) The air in the zone is fully mixed, so that the temperature distribution in each zone is uniform;
2) The density and flow rate of the air in the zones are constant and not influenced by the temperature change.
3) The walls, floor and ceiling have the same effect on the zone temperature. The windows have negligible thermal capacitance.

Based on the above assumptions, energy and mass balance governing equations for zone 1 can be written as:

$$C_z \frac{dT_1}{dt} = \dot{m} C_a (T_{sa} - T_1) + \frac{T_{out} - T_1}{R_g} + \frac{T_w - T_1}{R_w} + \frac{T_2 - T_1}{R_c} + Q_r + Q_p, \quad (1)$$

$$C_w \frac{dT_w}{dt} = \frac{T_1 - T_w}{R_w} + \frac{T_{out} - T_w}{R_w}, \quad (2)$$

where C_z is overall thermal capacitance of the air and other fast-response elements, C_w is thermal capacitance of the interior-walls and ceiling , C_a is the specific heat of the air, $\dot{m}$ is the mass flow rate of the supply air, T_1 is temperature of the investigated zone, T_2 is temperature of the neighbouring zone, T_{out} is the outdoor air temperature, T_{sa} is the supply air temperature, T_w is mean surface temperature of the interior walls, ceiling and floor, Q_p is internal heat gain from occupants, and Q_r is heat gain from solar radiation. Q_p and Q_r are modelled by an affine function of carbon dioxide concentration (ppm) and global horizontal irradiation (W/m^2), respectively [16]. The parameters associated with these two variables (α and β) are identified together with (1).

Equations. (1) and (2) are discretized by using the Euler method to obtain an innovation represenationt state-space form [17] as:

$$x_{k+1} = Ax_k + Bu_k + Ed_k + ke_k, \quad (3)$$

where $x = [T_z, T_w] \in \mathbb{R}^2$ is the states vector, $u = [T_{sa}, \dot{m}] \in \mathbb{R}^2$ is the input vector, $d = [T_{out}, Q_p, Q_r] \in \mathbb{R}^3$ denotes environmental variables, $e_k \in \mathbb{E}^2$ denotes the feedback vector which contains the estimated unmeasured disturbances (such as heat gain due to door opening), and k is the Kalman gain. During the identification process, the initial values of [R_w, R_g, R_c, C_z, C_w, α, β] are estimated based on the material properties and geometry of the surfaces surrounding of the selected zone first. A nonlinear least squares algorithm is used to identify unknown parameters. The data used for training and validation were collected from BMS between the 1st and 31st of January 2013. The meteorological data obtained from the Bureau of Meteorology of Australia is used as a prediction for outdoor temperature and global horizontal irradiation. Fig. 3 shows that the RC model achieves a fitness of 72% (normalised relative mean squared error) when an open-loop simulation is conducted.

B. AHU system

The building is controlled by a Johnson Controls Australia Pty Ltd building management system (BMS). At the zone level, air handling units (AHUs) transfer the cooling energy from the chilled water circuit into airflows, and then supply to the local thermal zones. The AHUs are with a constant air volume (CAV). The control system uses a proportional control rule to regulate chilled water valves, to maintain the zone temperature at the desired value. The AHUs are installed with an economizer: when the zone temperature is lower than the return air temperature but higher than 12^o C, the economizer will open the outdoor air damper more widely to employ cool ambient air for free cooling.

To compute the energy consumption of the individual AHUs, we use a series of simplified models to estimate it:

$$\Delta T_c = (1 - D_{out}) T_r + D_{out} T_{out} - T_{sa}, \quad (4)$$

$$P_c = \frac{\dot{m} C_a \Delta T_c}{\mathrm{COP}}, \quad (5)$$

$$P_f = C_o + C_1 \dot{m} + C_2 \dot{m}^2, \quad (6)$$

where P_c is the power consumption related to the cooling energy consumed by the cooling coils, $\dot{m}$ is the flow rate of the air passing through the cooling coil, C_a is the specific heat of the supply air, ΔT_c is the temperature change of the supply air after the heat exchange occurred at the cooling coil, COP is coefficient of performance of the chiller plant, D_{out} is the opening level of the outdoor air damper, T_r is the return air temperature, P_f is the energy consumed by supply fan, and C_o to C_2 are parameters related to the fan energy.

C. Neural network feedback linearisation

The building model developed previously is nonlinear, because the supply air flow rate $\dot{m}$ is multiplied by the supply air temperature T_{sa}, making the energy input a bilinear term. Also, the supply air temperature is affected by the change of chilled water temperature and flow rate within the main water loop. This results in a non-convex optimisation problem which is hard to solve. To solve the issue, we employ a neural network feedback-linearization (NNFL) based MPC scheme [14]. The idea of this approach is to cancel the system nonlinearity using neural network through feedback, so that the problem can be solved using a reliable and fast linear MPC. The design of MPC consists of two steps: nonlinear functions approximation and controller design. The control is illustrated in Fig. 4. We firstly linearise the system input as:

$$Q_u = \dot{m}C_a(T_{sa} - T_1), \tag{7}$$

where Q_u denotes the thermal energy supplied to the room. However, Q_u is not the actual cooling energy consumed by the AHU, because when economizer is activated, cool ambient air will contribute fully or partially to the cooling load. The actual energy consumed by cooling coil is:

$$Q_r = Q_u - D_{out}\dot{m}C_a(T_r - T_{out}), \tag{8}$$

where Q_r denotes the actual energy consumed by the AHU system. From Eq. 8, it can be seen that when the ambient temperature is lower than the return air temperature, it is desired to open outdoor damper as widely as possible, so that Q_r is minimised. Therefore, in this study, we set the outdoor air damper to be fully open after the economizer is activated, to maximise the use of free cooling energy.

We then use an affine neural network model to approximate the system dynamics [14], [15]. In an affine neural network model, the input-output relationship appears linearly as a state-space description. In such a way the neural model has a similar model structure to a linear state-space model. The building process can be modelled by an affine recurrent neural network (ARNN) in discretized form as

$$\begin{aligned} y(k+1) = f_n(y_k, .., y_{k-n_a}) + g_n(y_k, .., y_{k-n_b})u(k) \\ + h_n(y_k, .., y_{k-n_c})d(k), \end{aligned} \tag{9}$$

where y, u and d denote the output, controllable input and measurable disturbance, respectively. f_n, g_n and h_n are three neural networks with the orders of n_a, n_b and n_c, respectively. All the neural networks use multilayer perceptron (MLP) neural network function. The hidden layer uses tangent hyperbolic as the activation function. We trained the ANN model using a Levenberg-Marquard algorithm to minimise the mean squared errors (MSEs) between the predicted and actual values. After several trails of training, the final model was determined with the orders: n_a=3, n_b=2, and n_c=1. The simulation results of applying the ARNN for temperature prediction is illustrated in Fig. 5. One can observe that the simulation error (82% of fitness) is smaller than the RC model. By equalising (3) to (9), the following equation is built:

$$\begin{aligned} f_n(y_k, y_{k-1}, y_{k-2}) + g_n(y_k, y_{k-1}, y_{k-2})u_k \\ + h_n(y_k, y_{k-1})d_k = Ay_k + Bv_k + Ed_k, \end{aligned} \tag{10}$$

where u_k denotes the real control input (T_{sp}), d_k is a vector containing disturbance variables. After the control signal v is calculated by a standard MPC, the real control signal u can then be calculated by (10). Fig. 6 shows the result of applying this approach to calculate u (set point temperature) from v (supply cooling energy). It can be seen that the estimated u matches the recorded set point values with good accuracy.

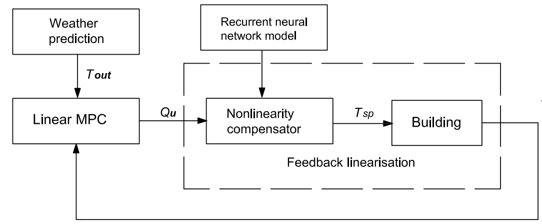


Fig. 4: Hybrid MPC control scheme based on recurrent neural network

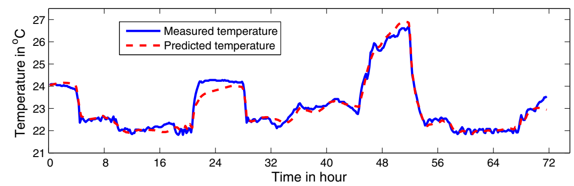


Fig. 5: Simulation results of using ANN model

III. Control Design

A. MPC design

The ASHARE standard 55 [18] defines comfortable temperature as a range of temperature values instead of a fixed value. Therefore, the cost function should allow indoor temperature to fluctuate within a specific range during the occupied hour. In this study, MPC is designed as a linear

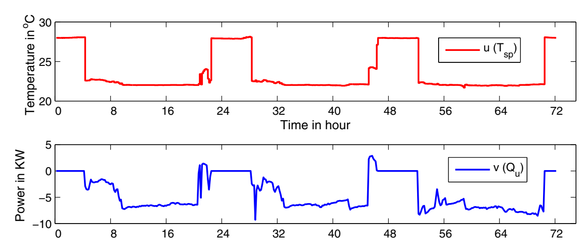


Fig. 6: Mapping from v to u using feedback linearisation

programming problem with time-varying constraints on thermal comfort and energy costs. The cost function used in this paper is employed from [7], [16]. The following optimisation problem is considered:

$$J(k) = a\sum_{k=0}^{N-1} p_e|(v_{k+j|k}) + P_{f(k+j|k)}| + b\sum_{k=1}^{N} |\hat{y}_{k+j|k} - r_{k+j|k}| + c\sum_{k=1}^{N} (|\underline{e}_{k+j|k}| + |\overline{e}_{k+j|k}|), \quad (11)$$

subject to:

$$\begin{aligned} x_{k+j+1|k} &= Ax_{k+j|k} + Bv_{k+j|k} + Ed_{k+j|k}, \quad \forall j = 0,...,N-1 \\ y_{k+j|k} &= Cx_{k+j|k}, \quad \forall j = 1,...,N \\ \underline{T}_{k+j|k} - \underline{e}_{k+j|k} &\leq y_{k+j|k} \leq \overline{T}_{k+j|k} + \overline{e}_{k+j|k}, \quad \forall j = 1,...,N \\ U_{k+j|k} &\leq v_{k+j|k} \leq 0, \quad \forall j = 0,...,N-1 \\ \underline{e}_{k+j|k} &> 0, \overline{e}_{k+j|k} > 0, \quad \forall j = 1,...,N \end{aligned} \quad (12)$$

where N is the prediction horizon, v is a vector of the control inputs within the prediction horizon, $\hat{y}_{k+j|k}$ is the predicted output at time k, which is obtained by iteratively solving (12) using the control input vector v, P_f is the energy consumed by the supply fan, and r is the reference temperature. $\underline{e}$ and $\overline{e}$ are the temperature violations from the lower and upper comfortable temperatures, respectively, and $\underline{T}$ and $\overline{T}$ are the lower and upper comfortable temperatures, respectively. U denotes the maximum cooling energy that the system can supply, which is a negative value as a cooling system is considered. p_e denotes the time-varying electricity price in dollars per kWh. The cost function (11) minimises a weighted sum of the energy costs, the deviations from the set point temperature, and the deviations from the comfortable bands. These terms are penalised by the weighting coefficients a, b, and c, respectively.

The constraints are:

1) $T_{oc} \in$ [21.5 ^{o}C, 24 ^{o}C] Thermal comfort during occupied hours.
2) $T_{uo} \in$ [19.5 ^{o}C, 26 ^{o}C] Thermal comfort during unoccupied hours.
3) $Q_u \in$[0, 12 kW] Maximum cooling energy that can be supplied to each zone.
4) $D_{out} \in$[20%,100%] Outdoor damper should meet the minimum amount of supply air requirement.

The constraints for the refined control inputs Q_u are calculated by (4) and (5) using the historical data. As the time step progresses, the time-varying constraints on thermal comfort shift forward. This guarantees a smooth transition from occupied hour to unoccupied hour, without violating temperature constraints. The linear programming optimisation problem (12) is solved using Yalmip [19], which generates an optimised input variable trajectory. The first control signal $u_{1|k}$ is applied to the building, and the rest are disposed. When a new time interval starts, the optimisation problem is repeated again with the updated initial condition x_{k+1} and shifted constraints.

TABLE I: Electricity rate of a T-1 building

	Time of the day	Energy charge($/kWh)
peak	7:00 am-9:00 pm	0.090
off-peak	All other	0.024

IV. Results and discussions

A. Simulation setup

Before incorporating the proposed control method in an on-line experiment, we first tested it under a simulation environment. The objective is to properly calibrate the parameters of MPC to reduce the possible errors of running the experiment. A commonly applied approach to conduct simulation is to replace the real building by a detailed physical model [6]. In this study, we use a feed-forward neural network model to achieve the same purpose. Since the output of the neural network is very similar to the real output, the simulation result will be close to the experiment. This enables a reliable tuning to be performed before the algorithm is applied to the real building.

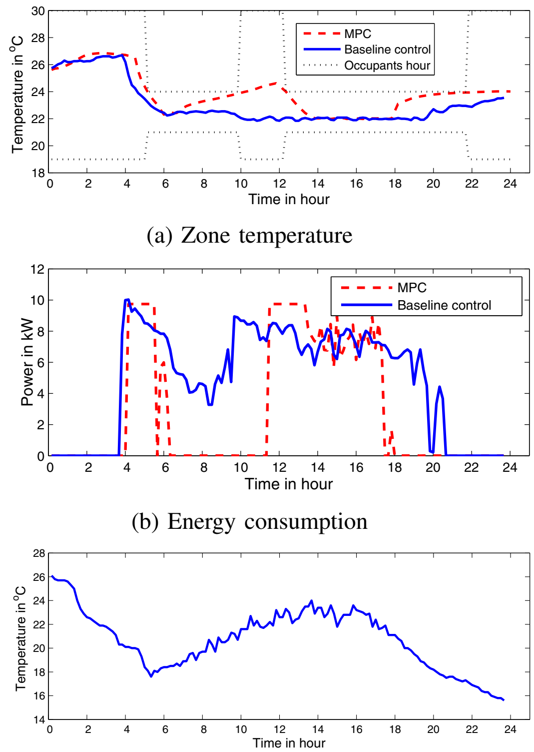
Beforehand, some key parameters were configured: The basic requirement of MPC is to maintain the indoor temperature between 21.5 ^{o}C to 24 ^{o}C during the occupancy hours, which is from 5:00 am to 9:30 pm. The building uses two electricity rates, which are shown in Table I. The weighting coefficients of the cost function were found empirically as: a=0.8, b=1, and c=1. The prediction horizon was set to be 6 hours (36 steps). We chose the historical profile from 24th to 25th January, 2013 for comparison purposes. In particular, we compare the following two types of MPC with the baseline control method:

1) Optimal start-stop MPC considers time-varying constraints and real-time flight schedules.
2) Pre-cooling MPC (PMPC) considers time-varying constraints and time of use (TOU) electricity price.

B. Simulation results

Fig. 7(a) shows the control results of applying the optimal start-stop MPC. It illustrates that, in the morning, MPC turns on the AHUs later than the baseline control so that the temperature reaches the upper comfortable temperature at the start of occupancy. Similarly, before the end of the occupancy, MPC turns off the AHUs earlier, but the temperature does not exceed the upper temperature band. Whenever the building is unoccupied during the daytime, the cooling supply of the AHUs will also be turned off to conserve energy. This explains why the AHU stops supplying cooling energy between 6:00 am and 12:00 pm. It is shown that this simple optimal start-stop MPC achieves up to 41% of energy savings compared to the baseline control method.

The second simulation considers PMPC. Fig. 8(e) shows that the morning ambient temperature was low on the investigated day. As a consequence, the baseline control waited passively until the zone temperature started to increase to



(a) Zone temperature

(b) Energy consumption

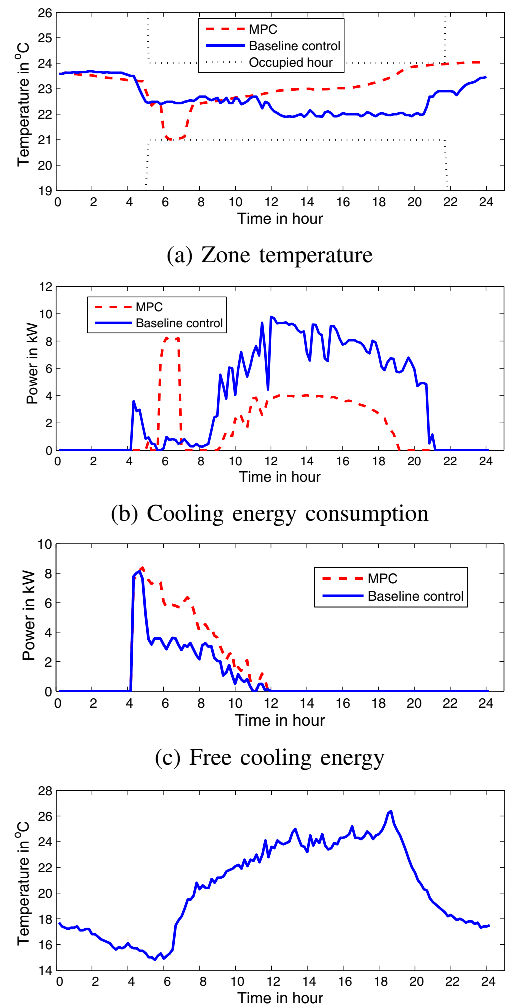
(c) Outdoor temperature

Fig. 7: Comparison results between baseline control and optimal start-stop MPC.

provide cooling energy. In opposition, PMPC pre-cools the temperature to 21 ^{o}C after the AHU was turned on, as shown in Fig. 8(a). Obviously, the PMPC takes advantage of cheap off-peak electricity prices to precool the building's thermal mass and to store cooling energy. Fig. 8(b) shows that the pre-cooling process ended before the peak hour started (7:00 am). After 7:00 am, the AHUs were turned off and the stored cooling energy started to release, so that very little cooling energy was needed to compensate for the increasing heat gains during the daytime. Fig. 8(c) shows that the process of pre-cooling does not require too much cooling energy, because more free cooling energy has been employed by the MPC.

C. Experiment results

After the simulation, a filed experiment was conducted to test the functionality of PMPC. The optimal start-stop MPC was not tested experimentally because its benefits can be directly observed from the simulation study. The real-time experiment was executed over a period of 4 days, from 23th January to 27th January, 2014. During the experiment, the data used for training were downloaded from the BMS a day ahead of the experiment. After the data were obtained, the MPC algorithm was executed to calculate the optimal set point temperature trajectory, based on the weather forecast data. The new set point was then scheduled to the BMS by the building manager. To better compare the performance of MPC with the baseline control method, we chose 10th



(a) Zone temperature

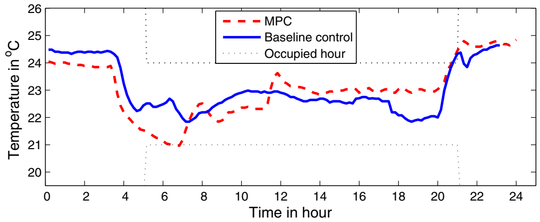
(b) Cooling energy consumption

(c) Free cooling energy

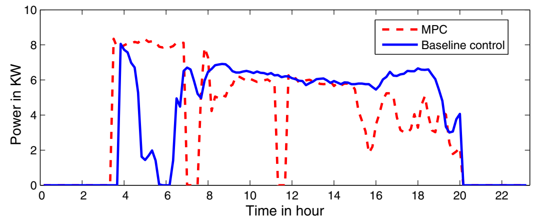
(d) Outdoor air temperature

Fig. 8: Comparison results between baseline control and PMPC

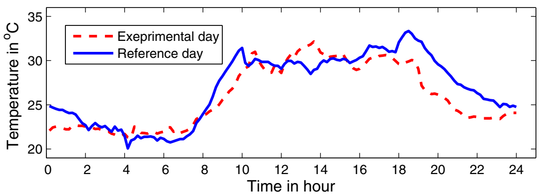
January, 2014 as the reference day. On this day, the outdoor temperature, initial zone temperature, and occupancy level were all similar to 23th January, 2014; therefore, we can make a fair comparison between the two control methods. We only compare the energy use between 3:00 am and 6:00 pm, considering there is a big difference in outdoor temperature starting from 6:00 pm (see Fig. 9(c)). Fig. 9(a) compares the zone temperature trajectory between the baseline control and MPC. Similar to the simulation, MPC decreased the temperature to the lower band of 21 °C before the start of the peak hour. Fig. 9 (b) depicts the cooling energy consumed by the AHUs. It can be seen that MPC has partially shifted the cooling load from on-peak hours to off-peak hours, at the cost of consuming more off-peak energy. As the ambient temperature was not too low (22 °C), the benefit from applying free cooling was relatively smaller than simulation. However, the cheap off-peak electricity price allows the pre-cooling process to be performed at a very low cost. By calculation, it is estimated that 13% of cost savings were



(a) Zone temperature



(b) Power consumption



(c) Outdoor air temperature

Fig. 9: Control performance between two homogeneous days

TABLE II: Comparison of performance between baseline control and PMPC from 3:00 am to 6:00 pm

Controller	Total energy input (kWh)	Utility costs ($)
Baseline control(10th Jan.)	423	12.3
MPC(23rd Jan.)	447	10.7

achieved as compared to the baseline control when MPC was used during the comparison period (see Table II).

V. CONCLUSIONS

This study proposes an neural network feedback linearisation-based MPC to achieve energy-saving for a commercial building. An affine neural network model is feedback-linearised through a state feedback, which converts a nonlinear control problem into a linear control problem. Two types of MPC schemes are investigated. The simulation result shows that optimal start-stop MPC is an effective method to save energy for the investigated building. This method is especially useful for a building with several occupancy sections within a day. On the other hand, when the TOU electricity price is considered, the utility costs can be reduced by using the PMPC. The savings mainly come from the use of cheap off-peak electricity price and free cooling energy. The experiment also shows that, due to the lightweight property of the building, the benefits of applying a pre-cooling strategy seem less significant, as compared to other similar studies. Future work will investigate the robustness of the control method to the modelling errors.

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